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G.P. Korchemsky

PARITY ANOMALY IN D=3 CHERN-SIMONS GAUGE THEORY

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1. INTRODUCTION

It was realized some time ago [1]-[5] that there is a unique possibility to define gauge theory in odd dimensions by adding the Chern-Simons terms to the nonabelian Yang–Mills action. In $D = 3$ Euclidean space-time the action of this theory called topological massive gluodynamics is defined as [2,3,5]

$$S = S_{CS} + S_{YM} , \tag{1}$$

where S_{CS} is the action of the $D = 3$ nonabelian Chern-Simons gauge theory

$$S_{CS}[A] = i\frac{k}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2}{3}iA_\mu A_\nu A_\rho\right). \tag{2}$$

Here $A_\mu = A^a_\mu t^a$ is the gauge field, t^a are Hermitian generators of the fundamental representation of nonabelian group G:

$$[t^a, t^b] = if^{abc}t^c , \qquad \mathrm{Tr}(t^a t^b) = \frac{1}{2}\,\delta^{ab}$$

and metric is chosen to be $g_{\mu\nu} = \mathrm{diag}(1,1,1)$. S_{YM} is the standard action on gluodynamics

$$S_{YM}[A] = \frac{1}{2M}\int d^3x\, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) ,$$

where $F_{\mu\nu}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ and the parameter $M > 0$ scales like a mass.

Topological massive gluodynamics has interesting properties [2,3,5,6]. In particular, for this theory be gauge invariant the dimensionless coupling constant k must be quantized [3]: $k \in \mathbf{Z}$. It turns out [6] that calculation of quantum corrections within the framework of perturbation theory leads to integer-valued additive renormalization of the bare parameter k in the limit $M \to \infty$:

$$k_{ren} = k + c_v(G) , \tag{3}$$

where $k > 0$ and $c_v(G)$ is the quadratic Casimir operator of the group G in the adjoint representation.

The interest in the Chern-Simons gauge theory was renewed after it was found that the statistics of fermions interacting with the abelian Chern-Simons gauge field transmutates [7,8]. Then, Witten [9] considered the quantum field theory defined by the nonabelian Chern-Simons action (2) and found that it was exactly solvable and had intrinsic relation to two dimensional conformal field theories and geometry of three-dimensional manifolds. The main results of his paper were developed and confirmed later on [10,11].

One of the interesting properties of nonabelian Chern-Simons gauge theory predicted by Witten is the following: quantum corrections have the effect of replacing k by $k+c_v(G)$. It is well-known that an analogous effect occurs in the Wess-Zumino-Witten model [12] and some other integrable models in two dimensions. Moreover, topological massive gluodynamics possesses the same property (3) in the limit $M \to \infty$ when only Chern-Simons term survives in the classical action (1).

To verify this conjecture, perturbation theory calculations were performed [13,14] and the results contradicting each other were obtained. It has been shown in [13] that there is no shift of coupling constant in the nonabelian Chern-Simons gauge theory, but in [14] it has been demonstrated that the Witten conjecture is true under proper regularization of theory.

The main result of the present paper is that the shift of the coupling constant indeed occurs in the $D = 3$ nonabelian Chern-Simons gauge theory and this property is a consequence of the appearing of the parity anomaly in the theory well known from the study of odd dimensional fermions [4].

2. PARITY OF THE EFFECTIVE ACTION

Let us examine symmetry of the Chern-Simons gauge theory. Under gauge transformations

$$A_\mu \to A_\mu^U = U^{-1} A_\mu U + iU^{-1}\partial_\mu U$$

of the gauge field vanishing at infinity $|x| \to \infty$ faster than $\frac{1}{|x|}$, the classical action of the theory transforms as follows:

$$S[A] \to S[A^U] = S[A] + 2\pi i\, k\, W[A] \ ,$$

where $W[A] = \frac{1}{24\pi^2}\int d^3x \epsilon^{\mu\nu\rho}\,\mathrm{Tr}(U^{-1}\partial_\mu U U^{-1}\partial_\nu U U^{-1}\partial_\rho U)$ is the winding number of gauge transformations that takes integer values for continuous transformations $U(x)$. Hence, the partition function of the Chern-Simons gauge theory

$$Z = \int \mathcal{D}A_\mu\, e^{-S_{CS}[A]}$$

is invariant under both infinitesimal ("small") and topologically nontrivial ("large") gauge transformations provided the coupling constant k takes integer (positive or negative) values [3]:

$$k \in \mathbf{Z} \ .$$

It must be noticed that the classical action possesses the following important property: it is pseudoscalar. It means that under discrete parity transformation P defined as

$$\begin{aligned} P: \quad x_\mu &\to x_\mu^P = (-x_1, x_2, x_3) \\ A_\mu(x) &\to A_\mu^P(x) = (-A_1(x^P), A_2(x^P), A_3(x^P)) \end{aligned}$$

the action changes the sign:

$$P: \quad S_{CS}[A] \to S_{CS}[A^P] = -S_{CS}[A] \ . \qquad (4)$$

The main object under consideration in quantum Chern-Simons theory is the effective action. We divide the gauge field into the sum of the background field $a_\mu(x)$ and quantum fluctuation $B_\mu(x)$ and define the effective action $\Gamma_{CS}[a]$ as

$$e^{-\Gamma_{CS}[a]} = \int \mathcal{D}B_\mu\, \delta(D^\mu B_\mu) e^{-S_{CS}[a+B]} \det\left(D^\mu(D_\mu - i[B_\mu, \ ])\right),$$

where the background field gauge was fixed

$$D^\mu B_\mu \equiv \partial^\mu B_\mu - i[a^\mu, B_\mu] = 0 \ .$$

The effective action is invariant under large and small gauge transformations of the background field $a_\mu(x)$. As to parity properties, using relation (4) and taking into account that the classical action (2) is a pure imaginary quantity, one easily obtains a transformation law of the effective action

$$P: \qquad \Gamma_{CS}[a] \to \Gamma_{CS}[a^P] = (\Gamma_{CS}[a])^* . \tag{5}$$

This relation implies that the real and imaginary parts of $\Gamma_{CS}[a]$ are scalar and pseudoscalar, respectively, and therefore, $\mathrm{Im}\,\Gamma_{CS}[a]$ is proportional to $\epsilon^{\mu\nu\rho}$-symbol.

In the weak coupling region (or,equivalently, for large k) we restrict ourselves to the one-loop approximation of the effective action

$$\begin{aligned} e^{-\Gamma_{CS}[a]} &= e^{-S_{CS}[a]} \int \mathcal{D}B_\mu \mathcal{D}\phi \mathcal{D}c \mathcal{D}\bar{c} \\ &\times \exp\left(-i\,\mathrm{Tr}\int d^3x \left(B_\mu \frac{k}{4\pi}\epsilon^{\mu\nu\rho} D_\nu B_\rho + 2\phi D^\mu B_\mu + \bar{c} D^\mu D_\mu c\right)\right), \end{aligned} \tag{6}$$

where the background field a_μ for $k \gg 1$ is the stationary point of the classical action

$$F_{\mu\nu}[a] = 0 \ . \tag{7}$$

Let us introduce an auxiliary field $B_A = \left((\frac{k}{4\pi})^{1/2} B_\mu, (\frac{k}{4\pi})^{-1/2}\phi\right)$ and represent $e^{-\Gamma_{CS}[a]}$ in the form

$$\begin{aligned} e^{-\Gamma_{CS}[a]} &= e^{-S_{CS}[a]} \int \mathcal{D}B_A \exp\left(i\,\mathrm{Tr}\int d^3x B_A \Delta^{AB}[a] B_B)\right) \det(D^\mu D_\mu) \\ &= e^{-S_{CS}[a]}\, \frac{\det(D^\mu D_\mu)}{(\det \Delta[a])^{1/2}} \ , \end{aligned} \tag{8}$$

where the operator $\Delta[a]$ is defined as

$$\Delta_{AB}[a] = \begin{pmatrix} -\epsilon_{\mu\rho\nu} D^\rho & D_\mu \\ -D_\nu & 0 \end{pmatrix}, \quad A = (\mu, 4),\ B = (\nu, 4) \ .$$

The ghost functional $\det(D^\mu D_\mu)$ takes real positive values whereas $\det \Delta[a]$ has the non-vanishing imaginary part and it is this imaginary part that is the source of the renormalization of the coupling constant. To prove this statement, we will demonstrate that for an arbitrary gauge field $A_\mu(x)$ the following relation holds:

$$(\det \Delta[A])^{1/2} = \det i\hat{D} \ , \tag{9}$$

where $\hat{D} = \gamma^\mu(\partial_\mu - i[A_\mu, \ ])$ is the three dimensional Dirac operator in the adjoint representation of gauge group and the Dirac matrices γ^μ coincide with the Pauli matrices.

Let $\{\lambda\}$ be eigenvalues of the operator $\Delta[A]$

$$\Delta^{ab}_{AB}[A]\psi^{B,b} = \lambda\psi^a_A, \quad \psi_A = (\psi_\mu, \psi_4) \ , \tag{10}$$

where a, b are "color" indices. This relation is equivalent to the following system:

$$-\epsilon_\mu{}^{\nu\rho} D_\nu \psi_\rho + D_\mu \psi_4 = \lambda \psi_\mu, \quad D^\mu \psi_\mu = -\lambda \psi_4 \ . \tag{11}$$

It turns out that $\{\lambda\}$ coincide with the eigenvalues of the Dirac operator

$$i\hat{D}\varphi = \lambda\varphi \ .$$

Indeed, multiplying both sides of this equation by $\bar{\chi}\gamma^\mu$ with $\bar{\chi}$ being constant spinor and using the identity $\gamma^\mu\gamma^\nu = g^{\mu\nu} + i\epsilon^{\mu\nu}{}_\rho\gamma^\rho$, one obtains the relation which after the replacement of variables

$$(\bar{\chi}\gamma_\mu\varphi^a) = \psi^a_\mu, \quad i(\bar{\chi}\varphi^a) = \psi^a_4$$

identically coincides with the first equation of the system (11). The fulfillment of the second equation (11) may be easily checked:

$$D^\mu\psi_\mu + \lambda\psi_4 = -i\bar{\chi}(i\hat{D} - \lambda)\varphi = 0 \ .$$

Thus, for an arbitrary gauge field $A_\mu(x)$ we have:

$$\det \Delta[A] = \left(\prod_k \lambda_k\right)^4 = (\det i\hat{D})^2 .$$

3. PARITY ANOMALY

The properties of the effective action of three dimensional Dirac fermions in the r.h.s. of (9) are well-known [4]. In particular, it contains nonzero imaginary part whose dependence on gauge field $A_\mu(x)$ is nonanalytic. It means that after expansion of the imaginary part in perturbative series near $A_\mu(x) = 0$ some diagrams contributing to the effective action have ultraviolet divergences. Therefore, the calculation of the effective action of fermions within the framework of perturbation theory requires regularization. As usual, one chooses the Pauli-Villars gauge invariant regularization where

$$e^{-\Gamma_f^{reg}[A]} \equiv \det \frac{i\hat{D}}{i\hat{D} + iM} = \left(\prod_k \frac{\lambda_k}{\lambda_k + iM}\right)^2 = \left(\prod_k \frac{\lambda_k^2}{\lambda_k^2 + M^2}\right) \exp\left(-2i\sum_k \arctan\frac{M}{\lambda_k}\right).$$

Hence, in the limit $M \to \infty$ we find [4]

$$\begin{aligned} \mathrm{Im}\Gamma_f[A] &= \pi\,\mathrm{sign}(M)\left(\sum_{\lambda_k>0} 1 - \sum_{\lambda_k<0} 1\right) \\ &= \frac{\pi}{2}\,\mathrm{sign}(M)\eta[A] \\ &= -\mathrm{sign}(M)\frac{c_v(G)}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2}{3}iA_\mu A_\nu A_\rho\right), \end{aligned}$$

where $\eta[A]$ is the Atiyah-Patodi-Singer function [15] for the Dirac operator in the adjoint representation of G.

To make sense of (9), the Chern-Simons theory must be regularized as well. We have to provide regularization that does not spoil the invariance of the theory under large and small gauge transformations. There are problems with dimensional regularization since the action (2) contains ϵ-symbol. Therefore, we are forced to use higher derivative regularization [16] which consists of the introduction to the action of additional explicitly gauge invariant terms with second and higher powers of derivatives. The regularized action cannot contain terms proportional to the Chern-Simons lagrangian since it transforms nontrivially under gauge transformations. The only allowed terms have the form of the product of the strength tensor and covariant derivatives and among them the simplest one is $\frac{1}{2M}\int d^3x\,\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$. Adding this term to (2) we have found that the regularized Chern-Simons action is equal exactly to the action (1) of topological massive gluodynamics with M being the regularization parameter. The removal of regularization corresponds to the limit $M \to \infty$. It must be noted that introduction of the term with higher derivatives, which is scalar, does not spoil gauge invariance of the Chern-Simons theory but parity symmetry (4) of the classical action is broken explicitly. This is the manifestation of the general property: in the quantum Chern-Simons gauge theory it is impossible to maintain simultaneously gauge invariance of the partition function and parity of the action.

Repeating previous consideration of the one-loop effective action we are led to the regularized functional

$$\det\Delta[A] \to \det\Delta^{reg}[A] = \det\left|\Delta_{AB}[A] - \frac{4\pi i}{kM}\begin{pmatrix}(D^2 g_{\mu\nu} - D_\nu D_\mu) & 0\\ 0 & 0\end{pmatrix}\right|, \qquad (12)$$

where $A=(\mu,4)$, $B=(\nu,4)$. To calculate this expression we assume, for simplicity, that the gauge field $A_\mu(x)$ satisfies the classical equation of motion (7): $A_\mu(x)\equiv a_\mu(x)$. Let $\bar\lambda$ be eigenvalues of the following operator

$$L_\mu{}^\rho\varphi_\rho \equiv -\epsilon_\mu{}^{\nu\rho}D_\nu\varphi_\rho = \bar\lambda\varphi_\mu \ ,$$

where $D_\mu = \partial_\mu - i[a_\mu, \ ]$. Some of $\bar\lambda$ are equal to zero with zero modes being D_μ but the remaining nonvanishing ones coincide with the eigenvalues defined in (10). Indeed, eliminating ψ_4 from the system (11) one obtains

$$\left(\epsilon_\mu{}^{\nu\rho}D_\nu + \frac{1}{\lambda}D_\mu D^\rho\right)\psi_\rho = -\lambda\psi_\mu \ . \qquad (13)$$

Multiplying both sides of this equation by $\epsilon_{\alpha\beta}{}^\mu D^\beta$ we derive

$$\epsilon_{\alpha\beta}{}^\mu D^\beta\epsilon_\mu{}^{\nu\rho}D_\nu\psi_\rho = -\lambda\epsilon_{\alpha\beta}{}^\mu D^\beta\psi_\mu$$

since $\epsilon_{\alpha\beta}{}^\mu D^\beta D_\mu = -i\epsilon_\alpha{}^{\beta\mu}F_{\beta\mu}[a] = 0$. In the matrix notation this relation may be rewritten as

$$L\,L|\psi\rangle = \lambda\,L|\psi\rangle \ .$$

Hence, either $L|\psi\rangle$ is null-vector or λ is equal to the eigenvalue $\bar\lambda$ of the operator L:

$$\lambda = \bar\lambda$$

This relation allows us to find the eigenvalues of the regularised operator

$$\Delta^{reg}_{AB}[A]\,\psi^{reg,B} = \lambda^{reg}\psi^{reg}_A, \quad \psi^{reg}_A = (\psi^{reg}_\mu, \psi^{reg}_4)\,.$$

Now ψ^{reg}_4 obeys the equation analogous to (13)

$$\left(\epsilon_\mu{}^{\nu\rho} D_\nu + \frac{4\pi i}{kM}(D^2 g^\rho_\mu - D_\rho D_\mu) + \frac{1}{\lambda} D_\mu D^\rho\right)\psi^{reg}_\rho = -\lambda^{reg}\psi^{reg}_\mu\,.$$

Multiplying as before both sides of this relation by $L^{\alpha\mu}$ and using the identity $D^2 g_{\mu\rho} - D_\rho D_\mu = -L_{\mu\alpha} L^\alpha{}_\rho$ we conclude that

$$\left(L + \frac{4\pi i}{kM} L^2\right) L|\psi^{reg}\rangle = \lambda^{reg}\, L|\psi^{reg}\rangle\,.$$

Thus either $L|\psi\rangle$ is null-vector or the eigenvalues of λ^{reg} are equal to

$$\lambda^{reg} = \bar\lambda + \frac{4\pi i}{kM}\bar\lambda^2 = \lambda + \frac{4\pi i}{kM}\lambda^2\,,$$

where $\bar\lambda$ are nonzero eigenvalues of the operator L.

As a result, the regularised determinant is

$$\begin{aligned}
\det\Delta^{reg}[A] &= \prod_n \left(\lambda_n + \frac{4\pi i}{kM}\lambda_n^2\right)^4 \\
&= \prod_n \left(\lambda_n^2 + \frac{16\pi^2\lambda_n^4}{k^2M^2}\right)^2 \exp\left(4i\arctan\frac{4\pi\lambda_n}{kM}\right) \\
&= \prod_n \left(\lambda_n^2 + \frac{16\pi^2\lambda_n^4}{k^2M^2}\right)^2 \exp\left(-4i\arctan\frac{kM}{4\pi\lambda_n}\right)
\end{aligned}$$

and has a nonvanishing imaginary part.

There is a simple relation (9) between bare values of functionals. Moreover, in the limit $M\to\infty$ when regularization is removed the imaginary part of $\log\det\Delta[a]$ is

$$\begin{aligned}
-\mathrm{Im}\log(\det\Delta[a])^{1/2} &= \pi\,\mathrm{sign}(k)\left(\sum_{\lambda_k>0} 1 - \sum_{\lambda_k<0} 1\right) \\
&\equiv \frac{\pi}{2}\,\mathrm{sign}(k)\,\eta[a] \\
&= -\mathrm{sign}(k)\,\frac{c_v(G)}{4\pi}\int d^3x\,\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(a_\mu\partial_\nu a_\rho - \frac{2}{3} i a_\mu a_\nu a_\rho\right)
\end{aligned}$$

and up to unessential factor it is equal to the imaginary part of the effective action of fermions.

Now, we substitute the received relations into the one-loop expression (8) for the effective action and take into consideration that the ghost functional $\det D^\mu D_\mu$ as well as $|\det\Delta[a]|$ are gauge invariant explicitly and are proportional to the product of the strength tensor and covariant derivatives. Then, with the equation of motion (7) the one-loop effective action of the Chern-Simons gauge theory has a simple form

$$e^{-\Gamma_{CS}[a]} = \exp\left(-(k+\mathrm{sign}(k)c_v(G))\frac{i}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(a_\mu\partial_\nu a_\rho - \frac{2}{3}ia_\mu a_\nu a_\rho\right)\right) \tag{14}$$

in accordance with the Witten conjecture [9].

The following question araises: does this result is changed in higher orders of perturbation theory? There is a simple argument [6] that eq.(14) is exact to all orders of perturbation theory. In the weak coupling region $1/k$ is a small parameter. Calculation of quantum corrections lead to renormalization of the coupling constant

$$k \to k_{ren} = Zk \ ,$$

where $Z = 1 + \frac{z_1}{k} + \frac{z_2}{k^2} + \cdots$. The invariance of the effective action under large gauge transformations implies that $k_{ren} \in \mathbf{Z}$ for arbitrary large k but this condition is fulfilled only for $z_i = 0,\ i \geq 2$.

What do the derived results mean? To calculate the effective actions of the Chern--Simons theory and massless Dirac fermions regularization is needed. It turns out that it is impossible to introduce regularization and maintain all symmetries of the classical theories: gauge invariance and parity (the classical actions of the Chern-Simons theory and massless Dirac fermions are pseudoscalar and scalar, respectively). Under regularization we keep gauge invariance and spoil parity of the regularized action: $S_{reg} = scalar + pseudoscalar$ (term with higher derivatives and massive term of Pauli-Villars regulator are scalar and pseudoscalar, respectively).

In the regularized effective action, the ultraviolet divergences are replaced by the singular dependence on M in the limit $M \to \infty$. However, it was demonstrated that the effective actions in both the theories are finite [4,17], that is ultraviolet divergences are cancelled as $M \to \infty$ but the nonvanishing finite part survives. It is natural to expect that the finite part of the regulator contribution to the effective action has the form of a local operator with dimension 3. Gauge invariance fixes unambiguously the following operator[1]

$$-C\frac{i}{4\pi}c_v(G)\int d^3x\, \epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2}{3}iA_\mu A_\nu A_\rho\right) . \tag{15}$$

In the Chern-Simons nonabelian theory $C = \mathrm{sign}(k)$ and inclusion of (15) leads to additive renormalization of the coupling constant. At the same time, for the Dirac fermions we have $C = -\mathrm{sign}(M)$ and appearance of (15) leads to violation of the parity of the classical action of massless fermions [4].

However, in both cases the parity properties of the effective actions are not spoiled since the definitions (4) and (5) of the parity are different for classical and effective actions. It follows from (5) that the real and imaginary parts of the effective action must be scalar and pseudoscalar, respectively. We note from (15) that the parity anomaly contribution is pseudoscalar and takes pure imaginary values.

In conclusion, it would be interesting to reproduce all the above results performing one-loop calculation of the Feynman diagrams.

[1] There is second permissible operator $\int d^3x\, \epsilon^{\mu\nu\rho}\,\mathrm{Tr}(D_\mu F_{\nu\rho})$ but it is equal to zero due to the Bianchi identity.

4. PERTURBATION THEORY CALCULATIONS

The regularized effective action of massless Dirac fermions is given by

$\Gamma_f[A] =$  $+ \dots$.

To remove regularization we set $M \to \infty$

$$\begin{aligned} \Gamma_f[A] &\equiv -\log\det\frac{i\hat{D}}{i\hat{D}+iM} \\ &= c_v(G)\,\mathrm{Tr}\int\frac{d^3p}{(2\pi)^3}A_\mu(-p)\left(\frac{1}{4\pi}\epsilon^{\mu\nu\rho}p_\nu\,\mathrm{sign}(M)+\frac{g^{\mu\rho}p^2-p^\mu p^\rho}{16\sqrt{p^2}}\right)A_\rho(p)+\cdots \quad (16) \end{aligned}$$

where dots denote terms with higher powers of gauge field.

To calculate the Chern-Simons effective action let us perform power-counting analysis of the Feynman diagrams. In the pure Chern-Simons theory there are five different vertices: three-gluon (AAA), ghost-gluon ($\bar{c}cA$), ghost-two-gluon ($\bar{c}cAA$) and vertices corresponding to interaction of the Lagrange multiplier with the gauge fields (ϕA) and (ϕAA). A simple calculation of the superficial degree of divergence of a diagram G yields:

$$\omega(G) = 3 - L_A - \frac{1}{2}L_c \quad ,$$

where L_a denote the number of external lines of the corresponding type. So only two- and three-point functions with $\omega(G) \le 0$ are formally ultraviolet divergent to any order of the coupling constant. To regularize divergences, the term with higher derivatives is added in (2). Then, new three- and four- gluon vertices induced by the Yang-Mills action appear in diagrams and the ultraviolet behaviour of the gluon propagator is changed:

$$D_{\mu\nu}(p) = \frac{1}{p^2\left(p^2+(\frac{kM}{4\pi})^2\right)}\left(-\frac{kM^2}{4\pi}\epsilon_{\mu\rho\nu}p^\rho + M(g_{\mu\nu}p^2 - p_\mu p_\nu)\right) \stackrel{p^2\to\infty}{\longrightarrow} \frac{1}{p^2} \quad .$$

The superficial degree of divergence in the regularized theory is equal to

$$\omega(G) = 3 - \frac{3}{2}N^o_{AAA} - \frac{1}{2}N^e_{AAA} - \frac{1}{2}N_{\bar{c}cA} - \frac{1}{2}N_{\phi AA} - N_{\bar{c}cAA} - N_{AAAA} - \frac{1}{2}L_{ext} \quad ,$$

where L_{ext} is the number of external lines, N_a denotes the number of different vertices with N^o_{AAA} and N^e_{AAA} corresponding to "odd" (or Chern-Simons) and "even" (or Yang–Mills) three gluon vertices, respectively. Solving the equation $\omega(G) \le 0$ one finds that only a finite part of one- and two-loop diagrams are divergent. Moreover, N^o_{AAA} can take only two values: 0 and 1. At $N^o_{AAA} = 1$ the condition $\omega(G) \le 0$ fixes the following nonzero parameters of diagrams: $L_{ext} = 2$, $N^e_{AAA} = 1$ or $L_{ext} = 2$, $N_{\phi AA} = 1$. These diagrams have $\omega(G) = 0$ but a detailed calculation gives finite results. So to be potentially divergent the diagram has to have no Chern-Simons three gluon vertices and

$$\omega(G) = 3 - \frac{1}{2}N^e_{AAA} - \frac{1}{2}N_{\bar{c}cA} - \frac{1}{2}N_{\phi AA} - N_{\bar{c}cAA} - N_{AAAA} - \frac{1}{2}L_{ext} \quad . \qquad (17)$$

It may be easily seen that this expression is equal exactly to the superficial degree of divergence of diagrams contributing to the effective action of Yang-Mills gauge theory in the background field gauge. It is well known [5,18] that the three dimensional Yang-Mills theory is finite. Hence, to regularize one- and two-loop divergent diagrams whose superficial degree of divergence is given by (17) the dimensional regularization with $D = 3 - \epsilon$ may be used [6]. Then, ultraviolet poles in $1/\epsilon$ are cancelled in the sum of all diagrams and there is no any finite contribution induced by regulators.

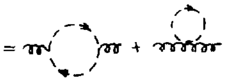
Now, to the lowest order of perturbation theory we have

$\log(\det \Delta[A])^{-1/2} =$  ,

where the internal wave and continuous lines denote gauge field and Lagrange multipliers, respectively, and the cross on the line corresponds to the interaction term $\phi\partial^\mu B_\mu$. In the limit $M \to \infty$ we get the final result in momentum representation

$$\log(\det \Delta[A])^{-1/2} = c_v(G)\,\mathrm{Tr}\int \frac{d^3p}{(2\pi)^3} A_\mu(-p)\left(\frac{1}{4\pi}\epsilon^{\mu\nu\rho}p_\nu\,\mathrm{sign}(k) + \frac{g^{\mu\rho}p^2 - p^\mu p^\rho}{16\sqrt{p^2}}\right)A_\rho(p). \tag{18}$$

Analogously, the ghost functional is equal to

$\log\det(D^\mu D_\mu) =$ 

$$= c_v(G)\,\mathrm{Tr}\int \frac{d^3p}{(2\pi)^3} A_\mu(-p)\frac{g^{\mu\nu}p^2 - p^\mu p^\nu}{16\sqrt{p^2}}A_\nu(p)\ , \tag{19}$$

where the dashed line denotes a ghost. Combining eqs. (18) and (19) one receives

$$\log\frac{\det(D^\mu D_\mu)}{(\det \Delta[A])^{1/2}} = c_v(G)\,\mathrm{Tr}\int \frac{d^3p}{(2\pi)^3} A_\mu(-p)\left(\mathrm{sign}(k)\frac{1}{4\pi}\epsilon^{\mu\rho\nu}p_\rho + \frac{g^{\mu\nu}p^2 - p^\mu p^\nu}{8\sqrt{p^2}}\right)A_\nu(p). \tag{20}$$

The second term in this expression is a nonlocal (in x-space) infrared singular functional whose properties were studied in [5,6,18]. With higher order corrections being included, it is proportional to the strength tensor, and therefore, does not contribute to the effective action. Substituting (20) into (8) we reproduce eq.(14). It is interesting to note that for an arbitrary gauge field and $M, k > 0$ relation (9) is fulfilled not only for bare expressions but for the renormalized functionals (16) and (18) also.

Thus, we conclude that perturbation theory calculations confirm the result obtained before.

5. CONCLUSION

The $D = 3$ Chern-Simons gauge theory is a finite theory and ultraviolet divergences are cancelled in the sum of diagrams contributing to the effective action. Nevertheless, for quantum theory to make sense it must be regularized. The regularization conserves gauge invariance but spoils inevitably the parity of the classical action. The regularized effective action depends on the regularization parameter M. In the limit $M \to \infty$ when reguiarization is removed, the singular dependence of the effective action on M disappears but the finite contribution induced by parity violating regulators survives. This contribution being added to the classical action leads to additive renormalization of the coupling constant.

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