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GLAUBER THEORY OF HIGH-ENERGY p-He SCATTERING WITH COMPOSITE NUCLEONS

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1. INTRODUCTION

The celebrated Glauber multiple scattering theory[1,2] is a generally accepted theory of high-energy hadron-nucleus and nucleus-nucleus diffraction scattering. Proton-helium scattering[3,4], in particular, has been the touchstone of the theory since its first derivation, showing a remarkable agreement between the predicted and experimental structure of the angular scattering distributions.

A detailed comparison with the most recent and precise data on elastic scattering[5,6], however, seems to display a small but definite discrepancy between the data and some characteristic features of the model, in particular the position of the first diffraction dip, the forward slope of the cross section and the relative height of the cross section of the optical point and after the dip. Dakhno and Nikolaev[7] have shown through a thorough and accurate analysis that Glauber theory with inelastic shadowing displays again a systematic disagreement with the data. They found that there is a persistent disagreement between theory and the data of high accuracy experiments, which cannot be eliminated in the conventional picture of the α particle made of four nucleons.

On the other hand, in ref.[8] the Glauber-like description of $p\alpha$ scattering was carried out and it was shown that the main discrepancies between the theory and the data, in particular the position of the diffraction dip, are corrected by the additional terms that contribute to the scattering amplitude when the nucleons are considered as composite objects.

With a somewhat different starting point, it is the purpose of this paper to show that the disagreement between the Glauber theory and the data disappears if the quark structure of nucleons is explicitly taken into account in the calculation of the differential cross section according to the multiple scattering theory. For this purpose we employ the non-relativistic quark-cluster model to describe a nucleon and treat the α-particle as a system of four clusters which is totally antisymmetrized with respect to the quark variables.

Our further presentation is organized as follows. We start with a conventional model of the α-particle made up of four nucleons and nucleons made of quarks. In sect.3 we expose the multiple scattering theory and describe direct and exchange integrals for all members of Glauber's series.

In sect.4 we discuss the numerical calculation. We shall see that the agreement with the experimental data of our model is rather good.

2. QUARK CLUSTER MODEL

A nucleus in the quark cluster model is described as a system of many clusters completely antisymmetrized with respect to the quark variables. Each cluster consists of three quarks and has the nucleon quantum number, namely it has /9/ symmetry for spin-isospin SU(4), $[1^3]$ symmetry for colour SU(3), and $[3]$ symmetry for the radial part

$$|N> = |q^3, [3]_{\sigma\tau} S = 1/2,\ T = 1/2,\ [1^3]_c > \Phi . \tag{1}$$

The radial wave function for a nucleon is taken to be a usual function of the oscillator-cluster model:

$$\Phi = \phi(r_1)\,\phi(r_2)\,\phi(r_3)\,\chi(R) , \tag{2}$$

where r_1, r_2, r_3 and R are the internal and center-of-mass coordinates, respectively,

$$R = \frac{(r_1 + r_2 + r_3)}{3} . \tag{3}$$

The wave function for the α-particle, $|\alpha>$ consisting of four clusters, is given by

$$|\alpha> = \mathcal{A}_\alpha \{(|N>^{(\sigma\tau c)})^4\}_{s=0,\tau=0}\ \Phi(R_1)\,\Phi(R_2)\,\Phi(R_3)\,\Phi(R_4)\ \times$$

$$\times\, \chi(\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4) , \quad N = 1/\sqrt{<\alpha|\mathcal{A}_\alpha|\alpha>} , \tag{4}$$

where $\mathcal{A}_\alpha$ is antisymmetrization operator among quarks in different clusters for the α-particle.

In eq. (1), we use a normalized gaussian:

$$\phi(r) = (\sqrt{\pi} R_h^2)\, e^{-r^2/R_h^2}. \tag{5}$$

Then for wave function (3) we have:

$$|a\rangle = \mathcal{A}_a \{ (|N^{(\sigma\tau c)}\rangle)^4 \} \frac{N}{(2\pi)^4} \exp\Big[-\frac{s_1^2 + s_2^2 + \ldots + s_{12}^2}{R_h^2} - 4\Big(\frac{1}{R_a^2} - \frac{1}{R_h^2}\Big)(R_1^2 + R_2^2 + R_3^2 + R_4^2) + i\vec{\xi}\,(\vec{s}_1 + \vec{s}_2 + \ldots + \vec{s}_{12})\Big]. \tag{6}$$

Next, using the representation:

$$e^{-a^2 R_i^2} = (\sqrt{\pi}/a)^{3/2} \int e^{-2i\vec{\beta}_i \vec{R}_i a - \beta^2 a} d^3\beta \tag{7}$$

we can write (6) in a formally factorized form for space coordinates.

In our case $\mathcal{A}_a$ consists of 12!/(3!) /4!=15400 terms, which are classified into 12 essentially different terms. The corresponding matrix elements of the exchange operators are given in the Table [9].

Table

term	factor	$p^{(\sigma\tau)}$	$p^{(c)}$	whole factor
1.	1	1	1	1
2.	-54	-1/27	1/3	2/3
3.	216	11/243	1/9	88/81
4.	648	-5/243	1/9	-40/27
5.	-144	0	0	0
6.	243	1/729	1/9	1/27
7.	-486	-25/2187	1/27	50/243
8.	-3888	7/2187	1/27	-112/243
9.	-648	-1/243	1/27	8/81
10.	-3888	-1/2187	1/27	16/243
11.	3888	0	0	0
12.	1296	1/1458	1/54	4/243

3. CALCULATION OF THE ELASTIC SCATTERING AMPLITUDE

According to the Glauber theory [1] the scattering amplitude of a composite object with A constituents on a system of B constituents is determined by the following expression:

$$\mathcal{F}_{AB}(q) = \frac{ik}{2\pi} \int d^2 b e^{i\vec{q}\vec{b}} \langle \psi_A \psi_B | [1 - \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j))] | \psi_A \psi_B \rangle, \quad (8)$$

where p is the momentum at the projectile system, q is the transverse momentum, ψ_A and ψ_B are wave functions of the target-projectile systems, $\{s_i\}$, $\{r_i\}$ are the coordinates of the constituents of projectiles and targets within the plane of the impact parameter b .

Inserting (6) and (7) into (8) and using parametrization

$$\gamma(\vec{b}) = \gamma(0)\, e^{-\frac{b^2}{2B}} \quad (9)$$

we have direct integral:

$$\mathcal{F}_{p\alpha}(q) = \frac{iP}{2\pi} e^{\frac{q^2 R_h^2}{12}} \sum_{i>j>k>\ell} N_i N_j N_k N_\ell \frac{N\tilde{N}}{\mathrm{Det}\, \mathcal{U}(i,j,k,\ell)} \times$$
$$\times (\gamma(0))^{I^T (Q_i + Q_j + Q_k + Q_\ell) B} \exp\left[-\frac{q^2}{4} \frac{\mathrm{Det}\, Q(i,j,k,\ell)}{\mathrm{Det}\, \mathcal{U}(i,j,k,\ell)} \right], \quad (10)$$

where

$$Q(i,j,k,\ell) = \begin{pmatrix} T_i & & & & Q_i & G_1 & \tilde{G}_1 & 1 \\ & T_j & & & Q_j & G_2 & \tilde{G}_2 & 1 \\ & & T_k & & Q_k & G_3 & \tilde{G}_3 & 1 \\ & & & T_\ell & Q_\ell & G_4 & \tilde{G}_4 & 1 \\ Q_i^T & Q_j^T & Q_k^T & Q_\ell^T & D & & & \\ G_1^T & G_2^T & G_3^T & G_4^T & & E & & \\ \tilde{G}_1^T & \tilde{G}_2^T & \tilde{G}_3^T & \tilde{G}_4^T & & & F & \\ 1 & 1 & 1 & 1 & & & & 0 \end{pmatrix}$$

$$T_i = \begin{pmatrix} \frac{1}{R_h^2} & 0 & 0 \\ 0 & \frac{1}{R_h^2} & 0 \\ 0 & 0 & \frac{1}{R_h^2} \end{pmatrix} + Q_i B \;; \qquad G_1 = \tilde{G}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \;;$$

$$G_2 = \tilde{G}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \qquad G_3 = \tilde{G}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix};$$

$$I = (1 \quad 1 \quad 1); \qquad B = (1/2B \quad 1/2B \quad 1/2B);$$

$$G_4 = \tilde{G}_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad D = \begin{pmatrix} \frac{1}{R_h^2} & 0 & 0 \\ 0 & \frac{1}{R_h^2} & 0 \\ 0 & 0 & \frac{1}{R_h^2} \end{pmatrix} + B^T(Q_i + Q_j + Q_k + Q_\ell);$$

$$E = F = \begin{pmatrix} \frac{1}{A} & 0 & 0 & 0 \\ 0 & \frac{1}{A} & 0 & 0 \\ 0 & 0 & \frac{1}{A} & 0 \\ 0 & 0 & 0 & \frac{1}{A} \end{pmatrix}; \qquad A = 3\left(\frac{1}{R_\alpha^2} - \frac{1}{R_h^2}\right);$$

and

$$\mathfrak{T}(i,j,k,\ell) = \begin{pmatrix} & & & & & t \\ & & & & & t \\ & & Q(i,j,k,\ell) & & & t \\ & & & & & t \\ & & & & & t \\ t^T & t^T & t^T & t^T & 0 & 12/R_h^2 \end{pmatrix}; \quad t^T = \left(\frac{1}{R_h^2} \; \frac{1}{R_h^2} \; \frac{1}{R_h^2}\right).$$

The structure of matrices Q_i and numbers N_i are given in the Appendix. By permutation elements of matrices G_i and $\tilde{G}_i$ we give an expression for exchange integrals. For example, for the matrix element:

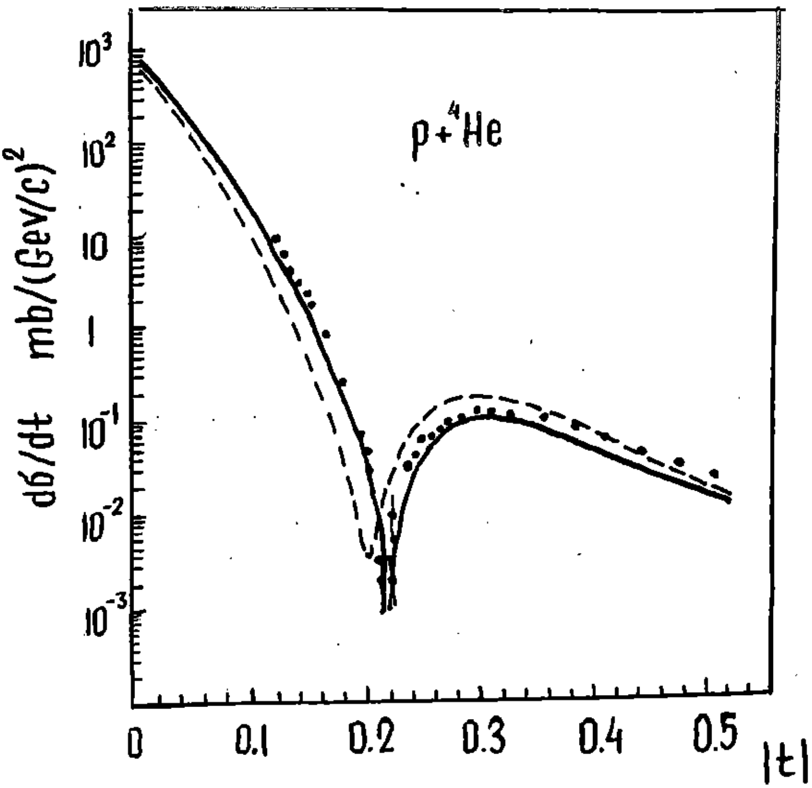
$$\langle \alpha \mid \hat{p}_{14} \mid \alpha \rangle \tag{12}$$

we have

$$G_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{13}$$

4. COMPARISON WITH THE EXPERIMENTAL DATA

The figure compares the differential cross section calculated from eq. (10) with the experimental data for $p\alpha$ scattering [6]. The solid line corresponds to the cross section calculated using eq. (10). The dashed line corresponds to the



cross sections calculated in ref. /7/ for the case of point-like nucleons.

In calculation of the elastic scattering amplitude (10) parametrizations (5,9) were used from ref. /8/ .

The figure shows that the quark cluster model gives good agreement and the Glauber approach extended to nucleus-nucleus scattering leads to satisfactory consistency of the calculated cross sections and their t-dependence with those obtained experimentally.

Our results can be summarized as follows:

1. Taking into account the quark structure of nucleons we have significantly improved the description of the data, thus restoring the agreement between the Glauber theory and the experiment.

2. The quark exchange effect is not negligible for this case.

3. The multiple scattering terms which were neglected in ref. /8/ are not unimportant.

APPENDIX

Matrices Q_i are the matrices which corresponded to the diagrams for essentially different term of (8) for the case $A = B = 3$ /10/ and N_i are the combinatorial coefficients:

$$Q_i = q_i \frac{1}{2B}$$

$$q_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$q_4 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_5 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_6 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$q_7 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_8 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_9 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};$$

$$q_{10} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_{11} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad q_{12} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix};$$

$$q_{13} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad q_{14} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_{15} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix};$$

$$q_{16} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_{17} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_{18} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix};$$

$$q_{19} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad q_{20} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad q_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix};$$

$$q_{22} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; \quad q_{23} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad q_{24} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix};$$

$$q_{25} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}; \quad q_{26} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}; \quad q_{27} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$q_{28}=\begin{pmatrix}1&0&1\\1&1&0\\1&1&0\end{pmatrix};\quad q_{29}=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix};\quad q_{30}=\begin{pmatrix}1&1&1\\1&1&1\\0&0&0\end{pmatrix};$$

$$q_{31}=\begin{pmatrix}1&1&0\\1&1&0\\1&1&0\end{pmatrix};\quad q_{32}=\begin{pmatrix}1&1&1\\1&1&1\\1&0&0\end{pmatrix};\quad q_{33}=\begin{pmatrix}1&1&1\\1&1&0\\1&1&0\end{pmatrix};$$

$$q_{34}=\begin{pmatrix}1&1&1\\1&1&0\\0&1&1\end{pmatrix};\quad q_{35}=\begin{pmatrix}1&1&1\\1&1&1\\0&1&1\end{pmatrix};\quad q_{36}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix};$$

$$N_1 = N_{36} = 1;\quad N_4 = N_5 = N_{12} = N_{18} = N_{35} = -9;$$
$$N_2 = N_{19} = N_{25} = N_{32} = N_{33} = 9;\quad N_6 = 6;\quad N_{26} = -6;$$
$$N_{10} = N_{11} = 3;\quad N_{30} = N_{31} = -3;\quad N_3 = N_{14} = N_{15} = N_{16} =$$
$$N_{17} = N_{28} = N_{29} = -18;\quad N_7 = N_8 = N_{20} = N_{21} = N_{23} = N_{24} =$$
$$= N_{34} = 18;\quad N_6 = N_{22} = 36;\quad N_{13} = -36;$$

REFERENCES

1. Glauber R.G. - In: Lectures in Theoretical Physics, vol.1 ed. W.E.Brittin and L.G.Dunham, Interscience, New York, 1959, p.315.
2. Sitenko A.G. - Ukrainian Phys.J. (In Russian), 1959, 4, p.152.
3. Bassel R.H., Wilkin C. - Phys.Rev., 1968, 174, p.1179.
4. Alkhazov G.D., Belostotsky, Vorobyov A.A. - Phys.Reports, 1978, 42C, p.89.
5. Burg J.P. et al. - Nucl.Phys., 1981, B187, p.205.
6. Bujak A. et al. - Phys.Rev., 1981, D23, p.1895.
7. Dakhno L.G., Nikolaev N.N. - Nucl.Phys., 1985, A436, p.653.
8. Forte S. - Nucl.Phys., 1987, A467, p.665.
9. Takeyschi S. et al. - Nucl.Phys., 1986, A449, p.617.
10. Omboo Z. - Preprint JINR E2-83-775, Dubna, 1983.

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