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A POTENTIAL QUARK MODEL WITH INSTANTON INTERACTION

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1 Introduction

As a rule, to describe hadron ground states within quark models one supposes that quarks in hadrons are bind by the rising scalar potential, and spin splitting of multiplets is explained by one gluon exchange. It implyes a rather large magnitude for hyperfine splitting constant α_s, and this fact makes exploration of perturbation theory methods daubtful. Moreover, general picture of observed exited hadron states does not correspond theoretical predictions of quark models in details.

Some years ago it was shown [1,2] that quark interactions via nonperturbative vacuum indeed is more essential for the hadron spectroscopy. In this paper we study the influence of interaction generated by vacuum fluctuation exchange on the excited baryon mass spectrum. To analyse this problem we use a potential quark model (Sec.2.) and calculate operator averages in the basis of harmonic oscillator wave functions . This approach is very fruitful for describing of ground and low-lying excited baryon systems with any scalar potential of confinement and allow to use nonrelativistic SU(6) classification in a natural way.[3,4] Follow this way we introduct three standart potential model parameters for unperturbed masses of ground and low-lying excited nonstrange multiplets (Sec.3.) . Further in Sec.4. we consider fine splitting structure of levels in spectrum. As a fine structure potential we use nonrelativistic expression for effective Lagrangian of one-instanton exchange and neglect any relativistic corrections. In Sec.4. we calculate also ground baryon octet masses. In this way we determine characteristic constant P of the one instanton exchange contributions and model standard constant E_0-energy of ground nonperturbed level N and Δ.To calculate strange members masses we use parameters: $x = m_u/m_s$, $\delta m = m_s$-m_u . In Sec.5. we calculate masses of excited states N and Δ and find others model constants from the mass fit.

2 Nonrelativistic quark model

As an effective Hamiltonian for baryons we use:

$$H^{eff} = \sum_i (m_i + \vec{p}_i^{\,2}) + \frac{K}{2}\sum_{i<j}(\vec{r}_{ij}^{\,2}) + U^1 + H^{inst}, \tag{1}$$

where $\vec{r}_{ij}$-is the separation between quarks, U^1-is the anharmonic part of the confining potential. Here we neglect the intrinsic mass difference between u and d quarks, so we are limited to the case

$$m_1 = m_2 = m, m_3 = m'. \tag{2}$$

If we define

$$\begin{aligned} &\vec{R} = m(\vec{r}_1 + \vec{r}_2) + m'\vec{r}_3/M, && M = 2m + m'; \\ &\vec{\rho} = (\vec{r}_1 - \vec{r}_2)/\sqrt{2}, && m_\rho = m; \\ &\vec{\lambda} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}, && m_\lambda = 3mm'/(2m + m'). \end{aligned} \tag{3}$$

then we obtain for two summs in (1) the following expressions

$$H_{HO} = m_1 + m_2 + m_3 + \frac{2}{3}K(\vec{\rho}^2 + \vec{\lambda}^2) + \vec{P}_R^2/2M + \vec{P}_\rho^2/2m_\rho + \vec{P}_\lambda^2/2m_\lambda. \quad (4)$$

Separation of the c.m. motions connected with the variable $\vec{R}$ leads to two decoupled oscillator equations in the variables ρ and λ. If the quark masses are equal $m_1 = m_2 = m_3 = m_d$ the H^{HO} in (4) transforms into

$$H_0^{HO} = -(\Delta_\rho + \Delta_\lambda)/2m_d + \frac{2}{3}K(\rho^2 + \lambda^2) + constant. \quad (5)$$

The classification of baryonic states in the quark model with Hamiltonian H_0^{HO} is well known (see [3,4] and references there in). Because the colour part is antisymmetric we look for solutions which are totally symmetric under permutation of SU(6) indices and spatial coordinates. For ground states the O(3) part is totally symmetric:

$$\Psi^{00} = \frac{\alpha^3}{\pi^{3/2}} exp(\alpha^2(\rho^2 + \lambda^2)/2); \qquad \alpha^2 = \omega m = \sqrt{Km}. \quad (6)$$

Denote H^{HO} in (4) for a system with one, two or three strange quarks by $H_1^{HO}, H_2^{HO}, H_3^{HO}$ respectively, so that

$$H_s^{HO} = H_0^{HO} + H_s^{corr}, \quad (7)$$

where H_s^{corr} vialates $SU(3)_{flavour}$ symmetry. Hence we have to correct unperturbed masses for Λ , Σ , Ξ and Ω in ground states:

$$E_s^0 = E^0 + < \Psi^{00} \mid H_s^{corr} \mid \Psi^{00} >. \quad (8)$$

A straightforward calculations lead to

$$\begin{aligned} E_1^0 &= E^0 + \delta_s \qquad for\ \Lambda,\ \Sigma; \\ E_2^0 &= E^0 + 2\delta_s \qquad for\ \Xi, \\ E_3^0 &= E^0 + 3\delta_s \qquad for\ \Omega, \\ where\ \delta_s &= \delta m + \omega\,(1-x)/2\,. \end{aligned} \quad (9)$$

3 Anharmonic part of the confining potential

Before we turn to the hyperfine interaction, we take into account that confining forces are really not harmonic. A detailed analysis on this problem was given in [3,4] we recall two rules from them:

- In first order perturbation theory any potential $U^1(r_{ij})$ splitts the N=2 harmonic oscillator energies in exactly the same manner:five degenerate multiplets $56^+(L = 0;2)$, $70^+(L=0;2)$, $20^+(L=1)$) are always ordered as:

$$\begin{aligned} E^2(56,0^+) &= E' - \Delta, \\ E^2(70,0^+) &= E' - \Delta/2, \\ E^2(56,2^+) &= E' - 2\Delta/5, \\ E^2(70,2^+) &= E' - \Delta/5, \\ E^2(0,1^+) &= E'. \end{aligned} \quad (10)$$

- And for $E^2(20,1^+)$, $E^1(70,1^-)$, $E^0(56,0^+)$ we have

$$E^2(20,1^+) - E^1(70,1^-) = E^1(70,1^-) - E^0(56,0^+). \quad (11)$$

4 Hyperfine interaction

In papers [1,2] it was shown that main features of spectrum of hadron ground states and π–ρ ; N–Δ ; η–η' splitting, quark–Dquark structure are explained by quark nonperturbative interaction via instanton. This interaction is nonvanish when the pair of quarks is in isoscalar state. In nonrelativistic limit, after averaging over collective coordinates of instanton effective Lagrangian in [1,2] transforms as follows:

$$H^{inst} = -(\eta/m_i m_j)\delta^3(\vec{\rho}_i - \vec{\rho}_j)((1 - \vec{\sigma}_i\vec{\sigma}_j)/4)((4/3 - \tau_i^a\tau_{aj})/4), \quad (12)$$

where σ^a and τ^a are the spin and flavour operators, normalised as $Sp(\sigma^a\sigma^a)=Sp(\tau^a\tau^a)=2$; η does not depend on individual quark properties. It should be noticed that (12) contains two projectors on spin and flavour states which are antisymmetric under permutations of i^{-th} and j^{-th} quark indices. Two kinds of symmetry correspond to the totally symmetric SU(6) part of ground-states baryon wave functions. They are $^4 10$ and $^2 8$. In the first case the spin and flavour parts are symmetric and masses of $^4 10$ states are nonperturbed by H^{inst} .In the second case the masses are lowered by instanton contributions. Formula (9) gives a good approximation for $^4 10$ baryon masses spectrum .(see [5] and Table 1). In our model mass difference between $\Lambda\frac{1}{2}^+$ and $\Sigma\frac{1}{2}^+$ is explained in a natural way because the H^{inst} contribution to $\Lambda\frac{1}{2}^+$ is greater then in $\Sigma\frac{1}{2}^+$ and has a negative sign.

$$m(\Sigma\tfrac{1}{2}^+) - m(\Lambda\tfrac{1}{2}^+) = P(1-x)/3 > 0$$

P–scale constant of instanton contributions:

$$P = 3\eta\frac{\alpha^3}{m_d^2\pi^{3/2}}.$$

We assume in Table 1.:

$$E^0 = 1230Mev, P = 580Mev, \delta = 150Mev, x = 0,7 \quad (13)$$

to calculate ground baryon octet masses, and get a good agreement with experimental data.

Table 1

Hadron	Strange	Rep. of SU(6)	E_s^0(Mev)	δE^{inst}(P)	M_{calc}
$N(939)1/2^+$	0	$^2 8$	1230	-1/2	940
$\Delta(1232)3/2^+$	0	$^4 10$	1230	0	1230
$\Lambda(1116)1/2^+$	-1	$^2 8$	1380	(2+x)/6	1119
$\Sigma(1193)1/2^+$	-1	$^2 8$	1380	x/2	1177
$\Sigma(1385)3/2^+$	-1	$^4 10$	1380	0	1380
$\Xi(1318)1/2^+$	-2	$^2 8$	1530	x/2	1327
$\Xi(1530)3/2^+$	-2	$^4 10$	1530	0	1530
$\Omega(1672)3/2^+$	-3	$^4 10$	1680	0	1680

5 Excited N and Δ states

Low-lying P-wave baryons are associated in this model with a 70-plet of the first excited level of system (5). Its nonstrange members are:

$$^2N(70,1^-)J^P = \frac{1}{2}^-, \frac{3}{2}^-, \tag{14}$$

$$^4N(70,1^-)J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \tag{15}$$

$$^2\Delta(70,1^-)J^P = \frac{1}{2}^-, \frac{3}{2}^-. \tag{16}$$

For (15,16) instanton contributions are vanish and these triplet and doublet are degenerated. For (14) we obtain

$$<^2 NJ^- \mid H^{inst} \mid^2 NJ^- >= -P/4 = -145mev, \tag{17}$$

so we are able to explain the main feature of the experimental situation: splitting between a doublet of $N\frac{1}{2}^+(1535)$, $N\frac{3}{2}^+(1520)$ and a set of N and Δ whose masses are about 1660Mev. So we put

$$E^1(70,1^-) = 1665Mev. \tag{18}$$

As mentioned above, the second excited level of oscillator model (5) contains five SU(6) multiplets unperturbed by H^{inst} and their masses depend on two parameters $E^2(20,1^+)$ and Δ'. From (11,13,18) we conclude that $E^2(20,1^+) \simeq 2000$ MeV.

Table 2

State	Instanton	contributions (P)
$^2N(56,2)5/2^+$	-1/4	$\sqrt{2}/8$
$^2N(70,2)5/2^+$	$\sqrt{2}/8$	-1/8
$^2N(56,2)3/2^+$	-1/4	$\sqrt{2}/8$
$^2N(70,2)3/2^+$	$\sqrt{2}/8$	-1/8
$^2N(56,0)1/2^+$	-5/8	$\sqrt{2}/8$
$^2N(70,0)1/2^+$	$\sqrt{2}/8$	-5/16

When Δ'=200Mev the calculated masses are arranged most close to experimental values.

6 Conclusion

In the last sections we obtain satisfactory results for ground baryon octet masses and for the masses of excited N and Δ with negative parity and for general picture for masses of positive parity baryons. Moreover in second excited level some calculated masses are very close to the masses of the observed states, for example, to $N\frac{1}{2}^+(1440)$,$N\frac{5}{2}^+(1680)$, $N\frac{1}{2}^+(1710)$,$N\frac{3}{2}^+(1710)$ which are well established. In Table 3 we distinguish the states of 20-plet and states with a high orbital momentum: the first are out of baryon-meson scattering product and masses of the last may be lowered by relativistic corrections to kinetic energy operator. Besides standard parameters of a potential model we use only the constant for instanton contributions. To our mind the obtained results confirm the hypothesis about dominant role of instanton interaction in the effects of hadron multiplet masses splitting.

Table 3

State	Calculation	Exp.mass^c	Status^c
$N5/2^-$	1665	1660-1690	****
$N3/2^-$	1665	1670-1730	***
	1520	1510-1530	****
$\Delta 3/2^-$	1665	1630-1740	***
$N1/2^-$	1665	1620-1680	****
	1520	1520-1560	****
$\Delta 1/2^-$	1665	1660-1650	****
$N7/2^+$	1960^b	1950-2050	**
$\Delta 7/2+$	1920^b	1910-1960	****
$N5/2^+$	1960^b	1880-2175	**
	1945^b	......	
	1715^b	1670-1690	****
$\Delta 5/2^+$	1960^b	2000-2200	**
	1920^b	1890-1920	****
$N3/2^+$	2000^a		
	1960^b		
	1945^b		
	1900		
	1715^b	1690-1800	****
$\Delta 3/2^+$	1960^b	1860-2160	****
	1920^b	......	
	1800	1600-1700	**
$N1/2^+$	2000^a		
	1960^b		
	1750	1680-1740	***
	1410	1400-1480	****
$\Delta 1/2^+$	1920^a	1850-1950	****
	1900	......	

- a—members of nonobserved 20-plet
- b—states with a high orbital angular momentum
- c—according to Particle Data Group [5]
- ...... — the same experimental value.

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