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*On leave of absence from Central Research Institute for Physics, Budapest, Hungary

In this paper we present a possible modification of parton models for inelastic processes at very large, but finite, energies. As a result, we find that the scaling terms in an asymptotic expansion for the deep inelastic structure function are only the zeroth order approximation with respect to a parameter which may possibly be a constant characteristic of processes at future energies.

The modification we want to propose is based upon the combination of the following two facts. It is known that the scaling property of the deep-inelastic structure functions can be derived in simple models which are assumed to have the symmetry properties of a two-dimensional non-relativistic (in what follows, 2-Galilean) theory [1]. The mapping of the originally three-dimensional relativistic (3-Poincaré covariant) theory onto the 2-Galilean one is defined by transforming the scattering system into the infinite momentum frame (IMF). A careful investigation of the problem of how to define the transformation properties of various physical quantities in the IMF [2,3] has led us to the conclusion that the transformation into the IMF is a hidden way of using the method of group contraction for establishing the above-mentioned mapping. On the basis of this conclusion we have been able to describe the mapping of 3-Poincaré covariant current matrix elements onto current matrix elements in a 2-dimensional relativistic (2-Poincaré covariant) theory /3/.

The symmetry group of the two-dimensional theory is generated by

$$K_1 = \lambda(M_2 + N_1) - \frac{1}{4\lambda c^2}(M_2 - N_1) \ ,$$

$$K_2 = -\lambda(M_1 - N_2) + \frac{1}{4\lambda c^2}(M_1 + N_2) \ , \qquad (1)$$

$$H = \frac{1}{4\lambda}(P_0 + P_3) + \lambda c^2 (P_0 - P_3)$$

and M_3 , $\underline{P}_\perp \equiv (P_1 \ , P_2)$, the generators M_i , N_i , i=1,2,3, and P_μ, $\mu = 0,1,2,3$, being those of the 3-Poincaré group, the symmetry group of the three-dimensional theory in the ordinary reference frame. No simple kinematical argument tells us the value of the parameter $c > 0$. It is a kind of "light velocity" from the point of view of the 2-Poincaré algebra:

$$[K_1 \ , K_2] = -i \frac{1}{c^2} M_3 \ , \quad [M_3 \ , K_i] = i\,\varepsilon_{ij} K_j \ , \ i,j=1,2,$$

$$[K_i \ , P_j] = i \frac{1}{c^2} \delta_{ij} H, \quad [K_i \ , H] = i P_i \ , \qquad (2)$$

$$[M_3 \ , P_i] = i\,\varepsilon_{ij} P_j \ , \quad [M_3 \ , H] = [P_i \ , P_j] = [H, P_i] = 0.$$

That Casimir operator of the 2-Poincaré algebra which corresponds to the "mass" can be written as

$$\frac{1}{c^2} H^2 - \underline{P}_\perp^2 = P_\mu P^\mu + \mu^2 c^2 \ , \qquad (3)$$

where

$$\mu = \lambda(P_0 - P_3) - \frac{1}{4\lambda c^2}(P_0 + P_3) \ . \qquad (4)$$

In the limit $c \to \infty$, for every fixed value of $\lambda > 0$, the formulas (1-4) reproduce the quantities of the 2-Galilei group advocated in ref. 1.

The other parameter, $\lambda > 0$, does not appear explicitly in the formulas (2) and (3). A detailed analysis of the contraction procedure shows that the arbitrariness of λ is an inherent freedom in the mapping of the three-dimensional theory onto the two-dimensional one /3/. It is natural to assume that the predictions of the two-dimensional theory are independent of λ . This is a natural generalization of the corresponding scaling property of the two-dimensional Galilean theory in the IMF /1/.

In what follows we consider the process

$$e + p \longrightarrow e + \text{anything} \ . \qquad (5)$$

We shall asume that for large incoming energies what actually happens is reflected by a two-dimensional theory, its symmetry group being generated by the operators (1). For simplicity, we assume that the protons have zero spin and a scalar current, $J(x)$, intermediates the interaction between the electrons and protons. Consequently, the inclusive cross section for process (5) is determined by the structure function

$$W(q^2 \ , pq) = \int e^{ixq} \langle p | J(x) J(0) | p \rangle \, d^4 x \ . \qquad (6)$$

Instead of (6) we shall consider the auxiliary quantity

$$\hat{W} = \int e^{ixq} \langle p | J(x) J(0, \underline{Q}_\perp, \mathfrak{z}') | p' \rangle \, d^4 x \, d\mathfrak{z}' \qquad (7)$$

in a reference frame, where

$$\underline{p}_\perp = \underline{p}'_\perp , \qquad q_- \equiv q_c - q_3 = 0 . \tag{8}$$

In eq. (7) the notations

$$\mathcal{T} = t + z , \qquad 2\mathfrak{z} = z - t , \qquad d^4x = d\mathcal{T}\, d^2\underline{x}_\perp d\mathfrak{z}$$

are used. It is easy to verify that, if (8) fulfills,

$$\hat{W} = 2\pi\,\delta(p_- - p'_-)\, W . \tag{9}$$

Let us notice that, since $xq = \frac{1}{2} q_+\mathcal{T} + \underline{q}_\perp \underline{x}_\perp$, $\hat{W}$ can be expressed in terms of the "transverse current"

$$j(\mathcal{T}, \underline{x}_\perp) \equiv \int_{-\infty}^{\infty} J(\mathcal{T}, \underline{x}_\perp, \mathfrak{z})\, d\mathfrak{z} \tag{10}$$

as follows:

$$\hat{W} = \int e^{i\hat{x}\hat{q}} \langle p | j(\hat{x}) j(\hat{0}) | p' \rangle\, d^3\hat{x} , \tag{11}$$

where $\hat{x} = (\mathcal{T}, \underline{x})$, $d^3\hat{x} = d\mathcal{T}\, d^2\underline{x}_\perp$, $\hat{x}\hat{q} = xq$.

Our basic assumption is that in the IMF (moving in the z-direction) the function $\hat{W}$ can be described in terms of 2-Poincaré covariant quantities. This description follows from the mapping of the transverse current matrix element

$$\mathcal{M} = \langle p' | j(\mathcal{T}, \underline{x}_\perp) | p \rangle = \\ = 2\pi\, e^{i\frac{1}{2}\mathcal{T}(p'_+ - p_+) + i(\underline{p}'_\perp - \underline{p}_\perp)\underline{x}_\perp} F(m', m, (p'-p)^2)\, \delta(p'_- - p_-) \tag{12}$$

onto the matrix element of a 2-Poincaré scalar current $S(\underset{\sim}{x})$, $\underset{\sim}{x} = (x^c, \underline{x}_\perp)$, between 2-Poincaré states $|m, \mu, \underset{\sim}{k}\rangle$ and $|m', \mu', \underset{\sim}{k}'\rangle$ of "mass" $\frac{1}{c}\left[(m^2 + \mu^2c^2)\right]^{1/2}$ and $\frac{1}{c}\left[(m'^2 + \mu'^2c^2)\right]^{1/2}$, respectively:

$$\mathcal{M}_{IMF} = \langle \underset{\sim}{k}', \mu', m' | S(\underset{\sim}{x}) | \underset{\sim}{k}, \mu, m \rangle = \\ = e^{i\underset{\sim}{x}(\underset{\sim}{k}' - \underset{\sim}{k})} F_{IMF}(m, m', \mu, \mu'; (\underset{\sim}{k} - \underset{\sim}{k}')^2). \tag{13}$$

Without repeating the argumentation, we cite the mapping of (12) onto (13), established in ref. 3. The current $S(\underset{\sim}{x})$ is related to $j(\mathcal{T}, \underline{x}_\perp)$ by

$$S(\underset{\sim}{x}) \equiv j(\frac{1}{2\lambda} x^\circ, \underline{x}) .$$

The state $|m, \mu, \underset{\sim}{k}\rangle$ is a spinless momentum eigenstate in that subspace of the irreducible 3-Poincaré representation space of mass m, which corresponds to the "mass"

$$\frac{1}{c^2}\underset{\sim}{k}^2 = \frac{1}{c^2}(\frac{1}{c^2}k_c^2 - \underline{k}_\perp^2) = \frac{1}{c^2}(m^2 + \mu^2c^2)$$

for the 2-Poincaré group generated by the operators (1). The mapping between the 3-Poincaré and 2-Poincaré momenta is defined as follows:

$$2\lambda c\, p_- = \mu c + \left[m^2 + \mu^2c^2\right]^{1/2} , \tag{14}$$

$$2\lambda c\, p'_- = \mu' c + \frac{m^2 + \mu^2c^2 + m'^2 + \mu'^2c^2 - (\underset{\sim}{k}' - \underset{\sim}{k})^2}{2\,(m^2 + \mu^2c^2)^{1/2}} , \tag{15}$$

$$(p - p')^2 = (\underset{\sim}{k}' - \underset{\sim}{k})^2 - (\mu' - \mu)^2 c^2 , \tag{16}$$

$$\underline{k}_\perp = \underline{p}_\perp , \qquad \underline{k}'_\perp = \underline{p}'_\perp . \tag{17}$$

Finally, the form factors" F_{IMF} and F are related in the following manner:

$$F_{IMF}(m, m', \mu, \mu'; (\underset{\sim}{k} - \underset{\sim}{k}')^2) = 2\pi F(m, m'; (p - p')^2)\, \delta(p_- - p'_-). \tag{18}$$

This mapping is a generalization of the usual formulas corresponding to 2-Galilean symmetry in the IMF /1/. It is easy to verify that the Galilean formulas are reproduced by eqs. (14-18) in the limit $c \to \infty$.

Now we transform the quantity $\hat{W}$ into the IMF, that is, we map $\hat{W}$ onto $\hat{W}_{IMF}$, $\hat{W}_{IMF}$ being given in terms of 2-Poincaré covariant quantities. In order to make our formulas plausible we repeat the recipe for finding out the mapping (12-18). First, one considers (12) in that special case, when $\underline{p}_\perp = 0$, $\check{\underset{\sim}{p}}' = (0, \check{p}'_\perp)$. In this case eqs. (13-18) follow simply from the definitions (1-4). Then one applies a 2-Poincaré transformation

$$\underset{\sim}{\Lambda} = e^{-i\Theta M_3}\, e^{-i\alpha K_2}\, e^{-i\Theta' M_3} ,$$

which gives the three-momenta $\underset{\sim}{k} = (c\sqrt{m^2 + \mu^2 c^2 + \underline{k}_\perp^2}, \underline{k}_\perp)$, $\underset{\sim}{k}' = (c\sqrt{m'^2 + \mu'^2 c^2 + \underline{k}_\perp'^2}, \underline{k}'_\perp)$, when applied to the ones $\check{\underset{\sim}{k}} = (c\sqrt{m^2 + \mu^2 c^2}, \underline{0}_\perp)$, $\check{\underset{\sim}{k}}' = (c\sqrt{m'^2 + \mu'^2 c^2 + \check{p}_\perp'^2}, 0, \check{p}'_\perp)$.

Applying this recipe to $\hat{W}$ one obtains:

$$\hat{W}_{IMF} = 2\lambda \int e^{i\underset{\sim}{x}\underset{\sim}{q}} \langle m, \mu, \underset{\sim}{k} | S(\underset{\sim}{x}) S(\underset{\sim}{0}) | m', \mu', \underset{\sim}{k}' \rangle \, d^3\underset{\sim}{x} , \tag{19}$$

where

$$2\lambda c p_- = \mu c + (m^2 + \mu^2 c^2)^{1/2} , \tag{20}$$

$$2\lambda c p'_- = \mu' c + (m^2 + \mu'^2 c^2)^{1/2} , \tag{21}$$

$$\underline{k}_\perp = p_\perp , \quad \underline{k}'_\perp = \left[\frac{m'^2 + \mu'^2 c^2}{m^2 + \mu^2 c^2}\right]^{1/2} \underline{p}_\perp , \tag{22}$$

$$\underset{\sim}{q} = \underset{\sim}{\Lambda}\, \check{\underset{\sim}{q}} , \quad \check{\underset{\sim}{q}} = \left(\frac{1}{4\lambda} q_+ , \underline{q}_\perp\right) . \tag{23}$$

The 2-Lorentz transformation $\underset{\sim}{\Lambda}$ in (23)

$$\underset{\sim}{\Lambda} = e^{-i\Theta M_3}\, e^{-i\alpha K_2}$$

is determined by the equation

$$\underset{\sim}{k} = \underset{\sim}{\Lambda}\check{\underset{\sim}{k}} , \quad \check{\underset{\sim}{k}} = (c\sqrt{m^2 + \mu^2 c^2}, \underline{0}_\perp) .$$

Inserting a complete set of states between the two currents in (19),

$$\int_{M_0}^{\infty} dM^2 \int_{-\infty}^{\infty} d\mu'' \int_{-\infty}^{\infty} c^2 \frac{d^2 \underline{k}''_\perp}{2k''_0} \left| M^2; \mu'', \underset{\sim}{k}'' \right\rangle \left\langle \underset{\sim}{k}'', \mu'', M^2 \right| ,$$

where M is the 3-Poincaré mass of the states, one can analyse $\hat{W}_{IMF}$ with the help of the matrix elements $\mathfrak{M}_{IMF}$. The calculation (most conveniently performed in a reference frame, where $\underline{k}_\perp = 0$) is, actually, integration by making use of Dirac-delta functions. The result is:

$$\hat{W}_{IMF} = 2\lambda \left(1 + \frac{\mu c}{\sqrt{m^2 + \mu^2 c^2}}\right)^{-1} \delta(\mu - \mu')\, \tilde{W}(c, \mu, \underset{\sim}{q}^2, \underset{\sim}{kq}), \tag{24}$$

where

$$\underset{\sim}{q}^2 = q^2 + \frac{(qp)^2}{(\mu c + \sqrt{m^2 + \mu^2 c^2})^2} , \tag{25}$$

$$\underset{\sim}{qk} = qp \left(1 + \frac{\mu c}{\sqrt{m^2 + \mu^2 c^2}}\right)^{-1} . \tag{26}$$

By comparison of (9) and (24) we can make the following identification:

$$W(q^2, pq) = \tilde{W}(c, \mu, \underset{\sim}{q}^2, \underset{\sim}{kq}) . \tag{27}$$

So far the only consequence of assuming that a two-dimensional framework is proper to calculate $W(q^2, pq)$ has been eq. (27), which says that W depends on q^2 and pq only via the combinations

(25), (26). By expanding into series in powers of $\frac{1}{c^2}$ one obtains:

$$W(q^2, pq) \approx \tilde{f}(\mu_0, q^2, \tfrac{1}{2}pq) + \frac{1}{c^2}\frac{(pq)^2}{4\mu_0}\frac{\partial \tilde{f}(\mu_0, q^2, \frac{1}{2}pq)}{\partial q^2} \tag{28}$$

$$+ \frac{1}{c^2}(pq)\frac{m^2}{4\mu_0^2}\frac{\partial \tilde{f}(\mu_0, q^2, \frac{1}{2}pq)}{\partial (pq)} - \frac{1}{c^2}\frac{m^2}{4\mu_0}\frac{\partial \tilde{f}(\mu_0, q^2, \frac{1}{2}pq)}{\partial \mu_0}$$

$$+ \frac{1}{c^2}g(\mu_0, q^2, \tfrac{1}{2}pq) + \dots\dots\dots ,$$

where

$$\mu = \mu_0 - \frac{1}{c^2}\frac{m^2}{4\mu_0} ,$$

and the notations

$$\tilde{W}(c, \mu, \underset{\sim}{q}^2, \underset{\sim\sim}{kq})\Big|_{c=\infty} \equiv \tilde{f}(\mu_0, q^2, \tfrac{1}{2}pq) ,$$

$$\frac{\partial^2 \tilde{W}(c, \mu, q^2, kq)}{\partial(\frac{1}{c^2})}\Bigg|_{c=\infty} \equiv g(\mu_0, q^2, \tfrac{1}{2}pq)$$

are introduced, and it is assumed that there are no terms of order $\frac{1}{c}$. Obviously, the function $\tilde{f}(\mu_0, q^2, \frac{1}{2}pq)$ coincides with $W(q^2, pq)$ if the 2-Galilei group is chosen as the symmetry group in the IMF. It is, by assumption, independent of λ, consequently, it does not depend on μ_0. Now we assume, that the proton is a composite object of some elementary constituents which transform with respect to the irreducible representations of the 2-Poincaré group. Moreover, we assume that in the limit $c \to \infty$ the dynamical properties of this composite system coincide with those of the Galilean parton model as formulated, e. g., by Kogut and Susskind in ref. 1. It follows that, when $|q^2| \gg m^2$, $|pq| \gg m^2$, for the function

$\tilde{f}(\mu_0, q^2, \frac{1}{2}pq)$ an approximation

$$\tilde{f}(\mu_0, q^2, \tfrac{1}{2}pq) \approx f(\omega) , \qquad \omega = \frac{pq}{2q^2} \tag{29}$$

is valid. Among the terms of order $\frac{1}{c^2}$ we find $g(\mu_0, q^2, \frac{1}{2}pq)$. For this function the Galilean model says nothing, it can be predicted only if more detailed properties of the 2-Poincaré covariant model are specified. Nevertheless, a reasonable approximation can be obtained from the requirement that also the terms of order $\frac{1}{c^2}$ must give a contribution independent of λ. It seems natural to assume that by an appropriate choice of μ_0 the term containing g can be made negligible in comparison with the other ones of order $\frac{1}{c^2}$. These assumptions yield the following approximate relation for $W(q^2, pq)$:

$$W(q^2, pq) \approx f(\omega) + \frac{m^2}{M^2}\omega f'(\omega) - 2\frac{(pq)}{M^2}\omega f'(\omega) + \dots \tag{30}$$

In this formula the quantity M^2 must be phenomenologically determined. It certainly depends on q^2 and pq. Obviously, eq. (30) has some phenomenological value only if this dependence is weak.

The ideas we presented in this paper serve only as an illustration for what happens when one property, the 2-Galilean symmetry, of the parton models is changed. Obviously, in order to obtain more definite results a more detailed model must be formulated.

References

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