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RADIATIVE DECAY $\tau\to\nu_\tau\pi\gamma$

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There are many papers dealing with both theoretical and experimental description of the $\pi\to e\bar{\nu}_e\gamma$ process (see [1,2] and references therein). The structural part of the amplitude is most attractive. It is characterized by the vector (h_V) and axial-vector (h_a) form factors. The former is determined by the $\rho\omega\pi$ vertex of the anomalous type. Even in the q^2 approximation this vertex describes the decays like $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$ и $\eta'\to\gamma\gamma$ in the energy region $0\le q^2\le 1$ GeV well. The other form factor is associated with a less studied vertex $a_1\pi\rho$ with the main contribution coming from q^2 terms as well [2]. We shall assume that in this case the q^2 approximation also holds good for energy region $0\le q^2\le 1$ GeV. It is proved by the calculations of different processes (see below). The expression obtained in [2] for the $a_1\pi\rho$ vertex allows one to explain the experimental value of the ratio $\gamma=h_a/h_V$ in the decay $\pi\to e\bar{\nu}\gamma$, which was a great difficulty in the standard quark models [3].

The $\tau\to\nu_\tau\pi\gamma$ decay is close to the $\pi\to e\bar{\nu}_e\gamma$ process in its nature. The invariant mass of the $\pi\gamma$ system can rather large because $m_\tau\gg m_\pi$. This leads to a large emission of photons from the structural part of the hadron vertex. These γ-quanta result from the resonant exchange with $\rho(770)$ and $a_1(1260)$ mesons saturating the weak vector and axial-vector hadron currents. They can be separated by registering the $\pi\gamma$ events with a large invariant mass. It becomes possible to study events of the structural origin at a cτ-factory [4]. According to our estimations at the luminosity $L=10^{33}cm^{-2}s^{-1}$, one can expect about 10^3 events/year from the structural part of the axial-vector vertex $a_1\pi\rho$.

In the theoretical consideration of the radiative decay $\tau\to\nu_\tau\pi\gamma$ one uses the same hadron vertices $\rho\omega\pi$ and $a_1\pi\rho$ as in the consideration of the $\pi\to e\bar{\nu}_e\gamma$ decay. Our analysis is based on the assumption that the functional form of the vertices studied does not practically change despite the higher energies transferred to the hadron block ($Q^2 \le m_\tau^2$). Indeed, as mentioned above, the anomalous vertices describe the meson processes at the energies of the order of 1 GeV well too. (See, for example, Ref.[5], where the decays of π^0, η and η' mesons are calculated.) We have already used the $a_1\pi\rho$ vertex in the q^2 approximation to describe a number of processes with the typical meson energies $\approx$1 GeV. This are, first of all, the $a_1\to\pi\rho$, $a_1\to\pi\gamma$ and $\tau\to\nu_\tau 3\pi$ decays [2,6,7]. Satisfactory agreement of the theoretical estimations for these processes with the experimental data indirectly justifies this approximation.

After these general remarks we turn to the discussion of the explicit form of the amplitude which we used for further calculations. The diagrams describing this process are given in Fig.1. The amplitude can be divided into two parts. One part describes the internal bremsstrahlung T^{IB} and the other is the structural part of special interest T^{SD}. *)

$$T=T^{IB}+T^{SD},$$

$$T^{IB} = -iem_\tau F_\pi G_F \cos\theta_C \varepsilon^*_\mu(q)\bar{\nu}_\tau(p')(1+\gamma_5)\left[\frac{k^\mu}{(kq)} - \frac{p^\mu}{(pq)} + i\gamma^{\mu\nu}\frac{q_\nu}{2(pq)}\right]\tau(p),$$

$$T^{SD} = ieF_\pi G_F \cos\theta_C \varepsilon^*_\mu(q)[\bar{\nu}_\tau(p')(1+\gamma_5)\gamma_\nu\tau(p)]$$
$$\left[h_a(Q^2)(k^\mu q^\nu - kq\cdot g^{\mu\nu}) + ih_v(Q^2)\varepsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma\right]. \qquad (1)$$

Here e is the electric charge of the particle; G_F is the Fermi constant; F_π=93 MeV is the weak pion decay constant; θ_C is the Cabbibo angle; p,p',k,q are the 4-momenta of the τ-lepton, τ-neutrino, pion and photon respectively; $\gamma^{\mu\nu}=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$; $\varepsilon^\mu(q)$

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*) We omit the details of the calculation. Partially they can be found in [7]. Completely they will be given in our separate paper.

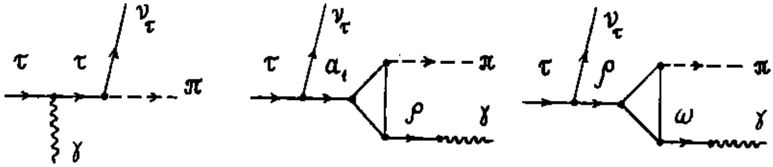

Fig. 1 Diagrams describing the radiative decay process $\tau\to\nu_\tau\pi\gamma$ in QMST.

is the photon polarization vector; the axial and vector form factors are equal: $h_a(Q^2)=m^2_{a_1}/[8\pi^2F^2_\pi Z(m^2_{a_1}-Q^2-im_{a_1}\Gamma_{a_1})]$, $h_V(Q^2)=m^2_\rho/[8\pi^2F^2_\pi(m^2_\rho-Q^2-im_\rho\Gamma_\rho)]$, and $Q=p-p'$. In the quark model of the superconducting type (QMST) [8], which we used to obtain formula (1), the constant Z enters into the Lagrangian that describes the interaction of the weak charges lepton current with hadrons [9]

$$L_W = \frac{G_F}{g_\rho}\cos\theta_C[\bar{\nu}\gamma^\mu(1-\gamma_5)\tau]\left[m^2_\rho\rho^+_\mu + Z^{-1}m^2_{a_1}a^+_{1\mu} - g_\rho F_\pi\partial_\mu\pi^+\right]+\text{h.c.}, \qquad (2)$$

and then goes to the form factor $h_a(Q^2)$. The coupling constant g_ρ characterizes the $\rho\to\pi\pi$ decay and is equal to $g^2_\rho/4\pi=3$. The value $Z=(1-6m^2/m^2_{a_1})^{-1}$ obtained in the QMST, where m is the mass of the constituent u-quark, can be expressed only through the observed quantities $2Z^{-1}=1+[1-(2g_\rho F_\pi/m_{a_1})^2]^{1/2}$.

The Lagrangian describing the radiative transitions $\rho^-\to\pi\gamma$ and $a_1^-\to\pi\gamma$ has the following form in the QMST [6]

$$L_R = \frac{eg_\rho}{32\pi^2F_\pi}\left[\varepsilon^{\mu\nu\kappa\sigma}\rho^-_{\kappa\sigma}(x)-2ia_1^{-\mu\nu}(x)\right]F_{\mu\nu}(x)\pi^+(x), \qquad (3)$$

where $F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)$, and $A_\mu(x)$ is the electromagnetic field. As a result, the model helps us to fix the values of the form factors at point $Q^2=0$: $h_a(0)=1/8\pi^2F^2_\pi Z$, $h_V(0)=1/8\pi^2F^2_\pi$.

Remember that formula (1) can also be obtained directly from the amplitude of the kindred process $\pi\to e\bar{\nu}_e\gamma$. In this sense the

QMST allows one only to fix the parameter Z and to understand why it appears. As seen, it is this parameter that determines the ratio $\gamma=h_a(0)/h_v(0)=Z^{-1}$ measured in the reaction $\pi\to e\bar{\nu}_e\gamma$.

If one performs the standard operations of averaging over the initial polarizations of the τ-lepton and summing over the final spin states, the matrix element squared will be equal to

$$
\begin{aligned}
|T|^2 = (2em_\tau F_\pi G_F\cos\theta_c)^2 & \left\{ \frac{qp'}{qp}\left[1-\frac{m_\tau^2-m_\pi^2}{2qk}\left(2-\frac{m_\tau^2}{qp}+\frac{m_\pi^2}{qk}\right)\right] - \right. \\
& - m_\tau(qp')\left[(h_a+h_a^*)\left(\frac{kp+kp'}{qp}-\frac{m_\pi^2}{qk}\right)-(h_v+h_v^*)\frac{qk}{qp}\right] + \\
& + 2(h_a h_v^*+h_v h_a^*)(qk)[(qp)(kp')-(kp)(qp')] + \\
& \left. + 2(|h_a|^2+|h_v|^2)\left[qk[(qp)(kp')+(kp)(qp')]-m_\pi^2(qp)(qp')\right]\right\}. \quad (4)
\end{aligned}
$$

Our next task is to calculate spectral distributions and to separate regions where the structural radiation dominates. For this purpose we analyse four spectra:

$$
\frac{d\Gamma_{\tau\to\nu\pi\gamma}}{dQ^2\,\Gamma_{\tau\to\nu\pi}}, \qquad \frac{d\Gamma_{\tau\to\nu\pi\gamma}}{d\cos\theta\ \Gamma_{\tau\to\nu\pi}}, \qquad \frac{d\Gamma_{\tau\to\nu\pi\gamma}}{dE_\gamma\Gamma_{\tau\to\nu\pi}}, \qquad \frac{d\Gamma_{\tau\to\nu\pi\gamma}}{dE_\pi\Gamma_{\tau\to\nu\pi}}.
$$

Here E_γ , E_π and θ are the photon energy, the pion energy and the angle of pion and photon emission in rest frame of a decaying τ-lepton. The resulting curves are shown in Fig.2-5 respectively. Before discussing them, we shall give the branching of τ decaying by this mode: $Br(\tau\to\nu\pi\gamma)=0.15\%$. Since the total probability diverges (the infrared catastrophy), we have done this estimation on the assumption that the registered photons have the energy not less than 50 MeV. The dependence on the minimal registered photon energy is shown in Fig.6. For the a_1-mesón parameters we used the following values: $m_{a_1}=1260$ MeV, $\Gamma_{a_1}=400$ MeV.

It is seen from Fig.2 that the contribution of the structural part of the axial-vector current has a pronounced resonant form and is not disguised by the internal bremsstrahlung. Its branching

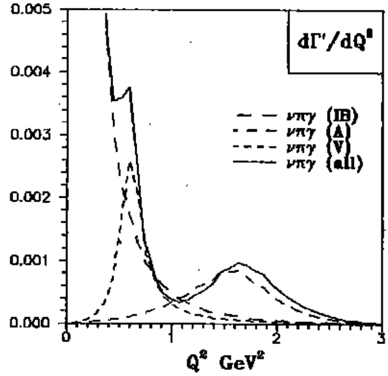

Fig. 2 The invariant mass spectrum of the $\pi\gamma$ system $d\Gamma_{\tau\to\nu_\tau\pi\gamma}/dQ^2\Gamma_{\tau\to\nu_\tau\pi}$. Here and in other figures the symbols IB, A, V, all denote the contributions from the internal bremsstrahlung, the axial-vector $a_1(1260)$ meson, the vector $\rho(770)$ meson, and their total contribution.

is $Br^{ASD}(\tau\to\nu\pi\gamma)=0.92\cdot10^{-2}\%$. If one takes into account that the number of $\tau^+\tau^-$-pairs produced at the cτ-factory is $5.6\cdot10^7$ pairs/year, one can expect about $5\cdot10^3$ events of this kind per year.

The majority of $\pi\gamma$-pairs produced by the axial-vector structural part fly away at an angle close to 180^0. Using Fig.3 one can find that the events with the angle θ in the interval $-1\le\cos\theta\le-0.7$ produced owing to T^{ASD} will amount to 10^3 per year of cτ-factory operation.

The photon energy spectrum shown in Fig.4 is sensitive to the structural part, though there is quite a noticeable admixture of the internal bremsstrahlung. Choosing the events at the end of the spectrum one can detect about 10^3 structural photons of high energy in a year of cτ-factory operation.

As seen from Fig.5, the pion spectrum is not worth studying, since here the structural contribution is completely screened by the internal bremsstrahlung events.

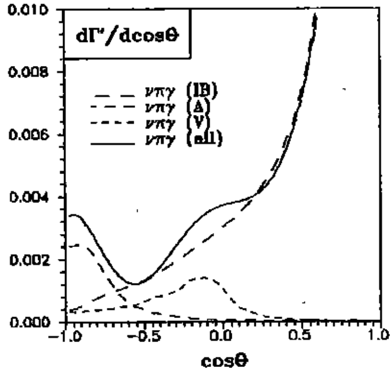

Fig. 3 The angular distribution $d\Gamma_{\tau\to\nu_\tau\pi\gamma}\ /d\cos\theta\Gamma_{\tau\to\nu_\tau\pi}$.

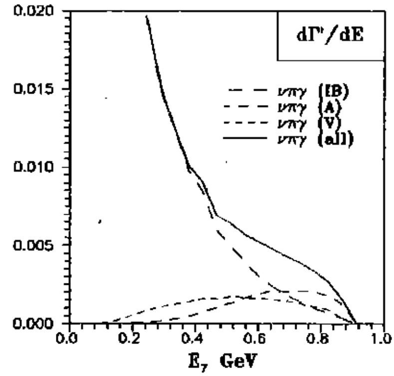

Fig. 4 The photon energy spectrum $d\Gamma_{\tau\to\nu_\tau\pi\gamma}\ /dE_\gamma\Gamma_{\tau\to\nu_\tau\pi}$.

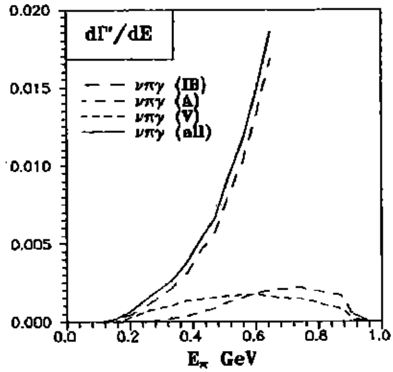

Fig. 5 The pion energy spectrum $d\Gamma_{\tau\to\nu_\tau\pi\gamma}\ /dE_\pi\Gamma_{\tau\to\nu_\tau\pi}$.

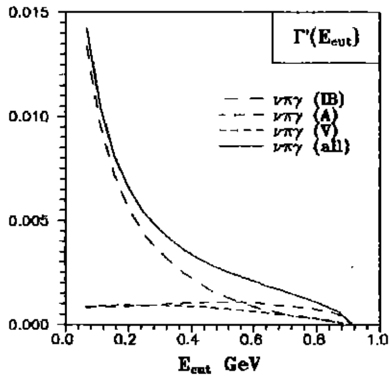

Fig. 6 The ratio $\Gamma_{\tau\to\nu_\tau\pi\gamma}/\Gamma_{\tau\to\nu_\tau\pi}$ plotted against the value of the minimal registered photon energy (E_{cut}).

In conclusion we would like to point out that the final conclusion on a possibility of studying this process at modern facilities of the cτ-factory can be drawn only after a thorough analysis of the background conditions. This work is well under way; its results will be reported separately.

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