# ОБЪЕАИНЕННЫЙ ИНСТИТУТ คАЕРНЫX ИССАЕАОВАНИЙ 

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DYNAMICAL AFFINE SYMMETRY
AND COVARIANT PERTURBATION
THEORY FOR GRAVITY

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## 1. Introduction

In the quantization procedure it is conventional to treat gravity as a variant of the gauge field theory $1,2,3$. In the present note another analogy chiral dynamics, $1 s$ used for covariant quantization of the gravitational field. As has been shomn ${ }^{4}$, the gravitational field theory is the theory of spontaneous breaking of affine and conformal symmetries like that the ohiral dynamics is the theory of spontaneous breaking of the chiral symmetry. It allows one to formulate perturbation theory with the most simple reduction properties. In the oh1ral dynamics, such a perturbation theory 5,6,7 simplifies considerably the calculation technique and is quite suitable for the use of regularization methods 8,9 based on summation of certain diagrams.

To make the statement of the problem more clear, consider the simple $\lambda \varphi^{4}$ theory. Together with the $\lambda \varphi^{4}$ Lagrongian there also exists a lot of equivalent on the mass shell Lagranglans obtained by the transformations

$$
\begin{equation*}
\varphi=\varphi^{\prime} f(\varphi) \quad ; f(\rho)-1 \tag{1}
\end{equation*}
$$

Constructive methods to fulfil the equivalence theorem are rearrangements of matrix elements 10 called reductions 10 (or contractions ${ }^{5}$ ). These reductions consist in transforring the vertex derivatives from one line to another and in reduoing certain propagators to $\delta$. functions.

Consideration of all possible reductions after the change of variables (1) is equivalent to the inverse transition to the $\lambda \varphi^{4}$ Lagrangian in terms of matrix el ements. The latter Lagrangian does not contain derivatives and, accordingly, perturbation theory has the most simple reduction properties.

Ls has been shown in refs. $10,5,6$, an analog of the $\lambda \varphi^{4}$ Lagrangian for nonlinear realization of the ohiral symmetry is a Lagrangian to mich there correspond the normal coordinates (along geodesics) in the Goldstone field space ${ }^{10}$. Here, to separate the fields into classical (background) and quantum ones the use should be made of the summation of vectors in the glven curved space ${ }^{5,6}$. For instanoe, for the ohiral $\operatorname{SU}(2) \times \operatorname{SU}(2)$ theory the Goldstone field space is the spaoe of constant ourvature and the summation of veotors is a displacement of the ooordinate origin on the sphere, that corresponds to the chiral transformation of quantum fields with parameters of classical plelds. For the latter, the coordinates may not be ifxed and in this sense the constructed perturbation theory is covariant.

The geometry of the Goldstone field space is determined by the dy namical group algebra. Standard group methods exist for constructing the normal $\infty$ ordinates ${ }^{10}$ given at an arbitrary point of the space ${ }^{6}$.

In the present note the group methods of constructing Lagrangians with the simplest reduction properties ${ }^{6}$ are
applied to the gravity theory as the theory of the dynamical affine symmetry. 4

In sect. 2 the main resilts of paper ${ }^{4}$ are presented and the role of general coordinate transformations is ascertalned.

In sect. 3 the covariant perturbation theory is formulated in terms of the Cartan forms.

## 2. Classical Theory

In paper ${ }^{4}$ it has been shown that the group of all innear transformations in a four-dimensional space

$$
A(4)=P_{4}(x L(4, R)
$$

may be the starting dynamical group of gravity theory. Its algebra consists of generators of the Lorentz group, $L_{\mu \nu}$ generators of the affine transformations $R_{\mu \nu}$, and those of translations $P_{\mu}$

$$
\begin{align*}
& \frac{1}{i}\left[L_{\mu v}, L_{\rho \tau}\right]=\delta_{\mu \rho} L_{\nu \tau}-\delta_{\mu \tau} L_{\nu \rho}-(\mu \leftrightarrow v) \\
& \frac{1}{i}\left[L_{\mu v}, R_{\rho \tau}\right]=\delta_{\mu \rho} R_{\nu \tau}+\delta_{\mu \tau} R_{\nu \rho}-(\mu \leftrightarrow \nu) \\
& \frac{1}{i}\left[R_{\mu v}, R_{\rho \tau}\right]=\delta_{\mu \rho} L_{\tau v}+\delta_{\mu \tau} L_{\rho \nu}+(\mu \leftrightarrow \nu)  \tag{2}\\
& \frac{1}{i}\left[L_{\mu v}, P_{\rho}\right]=\delta_{\mu \rho} P_{\nu}-\delta_{\nu \rho} P_{\mu} \\
& \frac{1}{i}\left[R_{\mu v}, P_{\rho}\right]=\delta_{\mu \rho} P_{\nu}+\delta_{\nu \rho} P_{\mu}
\end{align*}
$$

We consider the nonlinear transformations in the coset space $A(4) / L$ which parameters are the coordinates $x_{\mu}$ and ten Goldstone fields, $h_{\mu \nu}$ - gravitons, the invariants with respect to the inear transformations with constant parameters are $\infty$ nstructed with the help of the cartan forms:

$$
\begin{equation*}
G^{-1} d G=i\left[\omega_{\mu}^{P}(d) P_{\mu}+\frac{1}{2} \omega_{\mu \nu}^{R}(d) R_{\mu \nu}+\frac{1}{2} \omega_{\mu \nu}^{L}(d) L_{\mu \nu}\right] \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
G_{T}=e^{i p_{X}} e^{\frac{i}{2} h_{\alpha \beta} R_{\alpha \beta}} \tag{4}
\end{equation*}
$$

The form $w^{R}$ defines the covariant differential of fields $h$, and $\omega^{P}, \omega^{L}$ are used to define the covariant differentiation of fields $\psi$ transforming by representations of the Lorentz group with matrices generators $L_{\mu v}^{4}$

$$
\begin{equation*}
\nabla_{\lambda} 4=\frac{D)^{4}}{\omega_{\lambda}^{P}} \quad ; \quad D 4=\left(d+\frac{i}{2} \omega_{\mu \nu}^{L}(d) L_{\mu \nu}^{\psi}\right) 4 \tag{5}
\end{equation*}
$$

In what follows, the field 4 is understood to be a spinor field only. Let us find explicitly the Cartan forms for exponential parametrization (4) that corresponds to the choice of normai coordinates in the ten-dimensional space $h_{\mu v}$. Instering into (3) the parameter $t$ through the change $h \rightarrow t h$, differentiating both sides of (4) with respect to $t$, and using the commatation relations (2) we obtain the equations:

$$
\begin{align*}
\frac{\partial}{\partial t} \omega_{\mu}^{P}(d)= & h_{\mu \nu} \omega_{\nu}^{p}(d) ;\left.\omega_{\mu}^{p}(d)\right|_{t=:}=d x_{\mu} \\
\frac{\partial}{\partial t} \omega_{\mu \nu}(d)= & d h_{\mu \nu}-h_{\mu v} \omega_{z \nu}(d)+\omega_{\mu}(d) h_{\partial \nu} ;\left.\omega_{\mu \nu}(d)\right|_{t=0}=0 \\
& \omega_{v \nu}^{R}=\omega_{(\mu \nu)}(d)=\frac{1}{2}\left(\omega_{\mu \nu}+\omega_{\nu \mu}\right)  \tag{6}\\
& \omega_{\mu v}^{L}= \\
& \omega_{[, \mu \nu]}=\frac{1}{2}\left(\omega_{\mu \nu}-\omega_{\nu \mu}\right)
\end{align*}
$$

Solutions to these equations are the expressions
( at, $t=1$ ):

$$
\begin{align*}
\omega_{\mu}^{P}(d) & =\zeta_{\mu v} d x_{v} ; \quad \omega_{\mu \nu}(d)=\tau_{\mu b}^{-1} d \tau_{\Delta v}  \tag{7}\\
\tau_{\mu \nu} & =\left(e^{h}\right)_{\mu \nu} ; \quad \tau_{\mu v}^{-1}=\left(e^{-h}\right)_{\mu v} .
\end{align*}
$$

The invariant elements of length and volume are constructed on the basis of the Cartan forms $\omega^{\prime}$. Accordingly, we have ${ }^{4}$

$$
\begin{equation*}
d s^{2}=\omega_{\lambda}^{p} w_{1}^{p}=g_{\mu \nu} d x^{\mu} d x^{\nu} \tag{8}
\end{equation*}
$$

$$
g_{\mu \nu}=z_{\mu 2} I_{2 V} ; \quad d V=d_{e} t t_{d}{ }^{4} x .
$$

The requirement of minimum with in respect to the number of derivatives does not $f 1 x$ uniquely the theory because the transformation properties of the coveriant derivative (5) do not ohange if one adds several tems of the same order of derivative with arbitrary coefficients $C_{1}, C_{2}, C_{3}$ $\nabla_{\lambda} 4=\partial_{\lambda} \psi+\frac{i}{2} V_{\mu \nu, \lambda}\left(c_{1}, c_{2}, c_{3}\right) L_{\mu \nu}^{\mu} 4$ $V_{\mu \nu, \lambda}=\omega_{[\mu \nu]}\left(\partial_{\lambda}\right)+c_{\lambda}\left[\omega_{\left(\nu_{\lambda}\right)}\left(\partial_{\mu}\right)-\omega_{(\mu \lambda)}\left(\partial_{\nu}\right)\right]+$

$$
\partial_{\lambda}=\tau_{\lambda \gamma}^{-1} \partial_{\delta} ; \omega_{\mu \nu}\left(\partial_{\delta}\right)=\zeta_{\mu}^{-1} \partial_{\gamma} \gamma_{\partial \nu} .
$$

As has been shown in paper ${ }^{4}$, the parameters $C_{1}, C_{2}, C_{3}$ are uniquely defined by the requirement that the theory be simultaneously corresponding to the nonlinear realization of the conformal group

$$
\begin{equation*}
c_{1}=-1 ; \quad c_{2}=c_{3}=0 . \tag{10}
\end{equation*}
$$

This requirement leads to the tensor field theory which equations coincide with the binstein ones.

In this note we want to indicate that the ambiguity of the theory of nonlinear affine realization may be removed by requiring that in interaotions of the tensor Goldstone field there be only the particle with spin two. The interaction of the particie with spin one is complet ely ruled out by the invariance of Lagrangian with respect to the gauge transformations in the coset space, i.e., with respeot to the affine transformations with a parameter being a vector field gradient $C_{\mu}(x)$

$$
\begin{equation*}
e^{i \frac{h_{\mu}}{2} R_{\mu \nu}} \rightarrow e^{i \frac{\partial_{\mu} C_{\nu}(x)}{2} R_{\mu \nu}} \cdot e^{i \frac{h_{\alpha \beta}}{2} R_{\alpha \beta}} . \tag{iI}
\end{equation*}
$$

These transformations with $C_{\nu}(x)$ infinitesimal correspond to the transformations of ooordinates

$$
\begin{equation*}
x_{\mu} \rightarrow x_{\mu}+C_{\mu}(x) \tag{12}
\end{equation*}
$$

Provided the quantity $V_{\mu, \lambda}\left(c_{1} c_{1} c_{3}\right) 1_{3}$ transformed under transformations (11) like under constant transformations, we ootain the values or coefficients $C_{1}, G_{2}, C_{3}$ in (10) leading to the Einstein theory. Transformations (11), (12) oolucide with general ooordinate ones whicin in the giveri approsch acquire a strple phystoal and geometrical meaning as was mentioned above. A covarfant form for the Goldstone fields $h_{\mu \nu}$ may be found by consldering the commutator of covariant derivatives for any field $41(x)$

$$
\left(\nabla_{\lambda} \nabla_{\rho}-\nabla_{\rho} \nabla_{\lambda}\right) \Psi^{\mu}=\frac{i}{2} R_{\mu \nu, \lambda \rho} L_{\mu \nu}^{\mu}
$$

The contraotion $R=R_{\mu \nu, \mu \nu}$ is the scalar relative to the afflne group. To get a full colncldence with standard delinitions of gravity theory, one needs to introduce, by means of $\gamma_{\mu \nu}$, the ilnearly transforming quantities with spin integer, for instance:

$$
A_{\mu}=\tau_{\mu}-a_{\mu}
$$

$$
D_{\lambda} A_{\mu}=r_{\lambda \bar{\lambda}} \tau_{\mu \bar{\mu}} \nabla_{\bar{\lambda}}\left(r_{3 \mu}^{-1} A_{b}\right)=\left(\partial_{\lambda} A_{\mu}-\Gamma_{\lambda \mu}^{b} A^{2}\right)
$$

where $\Gamma_{\lambda \mu}^{2}$ are the Christoffel symbols. The quantities $g_{\mu \nu}$ (8) $\Gamma_{\lambda \mu}^{3}$ and $R$ are connected With each other by usual formulae of the Binstein gravity theory. The minimal interaotion 1 s described by the action

$$
S(h, 4)=\int d^{4} x \operatorname{det}\left[\mathcal{L}_{0}\left(4, \nabla_{\mu} 4\right)+\frac{F^{2}}{4} R\right]
$$

Here $\mathcal{L}_{0}(4,2,4)$ is the Lagrangian of free $\mathcal{P}$ ields, $\psi$; $F=\sqrt{k 4 \pi}^{-1} \sim 10^{19 / \Gamma e V} ; k$ is the Newton constant. The fields $h_{\mu \nu}$ are dimensionless and related to the usual dimensional Pields $h_{0}: h_{d}=F h$

## 3. quantum Theory

We will proceed from the generating functional for the Green functions written in the form of the continual integral ( see the review of Faddeer and Popor ${ }^{1}$ )

$$
\begin{align*}
& z(J, \psi)=1 / N \prod_{\mu<\nu, x} d h_{\mu \nu}(x) \prod_{x}  \tag{14}\\
& \delta(f(h(x))) \Delta_{f}(h) \exp \{i S(h, \psi)+ \\
&\left.+\int d_{x}^{4} J_{\mu \nu} h_{\mu \nu}\right\}
\end{align*}
$$

where $N$ is the normalization, $\mathcal{J}_{\mu \nu}$ - the source, $f(h)=0$ the equation fixing the gauge, $\Delta_{f}(h)$ - the Faddeer-Popor determinant, $S(h, \psi)$ is the action function ( 0 ). The fields 4 can be treated, without loss of generality, to be classical. The quantity $\Delta_{f}(h)$ is calculated by the formula ${ }^{1}$

$$
\begin{equation*}
\Delta_{f}(h) \cdot \int_{\mu, x} d c_{\mu}(x) \delta\left(f\left(h^{c}(x)\right)\right)=1 \tag{15}
\end{equation*}
$$

where $C$ is the general coordinate transformation (11), (12). In a quatsclassical expansion of the generating functional the following change of integration variables

$$
\begin{equation*}
h_{\mu \nu} \rightarrow \varphi_{\mu \nu}+h_{\mu \nu} \tag{16}
\end{equation*}
$$

1s conventional. In this way one extracts the classical (background) fields $\varphi_{\mu \nu}$ obeying the classical equations of motion

$$
\begin{equation*}
\frac{\delta S^{\gamma}(\varphi, 4)}{\delta \varphi_{\mu \nu}}=-J_{\mu \nu} \tag{17}
\end{equation*}
$$

and the "quantum" Pields $h_{\mu \nu}$ over which one integrates.
In paper ${ }^{6}$, where the nonlinear realizations for dynamical symmetries $G$ of the ohiral type were studied, it has been shown that for constructing lagrangians with the simplest reduction properties it is necessary to separate fields into the classical and quantum ones in the coset space $G / H$, where $H$ is the subgroup of transformations leaving vacuum invariant. The analogous separation of variables $\varphi$ and $h$ in the gravity theory

$$
\begin{equation*}
e^{i \frac{h_{\alpha \beta}}{2} R_{\alpha \beta}} \rightarrow e^{i \frac{\varphi_{\mu \nu}}{2} R_{\mu \nu}} e^{i \frac{h_{\alpha \beta}}{2} R_{\alpha \beta}} \tag{18}
\end{equation*}
$$

defines the system of normal coordinates in the space of Goldstone fields $h \quad w i$ th the coordinate origin at the point $\varphi$. Transformation (18) is a generalization of the summation of vectors (16) for a curved space

$$
h_{\mu \nu} \rightarrow \varphi_{\mu \nu}(+) h_{\mu \nu}
$$

Like in the case (16), the detemninant of this transformotion equals untty ${ }^{4}$. Thus, the generating functional (14) takes the forn

$$
\begin{aligned}
& Z(y, 4)=\frac{1}{N} \int_{w_{4}, x} d h_{4,}(x) / \prod_{x} \delta(f(\varphi(+) h)) \Delta f(\varphi(+) h) . \\
& \exp _{\mu}\left\{i S(\hat{y}(+) h, 4)+\int d^{\varphi} x J_{\mu \nu}\left(\varphi_{\mu \nu}+h_{\mu \nu \nu}\right)\right\} \text {. }
\end{aligned}
$$

Sore ft has seca conssidere that on the mass shell the equalaty.

$$
\gamma\left(\varphi_{1}+, j h\right)=y(\varphi+h)
$$

An? ?
U a boin the axsicit roxa of the action $S(\varphi(t) h, \psi)$.
$\because: \therefore \quad \therefore \quad \therefore$ suffichent to find new caxtan forma by



$$
\ddot{G}^{i}=e^{i x D^{i}} e^{\frac{i}{2} \varphi_{\alpha \beta} R_{\alpha \beta}} e^{\frac{i}{2} h_{\mu v} R_{\mu \nu}}
$$

Making the substitution $w 1$ th parameter $t: h \rightarrow t h$ and differentiating both sides of eq. (IO) With respect to $t$

[^0]We derive the differential equations for the cartan forms which are the same as in the classical oase (6) but with nonzero boundary conditions which are the Cartan forms of olassioal fields

$$
\begin{aligned}
& \frac{\partial}{\partial t} \bar{\omega}_{\mu}^{P}(d)=h_{\mu \nu} \bar{\omega}_{\nu}^{P}\left(d^{\prime}\right) ; \\
& \left.\bar{\omega}_{\mu}^{r}(d)\right|_{t=0}=\omega_{\mu}^{p}(d)=\tau_{\mu v}(\varphi) d x_{i} \\
& \frac{\partial}{\partial t} \bar{\omega}_{\mu \nu}(d)=d h_{\mu \nu}-h_{\mu} \bar{\omega}_{2 v}(d)+\bar{\omega}_{\mu_{3}}(d) h_{\Delta v} ;\left.\bar{\omega}_{\mu \nu}(d)\right|_{t=0}=\gamma_{\mu 2}^{-1}(\varphi) d \gamma_{\beta_{v}}(\varphi) \\
& \bar{\omega}_{\mu v}^{R}=\bar{\omega}_{(\mu v)} \\
& \bar{\omega}_{\mu \nu}^{L}=\bar{\omega}_{[\mu \nu \nu]} .
\end{aligned}
$$

Equations (20) describe the parallel displacement
of orthogonal moving lo-hedral along geodesics of the space of the Goldstone fields from the point $\varphi$ to the point $h$ and these are called fundamental ${ }^{12}$.

The solution of these equations at $t=1$ is

$$
\begin{gather*}
\bar{\omega}_{\mu}^{p}(d)=\left[e^{h} \tau(\varphi)\right]_{\mu v} d x_{v} \\
\bar{\omega}_{\mu \nu}(d)=\left[e^{-h} \tau^{-1}(\varphi)\right]_{\mu \partial} d\left[\tau(\varphi) e^{h}\right]_{\partial \nu} \tag{21}
\end{gather*}
$$

where $[A B]_{\mu v}=A_{\mu 3} B_{3 v}$.
In what follows the bar above will stand for the quantities Whioh simultaneousiy depend on $\varphi$ and $h$.

As is shown in paper ${ }^{6}$, forms (7) and (21) have the simplest reduction properties due to their oonstraction by means of the dynamical group structure oonstants antisymmetric in lower indices.

Fron (21) it follows that the trangformed metric tensor takes the porm

$$
\begin{align*}
& g_{\mu \nu}(\varphi(t) h)=\left[\tau(\varphi) e^{2 h} \tau(\varphi)\right]_{\mu v} \\
& g^{\mu v}(\varphi(\varphi) h) \because\left[\tau \zeta^{\prime}(\varphi) e^{-2 h} \tau^{-1}(\varphi)\right]_{\mu v} \tag{22}
\end{align*}
$$

Next, we write the action for spinor and gravitational fields

$$
\begin{aligned}
& \bar{R}=2 \bar{z}_{\mu} \bar{V}_{\mu \nu, \nu}+\bar{V}_{\nu, \gamma} \bar{V}_{\nu \gamma, \mu}-\bar{V}_{\mu \gamma, \mu} \bar{V}_{\nu \gamma, \nu} \\
& \nabla_{\lambda} H=\vec{x}_{\lambda} \mu+i-\theta_{\mu \nu} \vec{V}_{\mu \nu, \lambda} 4 \\
& \bar{V}_{\mu \nu, \lambda}=\bar{\omega}_{[, \alpha \nu]}\left(\bar{x}_{\lambda}\right)-\bar{u}_{\nu \nu \lambda}\left(\bar{x}_{\mu}\right)+\bar{\omega}_{(\mu \lambda}\left(x_{\nu}\right) ; \\
& \bar{\partial}_{\mu}=\left[e^{-h} r^{-1}(\varphi)\right]_{\mu \gamma} \partial_{\sigma} \\
& \overline{c i}_{\mu \nu}\left(\bar{\sigma}_{1}\right)=\left[e^{-h} \tau^{-1}(\varphi)\right]_{\mu \partial} \bar{z}_{\lambda}\left[\tau(\varphi) e^{h}\right]_{i \nu}= \\
& =\left(e^{-h}\right)_{,+2} \dddot{\partial}_{\lambda}\left(e^{h}\right)_{i v}+\left(e^{-h}\right)_{\mu \mu^{\prime}}\left[\tau^{-1}(\varphi) \bar{\partial}_{\lambda} \tau(\varphi)\right]_{\mu^{\prime} \nu^{\prime}}\left(e^{h}\right)_{\nu^{\prime} \nu} .
\end{aligned}
$$

Lagrangians for fields with integer spins are constructed by means of the Christoffel symbols $\vec{\sim} \rho$ in a standard way. The curvatwre $\vec{R}$ and Christoffel symbols $\overrightarrow{F_{\mu}} \boldsymbol{P}$ are connected with metric tensor (22) through the usual formulae

$$
\begin{gather*}
\bar{\Gamma}_{\mu v}^{2}=\bar{g}^{b \alpha}\left(\partial_{\mu} \bar{g}_{\alpha v}+\partial_{\nu} \bar{g}_{\alpha \mu}-\partial_{\alpha} \bar{g}_{\mu \nu}\right) \\
\bar{R}=\bar{g}_{\mu \nu}\left[\partial_{\nu} \vec{\Gamma}_{\mu} \rho-\partial_{\rho} \bar{\Gamma}_{\mu \nu}^{\rho}+\vec{\Gamma}_{\alpha} \rho_{\rho \nu}^{2}-\vec{\Gamma}_{\mu \nu}^{\rho} \vec{\rho}_{\rho}^{2}\right] . \tag{24}
\end{gather*}
$$

Formulae (22), (23) are the main results of this paper.
To obtain the generating functional directly for the

## S-matrix elements we use a prescription proposed

in papers 13,14 .
Let $S_{0}\left(\varphi_{0}\right)$ be the free aotion quadratic in $\varphi_{0}$ and the classical fields $\varphi$ obey the equation of motion

$$
\frac{\delta S(\varphi, 4)}{\delta \varphi}=\frac{\delta S_{0}\left(\varphi_{0}\right)}{\delta \varphi_{0}}
$$

Then the generating functional of the smatrix has the form ${ }^{14}$ : $Z\left(\varphi_{0}, 4\right)=1 / N \int_{\mu(\nu, x} d h_{\mu v}(x) \prod_{x} \delta^{4}\left(D_{\nu}(\varphi) g^{\mu \nu}(h)-\ell_{\mu}\right) \Delta(h, \varphi) \exp \{i S(\varphi(+) h, 4)\}$.

Here $D_{\nu}(\varphi) g^{\mu \nu}(h) \quad$ is the covariant derivative of $\mathcal{G N v}_{\sim}(h)$ with the Christoffel symbols dependent on the classical P1elds $\varphi$

$$
\Delta(h, \varphi) \cdot \int_{\mu_{x}} \nabla_{\mu} d c_{\mu}(x) \Pi_{x} \delta^{4}\left(\mathcal{D}_{\nu}(\varphi) g^{\mu \nu}\left(h^{c}\right)-\ell^{\mu}\right)=1
$$

The coefficient functions of the expansion of the functional $z$ in $\varphi_{0}$ ooincide $w 1$ th those of the expansion of the $S$-matrix over the normal products of asymptotical ifelds.

## Conclusion

We have formulated the perturbation theory for gravity， where the choice of fundamental flelds and their separation， in the generating functional，into classical（baokground） and quantum fields are defined by geometry of the geodesios of the space of gravitational fields．

The cholce of normal coorinates of this space and the separation of fields along geodesics lead to the perturbation theory with the simplest reduction properties，therefore the corresponding Lagrangian is an analog of the $\lambda \varphi^{4}$ lagrangian among all equivalent on the mass shell Lagrangians．For the choice of al arbitrary system of coordinates the consideration of all possible reductions of diagrams on the mass shell is equivalent to the covariant procedure of transition from this coordinate system to the normal one．

For nonlinear realizations of the chiral symmetry such an approach leads to the perturbation theory，which is the most simple for calculations，espocially，when one uses the regula－ rizations connected with summation of certain diagrams，the sumation being explicitly covariant with respect to the classical fields．

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[^0]:    $x$ ) For renormalization of wave functions in changing
    variables in the considered generating functional, see ref. ${ }^{11}$.

