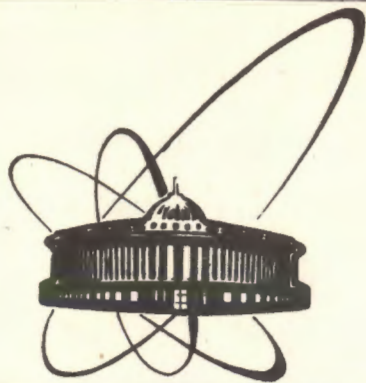


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THE $(\text{QED})_4$ HAMILTONIAN
UNBOUNDED FROM BELOW

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SEC.1 INTRODUCTION

The (QED)₄ Hamiltonian is

$$H = H_{0e} + H_{0ph} + H^{Tr} + H_Q + T + H'' . \quad (1.1)$$

Here H_{0e} is the Hamiltonian of a fermion field

$$H_{0e} = \sum E_p (a_{p\sigma}^* a_{p\sigma} + b_{p\sigma}^* b_{p\sigma}) , \quad (1.2)$$

$E_p = (p^2 + m^2)^{1/2}$, where m^2 does depend on the cut-off parameter L , a^* , b^* , a (and b) are the creation and annihilation electron (positron) operators, p is the momentum, and σ the spin projection. H_{0ph} is the free photon Hamiltonian

$$H_{0ph} = \sum_{k \neq 0, \lambda} |k| C_{k\lambda}^* C_{k\lambda} . \quad (1.3)$$

Here $C_{k\lambda}^*$ and $C_{k\lambda}$ are the creation and annihilation operators of a photon with the momentum k and polarization vector $\vec{e}_\lambda(k)$, $k \vec{e}_\lambda(k) = 0$, $\lambda = 1, 2$. The H^{Tr} term describes the interaction of fermions with transverse photons /1, 2/

$$H^{Tr} = e \int_V \psi^*(x) \vec{a} \psi(x) \vec{B}^{Tr}(x) d^3x , \quad (1.4)$$

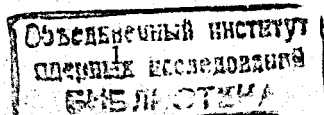
$$\psi(x) = \frac{1}{\sqrt{V}} \sum_{p, \sigma} (u_{p, \sigma}^+ a_{p, \sigma} + u_{p, \sigma}^- b_{-p, -\sigma}^*) e^{ipx} , \quad |p| < L , \quad (1.5)$$

$$\vec{B}^{Tr}(x) = \frac{1}{\sqrt{V}} \sum_{k \neq 0, \lambda} (C_{k\lambda} \vec{e}_\lambda(k) \frac{1}{\sqrt{2|k|}} e^{ikx} + \text{c.c.}) , \quad |k| < L , \quad (1.6)$$

here $u_{p\sigma}^\pm$ are the positive- and negative-energy solutions to the Dirac equation. H_Q is the Coulomb term,

$$H_Q = \frac{e^2}{2V} \sum_{k \neq 0} \rho_k \rho_{-k} / k^2 , \quad (1.7)$$

$$\rho_k = \int_V \psi^*(x) \psi(x) e^{ikx} d^3x ,$$



T and H'' are the terms associated with a zero mode of a Boson field ^{3/}

$$T = -\frac{(\partial \vec{s})^2}{2}, \quad \vec{s} = \int_V \vec{B}(x) d^3x / \sqrt{V}, \quad (1.8)$$

$$H'' = -\frac{e\vec{s}}{\sqrt{V}} \int_V \psi^*(x) \vec{a} \psi(x) d^3x = -\frac{e\vec{s}}{\sqrt{V}} [(11) + (20) + (02)], \quad (1.9)$$

$$(11) = \sum_{p\sigma} (a_{p\sigma}^* a_{p\sigma} + b_{p\sigma}^* b_{p\sigma}) \vec{p} / E_p, \quad (1.10)$$

$$(20) = \sum_{p, \alpha, \beta} a_{p, \alpha}^* b_{-p, -\beta}^* \left[\vec{\sigma} - \vec{p} \frac{(\vec{p}\vec{\sigma})}{E_p(E_p + m)} \right]_{\alpha\beta}, \quad (1.11)$$

$$(02) = (20)^*. \quad (1.12)$$

1. We shall consider the expectation value \bar{H} of the Hamiltonian H;

$$\bar{H} = \int \Omega^*(s) H \Omega(s) d^3s / N, \quad (1.13)$$

$$N = \int \Omega^*(s) \Omega(s) d^3s, \quad (1.14)$$

where $\Omega(s)$ is essentially the BCS probe function ^{4/}

$$\Omega(s) = \psi(s) \prod_p (1 + A_p |a_{p, \alpha}^* b_{-p, -\beta}^* f_{\alpha}(s) f_{\beta}^*(s)|) |0\rangle, \quad A_p^* = A_p, \quad (1.15)$$

$$\psi(s) = \exp[-(|\vec{s}| - n)^2], \quad (1.16)$$

$$A_p = (m/E_p)^4, \quad (1.17)$$

function $f(s)$ satisfy equations

$$(\vec{\sigma} \vec{s}) f(s) = |\vec{s}| \nu f(s), \quad f^*(s) f(s) = 1, \quad (1.18)$$

$$\nu = \text{sign } e. \quad (1.19)$$

The state $|0\rangle$ in eq.(1.15) is the state of the fermion and boson vacuum, so that

$$C_{k\lambda} |0\rangle = a_{p\sigma} |0\rangle = b_{p\sigma} |0\rangle = 0. \quad (1.20)$$

SEC. 2. EXPECTATION VALUE OF THE HAMILTONIAN

For the calculation of the quantity \bar{H} useful are some formulas of the Bayman's work ^{5/}. Equations (1.14), (1.15) and (1.18) give

$$N = Z \int |\psi|^2 d^3s, \quad Z = \prod_p (1 + A_p^2). \quad (2.1)$$

Analogously one gets

$$\bar{H}_{0e} = 2 \sum \frac{E_p A_p^2}{1 + A_p^2}, \quad (2.2)$$

$$\bar{H}_{0ph} = 0, \quad (2.3)$$

$$\bar{H}^{Tr} = 0, \quad (2.4)$$

$$\bar{H}'' = -\frac{e\nu}{\sqrt{V}} h'' K, \quad h'' = \sum \frac{2A_p}{1 + A_p^2} \left(1 - \frac{1}{3} \frac{p^2}{E_p(E_p + m)}\right), \quad (2.5)$$

$$K = \int |\psi(s)|^2 d^3s |s| / \int |\psi(s)|^2 d^3s. \quad (2.6)$$

Equation (1.16) implies the quantity K to tend to $+\infty$ as $n \rightarrow +\infty$. Equations (1.19) and (2.5) give $e\nu = |e|$, $h'' > 0$, h'' does not depend on s . Thus the expectation value (2.5) of the operator H'' tends to $-\infty$ as $n \rightarrow +\infty$ (if the quantities V and L as well as quantities A_p are fixed). Meanwhile, the expectation values (2.2)-(2.4) of the operators H_{0e} , H_{0ph} and H^{Tr} do not depend on n .

2. It is easy to see that expectation values of operators H_Q and T have finite limits as $n \rightarrow +\infty$ (these limits depend on V and L). Then it follows that the quantity \bar{H} (1.13) and the total operator (1.1) are not bounded from below. The only reason of this phenomenon is that the Hamiltonian contains the zero mode term H'' .

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