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E2-90-21

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APPLICATION
OF THE MULTIPLE SCATTERING THEORY
TO CALCULATION OF $\alpha^{12}C$ SCATTERING

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1990

1. INTRODUCTION

Glauber's multiple scattering theory [1] has become a standard mean of describing and understanding the experimental data on high energy hadron-nucleus diffraction scattering. Proton-helium scattering [2-4], in particular, has been the touchstone of the theory since its first derivation, showing a remarkable agreement between the predicted and experimental structure of the angular scattering distributions, even with rather simplified pictures of the nuclear structure and of the hadron-hadron scattering amplitudes.

A detailed comparison with the most recent and precise data on elastic scattering [5,6], however, seems to display a small but definite discrepancy between the data and some characteristic features of the model, in particular the position of the first diffraction dip, the forward slope of the cross section and the relative height of the cross section at the optical point and after the dip. Furthermore, ref. [7] has shown that the deviation of the Glauber calculation from, α particle scattering on C, Al, Cu and Pb at 3.64 GeV per nucleon experiment becomes obvious beyond $|t| \geq 0.25$ $(\mathrm{GeV/c})^2$. The inelastic cross section calculated in the model approximation of ref. [8] is clearly lower than the data points, and the discrepancies reach up to one order of magnitude at the largest $|t|$ measured for αAl and αCu scattering. Dakhno and Nikolaev [9], however, have shown through a through and accurate analysis that the Glauber theory with inelastic shadowing displays again a systematic disagreement with the data. They found that there is a persistent disagreement between theory and the data of high accuracy experiments, which cannot be eliminated in the conventional picture of the alpha particle mode of four nucleons.

With a somewhat different starting point, the purpose of this paper is to show that disagreement between the Glauber theory and the data disappears if the composite structure of (the bound) nucleons is explicitly taken into account in the calculation of the differential cross section according to the multiple scattering theory.

Three main purposes of the present work are the description of the antisymmetric oscillator wave function of $\alpha{}^{12}C$

in one-particle coordinates, the calculation of the scattering amplitude taking into account the nucleon exchange effects and comparison with $\alpha^{12}C$ diffractive scattering cross section data at high energies.

2. THE MODEL

According to Glauber's theory[1] the scattering amplitude of a pointlike projectile on a system of A constituents is given in terms of the wave function of the target and the elementary scattering amplitude off a constituent by

$$\mathcal{F}_{if}(q) = \frac{ik}{2\pi} \int d^2 b e^{i\vec{q}\vec{b}} \langle \psi_{A_f} | [1 - \prod_{i=1}^{A} (1 - \gamma(\vec{b} - \vec{s}_i))] | \psi_{A_i} \rangle , \quad (1)$$

where $\vec{q}$ is the momentum transfer, $\vec{b}$ is the impact parameter and s_i is the transverse position of the i-th constituent with respect to the collision axis, while $\langle\psi_{A_i}|$ and $\langle\psi_{A_f}|$ are the initial and final state wave functions of the target and $\gamma(\vec{b} - s_i)$ is the profile function of the i-th constituent, defined in terms of the elementary scattering amplitude on some constituent $f_i(q)$ as

$$\gamma(\vec{b} - s_i) = \frac{1}{2\pi ik} \int d^2 q e^{-iq(b - s_i)} f_i(q) . \quad (2)$$

If the projectile is a composite object with B constituents, the generalization of eq.(1) is

$$\mathcal{F}(q) = \frac{ik}{2\pi} \int d^2 b e^{i\vec{q}\vec{b}} \times$$

$$\times \langle \psi_{A_f} \psi_{B_f} | [1 - \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j))] | \psi_{A_i} \psi_{B_i} \rangle , \quad (3)$$

where the states $\langle\psi_{A_i} \psi_{B_f}|$, $\langle\psi_{A_i} \psi_{B_f}|$ are the wave functions of the target-projectile system.

A nucleus in the quark model is described as a system of many clusters, and each cluster consists of three quarks. Then a nuclear non-antisymmetrized wave function in the oscillator-cluster model can be written as [9]

$$\psi_{^{12}C} = \psi_{N_1}(r_1, r_2, r_3) \dots \psi_{N_{12}}(r_{34}, r_{35}, r_{36}) \chi(\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_{12}) , \quad (4)$$

where the nucleus is pictured as a bag (with radius R_A) located at $\vec{R}_{12}$ enclosing A nucleons with radii R_h located at R_i. Using the representation:

$$e^{-3\left(\frac{1}{R_A^2}-\frac{1}{R_h^2}\right)R_i^2}=\left[\frac{3(R_h^2-R_A^2)}{\pi B R_h^2}\right]^{3/2}\int e^{-6i\vec{\beta}_i\left(\frac{1}{R_A^2}-\frac{1}{R_h^2}\right)\vec{R}_i-3\left(\frac{1}{R_A^2}-\frac{1}{R_h^2}\right)\beta^2}\,d^3\beta \tag{5}$$

and the relation

$$R=\frac{r_{3i-2}+r_{3i-1}+r_{3i}}{3}, \qquad (i=1,2,\ldots,12)$$

we can write (4) in a formally factorized form

$$\psi_{^{12}C}=\prod_{j=1}^{12}\exp\left[-\frac{r_{3j-2}^2+r_{3j-1}^2+r_{3j}^2}{R_h^2}-6\vec{\beta}_j\,i\left(\frac{1}{R_A^2}-\frac{1}{R_h^2}\right)\times\right.$$
$$\left.\times(\vec{s}_{3j-2}+\vec{s}_{3j-1}+\vec{s}_{3j})-3\left(\frac{1}{R_A^2}-\frac{1}{R_h^2}\right)\beta_j^2\right]P_n(r_j)\,Y_{em}(\theta,\phi)\,. \tag{6}$$

Using factorized function (6) we can construct the totally antisymmetric wave function under permulation of nucleons, which can be written as Slater determinant.

Then scattering amplitude (3) may be written in the form

$$\mathcal{F}(q)=\frac{ik}{2\pi}\int d^2b\,e^{i\vec{q}\vec{b}}\left(\delta_{mn}\delta_{\bar{M}_m N_n}-\mathrm{Det}\,|\,\delta_{mn}\delta_{\bar{M}_m\bar{N}_n}-A_{mn}|\right) \tag{7}$$

with

$$A_{mn}=\langle\bar{M}_m|\prod_{i=1}^{3}\prod_{j=1}^{3}(1-\gamma(\vec{b}-\vec{s}_i+\vec{\tau}_j))\,|N_n\rangle \tag{8}$$

the matrix element of the profile function between the single particle states described by the quantum numbers M_m and N_n, respectively. Using the orthogonality condition of the oscillator wave function we find that all off-diagonal matrix elements in the determinant of eq.(7) are A_{mn} - I. In order to estimate the main effect of the Pauli principle we first ma-

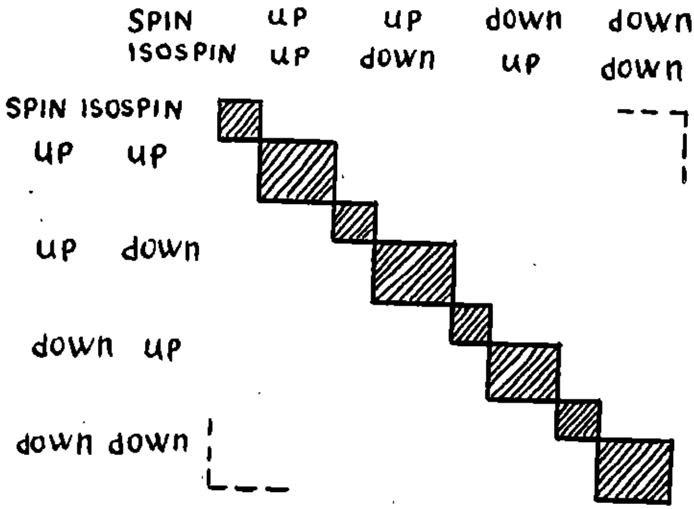

Fig.1

ke a simple estimation of the single particle matrix element of eq.(8). We consider the case where spin-flip and charge-exchange are neglected. Thus $A_{mn} = 0$ if only spin-flip or charge exchange could connect the two states involved. Similarly, since $\gamma(\vec{b})$ is two dimensional, Z direction orthogonality makes $A_{mn} = 0$ [10] if the Z direction quantum number differs. Going in detail one finds that A_{mn} may be rearranged to give exactly one non-zero submatrix per closed shell. Where closed shells are counted per spin up or down, per isospin up or down and per main quantum number (see Fig.1).

The Pauli principle means that two particles from the same shell cover each other very rarely when densities are projected onto the impact parameter plane and that this rare chance is exactly compensated for in the harmonic oscillator model by the Z direction orthogonality.

3. COMPARISON OF RESULTS OF MODEL WITH EXPERIMENTAL DATA

Figure 2 compares the results of the calculation based on formulae (7), with the experimental data for $\alpha^{12}C$ scattering [11]. The solid line corresponds to $d\sigma^{el}/dt + d\sigma^{q.el}/dt$ versus t-dependence calculated using amplitude (7) in Rigid projectile approximation [12]. The dashed line corresponds to $d\sigma^{q\cdot el}/dt$, for the case of structureless nucleons. This approximation yields too low values of $d\sigma/dt$ in the region of $\alpha\,^{12}C$ scattering in the large t region. The dash-dotted line corresponds to the sum $d\sigma^{el}/dt + d\sigma^{q\cdot el}/dt$ for the case of structureless nucleons. For calculation of the elastic scattering amplitude (7) the parametrization was used (ref.[13]). For calculating the differential cross section of quasi-elastic nucleus-nucleus scattering the analytical expression from ref.[7] was taken.

Figure 2 shows that the composite nucleon model yields better agreement than results of ref.[7], and the Glauber approach extended to nucleus-nucleus scattering leads to satisfactory consistency of the calculated cross sections and their t-dependence with those obtained experimentally. However, taking into account the exchange terms improved the agreement between theory and experiment.

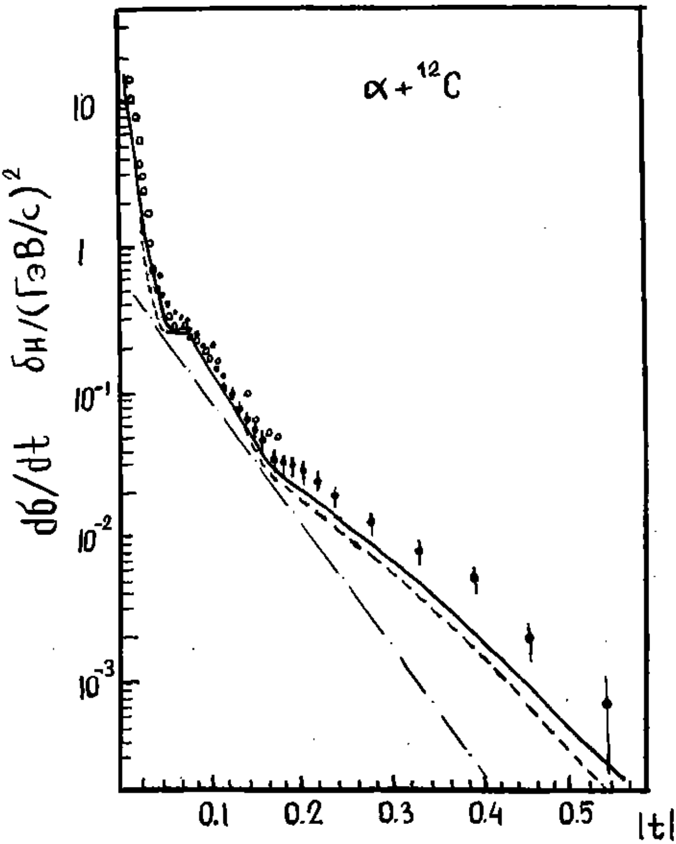

Fig.2

Thus, a quantitatively accurate self-contained Glauber-like description of the scattering process in terms of collisions among the elementary constituents is possible.

Furthermore, we conclude that the nucleon exchange effect is not negligible.

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Received by Publishing Department
on January 12, 1990.