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THE $(QED)_4$ HAMILTONIAN UNBOUNDED FROM BELOW

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SEC.1 INTRODUCTION

The $(QED)_4$ Hamiltonian is

$$H = H_{0e} + H_{0ph} + H^{Tr} + H_Q + T + H''. \qquad (1.1)$$

Here H_{0e} is the Hamiltonian of a fermion field

$$H_{0e} = \Sigma\, E_p (a^*_{p\sigma} a_{p\sigma} + b^*_{p\sigma} b_{p\sigma}), \qquad (1.2)$$

$E_p = (p^2 + m^2)^{1/2}$, where m^2 does depend on the cut-off parameter L, a^*, b^*, a (and b) are the creation and annihilation electron (positron) operators, p is the momentum, and σ the spin projection. H_{0ph} is the free photon Hamiltonian

$$H_{0ph} = \sum_{k \neq 0, \lambda} |k| C^*_{k\lambda} C_{k\lambda}. \qquad (1.3)$$

Here $C^*_{k\lambda}$ and $C_{k\lambda}$ are the creation and annihilation operators of a photon with the momentum k and polarization vector $\vec{e}_\lambda(k)$, $\vec{k}\vec{e}_\lambda(k) = 0$, $\lambda = 1,2$. The H^{Tr} term describes the interaction of fermions with transverse photons [1,2]

$$H^{Tr} = e \int_V \psi^*(x)\, \vec{\alpha}\psi(x)\, \vec{B}^{Tr}(x)\, d^3x, \qquad (1.4)$$

$$\psi(x) = \frac{1}{\sqrt{V}} \sum_{p,\sigma} (u^+_{p,\sigma} a_{p,\sigma} + u^-_{p,\sigma} b^*_{-p,-\sigma}) e^{ipx}, \quad |p| < L, \qquad (1.5)$$

$$\vec{B}^{Tr}(x) = \frac{1}{\sqrt{V}} \sum_{k \neq 0, \lambda} (C_{k\lambda} \vec{e}_\lambda(k) \frac{1}{\sqrt{2|k|}} e^{ikx} + c.c.), \quad |k| < L, \qquad (1.6)$$

here $u^\pm_{p\sigma}$ are the positive- and negative-energy solutions to the Dirac equation. H_Q is the Coulomb term,

$$H_Q = \frac{e^2}{2V} \sum_{k \neq 0} \rho_k \rho_{-k} / k^2, \qquad (1.7)$$

$$\rho_k = \int_V \psi^*(x)\psi(x) e^{ikx} d^3x,$$

T and H″ are the terms associated with a zero mode of a Boson field [3]

$$T = -(\frac{\partial}{\partial \vec{s}})^2/2, \quad \vec{s} = \int_V \vec{B}(x)\, d^3x / \sqrt{V}, \tag{1.8}$$

$$H'' = -\frac{e\vec{s}}{\sqrt{V}} \int_V \psi^*(x)\, \vec{\alpha}\, \psi(x)\, d^3x = -\frac{e\vec{s}}{\sqrt{V}}[\,(11) + (20) + (02)\,], \tag{1.9}$$

$$(11) = \sum_{p\sigma} (a^*_{p\sigma} a_{p\sigma} + b^*_{p\sigma} b_{p\sigma})\, \vec{p}/E_p, \tag{1.10}$$

$$(20) = \sum_{p,\alpha,\beta} a^*_{p,\alpha} b^*_{-p,-\beta} [\,\vec{\sigma} - \vec{p}\,\frac{(\vec{p}\vec{\sigma})}{E_p(E_p+m)}\,]_{\alpha\beta}, \tag{1.11}$$

$$(02) = (20)^*. \tag{1.12}$$

1.We shall consider the expectation value $\bar{H}$ of the Hamiltonian H;

$$\bar{H} \equiv \int \Omega^*(s)\, H\, \Omega(s)\, d^3s / N, \tag{1.13}$$

$$N \equiv \int \Omega^*(s)\, \Omega(s)\, d^3s, \tag{1.14}$$

where $\Omega(s)$ is essentially the BCS probe function [4]

$$\Omega(s) = \psi(s) \prod_p (1 + A_{|p|}\, a^*_{p,\alpha}\, b^*_{-p,-\beta} f_\alpha(s)\, f^*_\beta(s))\,|\,0>, \quad A^*_p = A_p, \tag{1.15}$$

$$\psi(s) = \exp[-(|\vec{s}| - n)^2], \tag{1.16}$$

$$A_p = (m/E_p)^4, \tag{1.17}$$

function $f(s)$ satisfy equations

$$(\vec{\sigma}\vec{s})\, f(s) = |\vec{s}|\, \nu\, f(s), \quad f^*(s)\, f(s) = 1, \tag{1.18}$$

$$\nu = \text{sign}\, e. \tag{1.19}$$

The state $|0>$ in eq.(1.15) is the state of the fermion and boson vacuum, so that

$$C_{k\lambda}|\,0> = a_{p\sigma}|\,0> = b_{p\sigma}|\,0> = 0. \tag{1.20}$$

SEC. 2. EXPECTATION VALUE OF THE HAMILTONIAN

For the calculation of the quantity $\bar{H}$ useful are some formulas of the Bayman's work [5]. Equations (1.14), (1.15) and (1.18) give

$$N = Z \int |\psi|^2 d^3s, \quad Z = \prod_p (1 + A_p^2). \tag{2.1}$$

Analogously one gets

$$\bar{H}_{0e} = 2 \sum \frac{E_p A_p^2}{1 + A_p^2}, \tag{2.2}$$

$$\bar{H}_{0ph} = 0, \tag{2.3}$$

$$\bar{H}^{Tr} = 0, \tag{2.4}$$

$$\bar{H}'' = -\frac{e\nu}{\sqrt{V}} h'' K, \quad h'' = \sum \frac{2A_p}{1 + A_p^2}(1 - \frac{1}{3}\frac{p^2}{E_p(E_p+m)}), \tag{2.5}$$

$$K = \int |\psi(s)|^2 d^3s\, |s| \,/ \int |\psi(s)|^2 d^3s. \tag{2.6}$$

Equation (1.16) implies the quantity K to tend to $+\infty$ as $n \to +\infty$. Equations (1.19) and (2.5) give $e\nu = |e|$, $h'' > 0$, h'' does not depend on s. Thus the expectation value (2.5) of the operator H″ tends to $-\infty$ as $n \to +\infty$ (if the quantities V and L as well as quantities A_p are fixed). Meanwhile, the expectation values (2.2)-(2.4) of the operators H_{0e}, H_{0ph} and H^{Tr} do not depend on n.

2. It is easy to see that expectation values of epartors H_Q and T have finite limits as $n \to +\infty$ (these limits depend on V and L). Then it follows that the quantity $\bar{H}$ (1.13) and the total operator (1.1) are not bounded from below. The only reason of this phenomenon is that the Hamiltonian contains the zero mode term H″.

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REFERENCES

1. Heitler W. - Quantum Theory of Radiation,Oxford, Clarendon Press, 1955.
2. Wentzel G. - Quantum Theory of Fields,Interscience Publ. Inc.,New York, 1949.
3. Zastavenko L.G. - JINR Preprint E2-90-280, Dubna, 1990.
4. Bardeen L., Cooper L.N., Schrieffer J.R. - Phys.Rev., 1957, 108, p.1175.
5. Baynman B.F. - Nucl.Phys., 1960, 15, p.33.

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