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ON THE PARADOXES OF LEWIS-TOLMAN AND EINSTEIN

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The first problem [1] which we are going to speak about was formulated more than 90 years ago. However, dissatisfaction of its initial solution led to a lively discussion at the beginning of the sixties*. The articles on this subject having been published until very recently (see, e.g. [3,4]) give evidence for that there is no full clearness yet here. In addition, we are going to touch upon the Einstein's little known remark [5] concerning solid body dynamics which is related to the indicated problem. The distinguishing feature of the approach presented below is its primarily relativistic-covariant character.

The essence of the known problem of right-angled lever equilibrium lies in the appearance of a torsion torque ($N_z \neq 0$) in a reference system (S) where a lever is moving (along the x axis) whilst in its proper system S*

$$N_z^* = \ell_x^* F_y^* - \ell_y^* F_x^* = 0. \tag{1}$$

Here ℓ_x^* and ℓ_y^* are the lever arms directed along the x and y axes, respectively; F_y^* and F_x^* forces applied to them. In so doing, for simplicity the vertex of the lever is at the origin: $\ell_x^* = \ell_y^* = \ell$ and $F_x^* = F_y^* = F^*$.

Taking into account the Lorentz contraction formula

$$\ell_x = \ell_x^* \sqrt{1-\beta^2} = \ell_x^* \gamma^{-1} \tag{2a}$$

and

$$\ell_y = \ell_y^* \tag{2b}$$

and also the transformation formulae of relativistic force components

$$F_x = F_x^* \gamma \text{ and } F_y = F_y^* , \tag{3}$$

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* Arzelies' article [2] can be particularly singled out among these papers where, to all appearances, a doubt about the validity of the Lorentz contraction formula has been first expressed.

we actually find

$$N_z = \ell_x F_y - \ell_y F_x = \ell^* \gamma^{-1} F^* - \ell^* F^* \gamma = -\beta^2 \ell^* F^* \gamma . \qquad (4)$$

Within the frame of 4-dimensional formulation the torque is described by an antisymmetrical 4-tensor of the second rank N_{ik} ("4-torque")

$$N_{ik} = -\Sigma \, \epsilon_{ik\ell n} x^\ell F^n , \qquad (5)$$

where i,k,... = 0,1,2,3; $\epsilon_{ik\ell n}$ is the Levi-Civita pseudotensor ($\epsilon^{0123} = 1$), c = 1 and, in particular, $x^1 = \ell_x$, $F^1 = F_x$ and so no. From the purely mathematical point of view, there is no difficulty here. Indeed, taking into account the transformation formulae, for components N_{ik} we have

$$N_z^* = N_{03}^* = (N_{03} + \beta N_{13}) \gamma = 0 \qquad (6)$$

or

$$N_{13} = -\beta^{-1} N_{03} .$$

From here it follows

$$N_{13}^* = (N_{13} + \beta N_{03}) \gamma = (-\beta^{-1} + \beta) N_{03} \gamma = \beta \ell^* F^* . \qquad (6')$$

In other words, one can say that the appearance of the driving torque N_{03} in the S-system is due to that the component N_{13}^* different from zero exists in the S*-system.

However, such a statement when the torque tensor is different from zero in the system at equilibrium cannot be admitted to be satisfactory. In order to clear up the situation, write out the expression for N_{13}^*. According to (5), it takes the form

$$N_{13}^* = \Sigma \, (x_*^2 F_*^\circ - x_*^\circ F_*^2) . \qquad (5')$$

As the lever is at rest in the S*-system, the force F_x^* does not execute any work, its power and hence the first term of (5') are equal to zero. Thus, the condition $N_{13}^* \neq 0$ can be fulfilled if only the point of application of the force F_y^* and the lever vertex (the origin) "are taken" at different instants of time. This statement is physically meaningless particularly if it is remembered that, for example, the forces pulsate. So, with necessity we should call for going N_{ik} to zero, i.e., in particular, the validity of the equality

$$N_{03} = 0 = x^1 F^2 - x^2 F^1 \qquad (7)$$

from whence we find

$$\ell_x = \ell_x^* \gamma \qquad (8)$$

taking (2b) and (3) into account for the transformation formula of longitudinal arm. This formula corresponds to the definition of relativistic length [6]* based on a direct use of clocks and light signals. The use of (8) allows one to abviate some other difficulties, namely: presence of the electromagnetic field momentum of an electron at rest, appearance of charge in a moving conductor with current and so on. Using (6) and (6') based on (7), we find $N_{13} = 0$ and $N_{13}^* = 0$. Thus, as the logic of events requires, we actually have

$$N_{ik} = 0 \qquad (9)$$

independent of reference system. In this case it is evident that the work of force F^1 in the S-system (its power $F^\circ = u^1 F^1 / u^\circ$, where u^i is 4-velocity) above the lever (to be more precise, the product of F° by the arm of x^2) is completely compensated by the product of F^2 by the instant of time $x^\circ = \beta x_*^1 \gamma$ of its action.

It should be especially underlined here that when the forces are applied at different n points of an extended body, the state of equilibrium in relativistic static should be defined just by the equality

$$N_{\alpha\beta}^{(n)} = 0 . \qquad (9')$$

with α, β = 1,2,3, instead of nonrelativistic $\sum_n F_{x^\alpha}^{(n)} = 0$,

i.e., in particular, the instants of force action should be accounted for.

The use of the last equation is actually illustrated using the example considered by Einstein at one time (see also [7]).

Let us visualize a rigid rod AB which is at rest in the S*-system (on the x*-axis). Let oppositely directed equal forces F_x^* be applied to the ends of the rod at some definite instant t* for a very short period of time, and the rod is unaffected by the forces over all the rest of time. It is clear

* An analog of the radar method of measuring distances.

that the described action on the rod at instant t^* does not give rise to motion. Now consider just the same event in the coordinate reference frame relative to which our rod is moving in the direction AB. The force impulses at points A and B are not simultaneous yet in the S-system; on the contrary, the impulse at point B will be delayed relative to that at point A by $\beta\ell_x^*\gamma$ time units, where ℓ_x^* is the length of one rod at rest. Thus, we have come to the following strange result. At first the impulse at point A and some time later the opposite impulse at point B are applied to the moving rod. These impulses conpensate each other so that motion under their action is not disrupted. The fact seems still stranger if we show interest in the energy of the rod at the instant when the impulse at point A has already come to an end, and the impulse at point B has not yet begun to act. The impulse at point A (in a conventional sence) does work above the rod (as the rod is moving); due to this work, the energy of the rod should be therefore increased. However, neither velocity nor other quantities relating to the rod, which its energy could depend on, are changeable. Thus, a seeming violation of the energy conservation law is available.

A principal overcoming of this difficulty is apparent. Based on our implicit supposition that an instantaneous state of the rod can be completely defined by the forces acting on it and by its velocity at the same instant, we have assumed the following. In consequence of applying the force to the body at some point, its velocity increases instantly, and so the propagation of the force acting at that point over all the body demands no time.

As noted by Einstein, the supposition of such a kind is incompatible with the relativity principle.

We add this qualitative explanation of Einstein to the following quantitative calculation.

So, in this case in the S^*-system we have

$$F_*^1 = F_A^* + F_B^* = F_x^* - F_x^* = 0, \quad F_*^\circ = F_*^2 = F_*^3 = 0, \qquad (10^*)$$

as the forces applied at points A and B are equal in value and oppositely directed; $N_{0\alpha}^* = 0$, $N_{12}^* = N_{13}^* = 0$,

$$N_{32}^* = t_A^* F_A^* + t_B^* F_B^* = t_{AB}^* F_x^* = 0. \qquad (11^*)$$

As the forces F_A^* and F_B^* are applied simultaneously, $t_{AB}^* = t_A^* - t_B^* = 0$.

From the viewpoint of the S-system we have

$$F^1 = F_A^*\gamma + F_B^*\gamma = F_x - F_x = 0, \quad F^\circ = \beta F^1 = 0, \quad F^2 = F^3 = 0, \qquad (10)$$

i.e. the rod energy is actually invariable. At the same time

$$t_{AB}F_x = (\beta\ell_x^*\gamma)(F_x^*\gamma) \neq 0.$$

However,

$$N_{32} = t_{AB}F^1 - x_{AB}F^\circ$$

and $x_{AB} = \ell_x^*\gamma$ according to the "elongation formula" (6) and $F^\circ = \beta F^1\gamma$. From here it follows that with respect to the S-system

$$N_{32} = 0. \qquad (11)$$

The result is evidently in full accordance with the motion conservation law of the centre inertia and, along with (10), gives evidence for that rod equilibrium is really not violated when passing to the S-system.

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