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EXTRA Z' BOSON AND TOP QUARK MASS

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Today almost nobody has doubts that the top quark exists. The determination of the top mass $m_{t}$ is one of the main tasks of the experiment. Unfortunately the search for the $t$ quark creation on contemporary colliders has not been successful, only low limits on $m_{t}$ were obtained. The indirect estimations [1-4] from the precise neutral current data analysis gave nearly 130 GeV for $m_{t}$ with accuracy of $30-40 \mathrm{GeV}$ and

$$
\begin{equation*}
\sin ^{2} \theta_{W} \equiv 1-m_{W}^{2} / m_{Z}^{2} \cong 0.2276 \pm 0.009 \tag{1}
\end{equation*}
$$

for the weak angle, where $m_{W}$ and $m_{Z}$ are the $W$ - and $Z$-boson masses. The minimal standard model (SM) was assumed to be correct with one-loop radiative corrections, which are very sensitive to the $t$ mass $m_{t}$ [5]. Roughly speaking, wanted $m_{t}$ is connected with the $X$-coordinate of the intersect point of two curves in fig.1 [1]. The first curve, marked by $\sin ^{2} \theta_{\mathrm{W}}(\nu-q)$ and almost independent of $m_{t}$, corresponds to $\sin ^{2} \theta_{W}$ obtained from the ratio:

$$
\begin{equation*}
\mathrm{R}_{\nu}=\sigma_{\nu N}^{N C} / \sigma_{\nu N}^{C C_{N}} \rho^{2}\left\{\varepsilon_{L}^{2}(u)+\varepsilon_{L}^{2}(d)+r\left[\left(\varepsilon_{R}^{2}(u)+\varepsilon_{R}^{2}(d)\right]\right\}\right. \tag{2}
\end{equation*}
$$

For $\quad \mathrm{SM} \quad \varepsilon_{\mathrm{L}}(q)=\mathrm{T}_{3 \mathrm{~L}}-Q^{\mathrm{em}} \sin ^{2} \theta_{W}, \quad \varepsilon_{R}(q)=-Q^{e m} \sin ^{2} \theta_{W}$ and $R_{\nu}=\rho^{2}\left[1 / 2-\sin ^{2} \theta_{W}+\right.$ $\left.5 / 9(1+r) \sin ^{4} \theta_{W}\right] ; \quad r=\sigma \frac{C N}{C D} / \sigma_{\nu N}^{C C_{N 0}} 0.4$ where $\sigma_{\nu N}^{N C}, \sigma_{\nu N}^{C C}$ are the neutrino scattering cross sections on the isoscalar target in neutral and charged currents (for simplicity the modification of relation (2) by means of radiative corrections and hadron structure is not: written). The second curve, marked by $\sin ^{2} \theta_{W}(V B M)$, was obtained from a relation between the $S M$ parameters $\alpha, G_{F}, m_{Z}$ and $\sin ^{2} \theta_{W}$ :

$$
\begin{equation*}
\sin ^{2} \theta_{W}=\frac{1}{2}\left[1-\left\{1-\frac{4 \pi \alpha}{\sqrt{2} G_{F} m_{Z}^{2}} \frac{1}{1-\Delta r}\right\}^{1 / 2}\right] \tag{3}
\end{equation*}
$$

The top mass $m_{t}$ arises in $\sin ^{2} \theta_{W}(V B M)$ by means of the loops contribution $\Delta r[4,5]$ :
$\frac{1}{1-\Delta r} \cong \frac{\alpha\left(m_{Z}\right)}{\alpha}\left\{1+\frac{\cos ^{2} \theta_{W}}{\sin ^{2} \theta_{W}}\left\{\frac{3 G_{F} m_{t}^{2}}{8 \sqrt{2} \pi^{2}}+\left(\frac{3 G_{F} m_{t}^{2}}{8 \sqrt{2} \pi^{2}}\right)^{2} \frac{19-2 \pi^{2}}{3}+\ldots\right\}\right\}^{-1}$,
thus $\sin ^{2} \theta_{W}$ (VBM) decreases if $m_{t}$ increases. So simultaneous precise determination of $\sin ^{2} \theta_{W}(\nu-q)$ and $\sin ^{2} \theta_{W}(V B M)$ allows indirect estimation of $m_{t}$ (and $\sin ^{2} \theta_{W}$ too). For more accurate determination of the central value $m_{t}$ of the top quark mass and the error $\Delta m_{t}$ it is necessary, however, to take into consideration the whole set of the neutral current data as done in [1-4].

The $m_{t}$-independent value of $\sin ^{2} \theta_{W}=0.220 \pm 0.009$, calculated with formula (1) using the experimental values of the mass ratio $m_{W} / m_{Z}$ [6] is shown in fig. 1 too. This value is essentially smaller than $\sin ^{2} \theta_{W}(\nu-q)$ but still has considerable experimental uncertainties, therefore we don't use this value here. However, more precise determination of the ratio $m_{W} / m_{Z}$ can yield a significantly larger extracted $m_{t}$.


Fig.1. $\sin ^{2} \theta$ obtained from $m_{Z}\left(\sin ^{2} \theta_{W}(V B M)\right), m_{W} m_{Z}$ and $\mathrm{R}\left(\sin ^{2} \theta_{W}(\nu-q)\right)$ as a function of top mass $m_{t}$. (Figure is taken from [1]).

There is only one vector $Z$-boson that transfers the neutral weak interactions in SM. Today the possible manifestations of the superstring-inspired extra $Z^{\prime}$ boson [7] in a large domain of neutral current processes are widely discussed [8-10]. It is suggested that after superstring compactification the effective theory with $\mathrm{E}_{6}$ gauge group arises. This group breaks down to the subgroup $G_{5}=S U_{3 C} \times S U_{2 L} \times U_{1 Y} \times U_{1 \eta}$ [rank 5] or $G_{6}=S U_{3 C} \times S U_{2 L} \times U_{1 Y} \times$ $\mathrm{U}_{1 \psi} \times \mathrm{U}_{1 \chi}$ [rank 6]. Examination of the rank 6 subgroup yields two extra neutral bosons $Z_{\psi}$ and $Z_{\chi}$. Assuming that $G_{6}$ spontaneously breaks on intermediate scale $M_{I} \approx 10^{10-11} \mathrm{GeV}$ one has only one "light" $z^{\prime}$-boson [7]. The corresponding $U_{1}^{\prime}$ charge is

$$
\begin{equation*}
Q^{\prime}\left(\vartheta_{E 6}\right)=Q_{\psi} \cos \vartheta_{E 6}+Q_{\chi} \sin \vartheta_{E 6} \tag{5}
\end{equation*}
$$

$Q_{\psi, \chi}$ are $U_{1 \psi, \chi}$-charges. The angle $\vartheta_{E 6}$ parametrises the scheme of the symmetry breaking at the intermediate scale. The charge $Q_{\eta}$ of subgroup $U_{1 \eta}$ corresponds to the fixed value of the angle $v_{E 6}=142.2^{\circ}$.

The $Z, Z^{\prime}$ mass matrix is non-diagonal in the general case. After diagonalization the $Z_{1}$ and $Z_{2}$ fields with the definite masses $m_{1}$ and $m_{2}$ arise. The $Z-Z^{\prime}$ mixing angle is determined by relation

$$
\begin{equation*}
\operatorname{tg}^{2} \theta=\frac{m_{0}^{2}-m_{1}^{2}}{m_{2}^{2}-m_{0}^{2}}, \tag{6}
\end{equation*}
$$

here $m_{0}=m_{W} / \cos \theta_{W}$ is a mass parameter coinciding with $S M z$ boson mass. For the $Z_{1}$ and $Z_{2}$ fields the Lagrangian takes the form [9]

$$
\begin{equation*}
L_{N C}=e_{1} \sum_{1}^{2} g_{1} z_{1}^{\mu}\left(\varepsilon_{L}^{1} \bar{f}_{L} \gamma_{\mu} f_{L}+\varepsilon_{R}^{1} \bar{f}_{R} \gamma_{\mu} f_{R}\right) \tag{7}
\end{equation*}
$$

$g_{1}=\cos ^{-1} \theta_{W} \sin ^{-1} \theta_{W}, \quad g_{2}=g_{1} W=g_{1} \frac{\sqrt{5}}{\sqrt{3}} \sin \theta_{W}[9]$ and $f_{L, R}=\frac{1}{2}\left(1 \mp \gamma_{5}\right)$. For $1 \neq 0$ and $B=L, R$ :

$$
\begin{align*}
& \varepsilon_{B}^{1}(f)=\varepsilon_{B}^{S m}(f) \cos \theta+W \varepsilon_{B}^{\prime}(f) \sin \theta, \\
& \varepsilon_{B}^{2}(f)=\varepsilon_{B}^{\prime}(f) \cos \theta-w^{-1} \varepsilon_{B}^{s m}(f) \sin \theta . \tag{8}
\end{align*}
$$

The chiral couplings $\varepsilon_{B}^{S m}(f)$ and $\varepsilon_{B}^{\prime}(f)$ are given in the table. Table

|  | $\varepsilon_{L}(u)$ | $\varepsilon_{R}(u)$ | $\varepsilon_{L}(d)$ | $\varepsilon_{R}(d)$ | $\varepsilon_{L}(v)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Z_{\operatorname{sm}}$ | $\frac{1}{2}-\frac{2}{3} \sin ^{2} \theta_{W}$ | $-\frac{2}{3} \sin ^{2} \theta_{W}$ | $-\frac{1}{2}+\frac{1}{3} \sin ^{2} \theta_{W}$ | $\frac{1}{3} \sin ^{2} \theta_{W}$ | $\frac{1}{2}$ |
| $Z_{\chi}$ | $-\frac{1}{2 \sqrt{10}}$ | $\frac{1}{2 \sqrt{10}}$ | $-\frac{1}{2 \sqrt{10}}$ | $-\frac{3}{2 \sqrt{10}}$ | $\frac{3}{2 \sqrt{10}}$ |
| $Z_{\psi}$ | $\frac{1}{\sqrt{24}}$ | $-\frac{1}{\sqrt{24}}$ | $\frac{1}{\sqrt{24}}$ | $-\frac{1}{\sqrt{24}}$ | $\frac{1}{\sqrt{24}}$ |
| $Z_{\eta}$ | $-\frac{1}{\sqrt{15}}$ | $\frac{1}{\sqrt{15}}$ | $-\frac{1}{\sqrt{15}}$ | $-\frac{1}{2 \sqrt{15}}$ | $\frac{1}{2 \sqrt{15}}$ |

The top quark mass $m_{t}\left(\right.$ and $\left.\sin ^{2} \theta_{W}\right)$ dependence upon extra ' $Z$ ' boson parameters is investigated in this paper. The $Z^{\prime}$ existence must manifest itself in deviations from $S M$ already at the tree level. Particularly, due to non-zero $Z-Z$ mixing the measured $Z$ boson mass $m_{1}$ has to be slightly less than the $Z$ boson mass $m_{0}$ expected in SM (when there is no mixing). So in formulae (1), (3) one has to take for $m_{z}$ not the experimentally determined mass $m_{1}$ but $m_{0}$ defined by formula (6). So one has a decrease of the ratio $m_{W} / m_{Z}$ and hence an increase of $\sin ^{2} \theta_{W}$ calculated by (1) as well as a decrease of $\sin ^{2} \theta_{W}(V B M)$ independently of $m_{t}$. The value of $\sin ^{2} \theta_{W}(\nu-q)$ will change too. Actually, to take $Z^{\prime}$ boson into consideration one must change the chiral couplings $\varepsilon_{\mathrm{L}, \mathrm{R}}^{\mathrm{sm}}(\mathrm{q})$ in formula (2) to the low-energy effective chiral couplings in the form [9]

$$
\begin{equation*}
E_{B}(q)=\frac{\sqrt{2}}{2 G_{F}} \sum_{i=1}^{2} \varepsilon_{L}^{i}(\nu) \varepsilon_{B}^{i}(q)^{g_{i}^{2}} / m_{1}^{2} \tag{9}
\end{equation*}
$$

As a result, the functional connection between $R_{\nu}$ and $\sin ^{2} \theta_{W}$ changes and another value will be extracted for $\sin ^{2} \theta_{W}(\nu-q)$, which depends on $Z^{\prime}$ parameters. The radiative corrections can give a slightly different contribution too, but we ignore their
modifications. The top mass decrease is expected owing to all these changes [10].

So two different $\sin ^{2} \theta_{W}$ as a function of $m_{t}$ take place. The first one, $\sin ^{2} \theta_{W}(\nu-q)$, is obtained from $\nu-q$ scattering using formulae (2) and (9). The second one, $\sin ^{2} \theta_{W}$ (VBM), is calculated using formulae (3) and (6) with measured $Z$ boson mass $m_{1}$. The condition $\sin ^{2} \theta_{W}(V B M)=\sin ^{2} \theta_{W}(\nu-q)$ gives the desired estimation for the top quark mass $m_{t}\left(\right.$ and $\left.\sin ^{2} \theta_{W}\right)$ as a function of $Z^{\prime}$ boson parameters.

We consider three more popular low-energy superstring inspired models usually marked as $\chi^{-}, \psi-, \eta$-models. The obtained


Fig.2a, The dependence of the central values of the top mass $m_{t}$ on $Z$ ' mass $m_{2}$ for some allowed $Z-Z^{\prime}$ mixing angles $\theta_{m i x}$ in the $\psi$ model.
results are shown in fig. 2 a for model $\psi$ and in fig. 2 b , c for models $\boldsymbol{\chi}$ and $\eta$. The top quark mass $m_{t}$ as a function of the $Z^{\prime}$ boson mass $m_{2}$ for some allowed [11] $Z-Z$ ' mixing angles $\theta$ is reported. The corresponding values of $\sin ^{2} \theta_{W}$, which practically uniformly varies


Fig.2b. The same as in Fig. 2 a but for the $\chi$ model.
along the curve, are plotted near the marked end points of each curves. The $m_{t}$ shows a distinct tendency to decrease as the $Z^{\prime}$ mass increases. Only for $m_{2}$ and $\theta$ near the boundary of the allowed region of parameters (low $m_{2}$ and big $|\theta|$ ) in some cases one can see a rather small increase of $m_{t}$ as compared with the value 130 Gev obtained in SM. [1]. This can be explained by the inevitable decrease of $\sin ^{2} \theta_{W}(V B M)$ and, on the contrary, by a possible increase of $\sin ^{2} \theta_{W}(\nu-q)$ as $\theta$ - and $m_{2}$-function for fixed $m_{t}$.


Fig. 2c. The same as in Fig. 2a but for the $\eta$ model.
So, despite the fact that the variations of $m_{t}$ and $\sin ^{2} \theta_{W}$ depending on the $Z^{\prime}$ boson are practically within accuracy region obtained in $S M$ [1] $\left(\sin ^{2} \theta_{W}=0.2276_{-0.034}^{+0.039}\right.$ and $\left.m_{t}=132_{-37}^{+31}\right)$ the inclusion of the $Z^{\prime}$ boson makes the top mass show a stable tendency to decrease and allows better co-ordination of $\sin ^{2} \theta_{W}$ obtained from different experimental data. For instance, if $\vartheta_{E 6}=90^{\circ}, m_{2}=500 \mathrm{GeV}$ and $\theta_{\mathrm{mix}}=-0.02$ then the central values of $\mathrm{m}_{\mathrm{t}}$ and $\sin ^{2} \theta_{W}$ are 85 GeV and 0.2293 respectively. Formula (1) in this case yields $\sin ^{2} \theta_{W}$ equal to 0.2290 . As pointed out in [1-4], the sensibility of $m_{t}$ and $\sin ^{2} \theta_{W}$ to the Higgs boson mass is rather weak, so we restrict ourselves to $m_{H}=100 \mathrm{GeV}$. Note that for the fixed $Z^{\prime}$ boson parameters the accuracy in definition of $m_{t}$ and $\sin ^{2} \theta_{W}$ does not differ very much from that obtained in [1]. Nevertheless the thorough investigation of the indirect determination of the top quark mass requires a comprehensive high-precision data analysis with inclusion of the $Z^{\prime}$ boson. Yu.P.Ivanov, S.G.Kovalenko, A.A.Pankov and I.S.Satsunkevich for useful discussions.

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