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HADRONIC CURRENTS IN THE INFINITE MOMENTUM FRAME

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S U M M A R Y

The problem of the transformation properties of hadronic currents in the infinite momentum frame (IMF) is investigated. A general method is proposed to deal with the problem which is based upon the concept of group contraction. The two-dimensional aspects of the IMF description are studied in detail, and the current matrix elements of a three-dimensional Poincaré covariant theory are reduced to those of a two-dimensional one. It is explicitly shown that the covariance group of the two-dimensional theory may either be a "non-relativistic" (Galilei) group, or a "relativistic" (Poincaré) one depending on the value of a parameter reminiscent of the light velocity in the three-dimensional theory. The value of this parameter cannot be determined by kinematical arguments. These results offer a natural generalization of models which assume Galilean symmetry in the infinite momentum frame.

I. Introduction

In this paper we are going to study the transformation properties of local interaction currents in the infinite momentum frame (IMF). There is no need for emphasizing the importance of the commonly used fact that all unknown dynamical details about the matrix element of the electromagnetic current between, for example, pion states can be condensed into one invariant function of the momentum transfer, and the relation

$$\int d^4x \, e^{iqx} \langle p | J_\mu(x) | p' \rangle = (p+p')_\mu F(q^2) \delta^4(p-p'+q) \qquad (I.1)$$

can be deduced from the transformation properties of the states and the current. Since IMF methods are quite useful in studying many physical problems we have found it attractive to investigate the transformation properties of various quantities in the IMF in order to be able to apply such powerful arguments as in (I.1). In a recent paper we examined such questions concerning the four-momentum and angular momentum tensors of a scalar field theory /1/. In the present paper we want to deal with the transformation properties in the IMF of currents integrated over the variable $\zeta = \frac{1}{2}(z-t)$,

$$\int j_\mu(t, \underline{x}_\perp, z) \, d\zeta \quad , \qquad (I.2)$$

which are often called the "transverse currents" /2,3/. Our investigations are motivated by the review paper of Kogut and

Susskind [3] in which the authors give a certain group-theoretical background of parton models. On the basis of some heuristically justified transformation properties of the current (I.2) in the IMF they give an elegant and compact derivation of the main features of deep-inelastic ep scattering. They emphasize the two-dimensional aspects of IMF theories and argue that a two-dimensional Galilei group is the symmetry group of these theories, the Galilean space being the transverse space, $\underline{x}_\perp=(x,y)$, and the Galilean "time" is obtained by a "rescaling" of the variable t+z [2,3]. Their argumentation for the Galilean symmetry has, beyond its intuitive beauty, two somewhat obscure points. One of them is the ambiguous role of z-boosts. In order to define the current in the IMF the infinite boost is followed by a change of the scale for lengths in the 3-direction. But this rescaling is not present in the case of successively applied "finite" z-boosts [2,3]. The second point concerns "time" in the IMF. The transverse current (I.2) appears to be a function only of $\underline{x}_\perp$ in the IMF. Its "time dependence", and the "time" itself, can be defined by means of the "time translation operator" after the symmetry group in the IMF has been introduced. The procedure described in refs. 2 and 3 to obtain the two-dimensional Galilei group is very much reminiscent of the contraction of the Poincaré group into a Galilean subgroup. It is well-known that group contraction is not a unique procedure, therefore the question arises whether the Galilean symmetry in the IMF is unique.

The purpose of this paper is to show that the definition of "relativistic" quantities is also possible in the IMF, and on purely kinematical grounds the "non-relativistic" transformation properties of the transverse current cannot be justified. The assumption of Galilean symmetry in the IMF may perhaps be supported by that it allows one to deduce many experimentally observed features of high energy collisions /3/. It is an interesting question whether these features rule out "relativistic" currents in the IMF.

In what follows we give a detailed derivation of the transformation properties of "transverse currents" in the IMF. Our method will be the contraction of the Poincaré representations defined on the matrix elements of currents between physical states. The crucial steps of this method are the contraction of the Poincaré group and the "contraction" of the representation space by means of appropriately defined integrals of the above matrix elements. Only after performing both parts of the programme we can deduce the transformation properties of the "transverse current" in the IMF. In Sect.II we summarize the basic concepts needed in the subsequent parts of the paper and formulate our programme for the IMF description. In Sect.III we describe contraction schemes of the Poincaré group which we will be interested in. In Sect.IV these contraction schemes are used to construct representation spaces for the group obtained by the contractions and the corresponding transformation rules are deduced for the "transverse currents" in the IMF. In Sect.V. we apply the results to matrix elements between momentum eigenstates.

II. Formulation of the IMF problem

In general, we shall be engaged in the properties of a scalar current $s(x) \equiv s(t,\underline{x}_\perp,z)$, but none of the forthcoming conclusions depends on this choice. We shall assume that we are given all states, ϕ , of a physical system, the physical observables being the matrix elements of $s(x)$ between these states:

$$f(x) \equiv f(t,\underline{x}_\perp,z) \equiv (\phi_\beta\, ,\; s(x)\phi_\alpha) \; . \qquad \text{(II.1)}$$

As usual, we assume that on the states ϕ a unitary, irreducible representation of the Poincaré group is given:

$$U(a,\Lambda)\phi = \phi' \; , \qquad \text{(II.2)}$$

$$U(a_1,\Lambda_1)U(a_2,\Lambda_2)\phi = U(a_1\Lambda_2+a_2,\; \Lambda_1\Lambda_2) \; .$$

The relation between matrix elements given in two different Lorentz reference frames comes from the principle of relativistic covariance, which says:

$$\left(U(a,\Lambda)\phi_\beta\, ,\; s(x)U(a,\Lambda)\phi_\alpha\right) = \left(\phi_\beta,\; s(x\Lambda+a)\phi_\alpha\right) \; . \qquad \text{(II.3)}$$

The relations (II.1-3) can be converted into a representation of the Poincaré group on all functions $f(x)$, which are all matrix elements of $s(x)$ between the states ϕ:

$$T(a,\Lambda)f(x) = f(x\Lambda+a), \qquad (II.4)$$

or, in the infinitesimal form:

$$M^{\mu\nu}f(x) = -i\left[x^{\mu}\frac{\partial}{\partial x_{\nu}} - x^{\nu}\frac{\partial}{\partial x_{\mu}}\right]f(x), \qquad (II.5)$$

$$P^{\mu}f(x) = -i\frac{\partial}{\partial x_{\mu}}f(x). \qquad (II.6)$$

This representation corresponds to a scalar current and the relations (II.4-6) are to be compared with the usual operator relations:

$$U^{-1}(a,\Lambda)s(x)U(a,\Lambda) = s(x\Lambda+a), \qquad (II.4')$$

$$\left[M^{\mu\nu}, s(x)\right] = -i\left[x^{\mu}\frac{\partial}{\partial x_{\nu}} - x^{\nu}\frac{\partial}{\partial x_{\mu}}\right]s(x), \qquad (II.5')$$

$$\left[P^{\mu}, s(x)\right] = -i\frac{\partial}{\partial x_{\mu}}s(x). \qquad (II.6')$$

The detailed properties, like unitarity, etc., of the representation (II.4-6) depend very much on the operator $s(x)$. These properties will not be important in this paper. But we shall need the following properties of the functions $f(x)$:

1. They are infinitely differentiable with respect to any of their four variables;
2. Such series like

$$T(e^{-i\alpha G_k})f(x) \equiv \sum_{n=0}^{\infty}\frac{1}{n!}(-i\alpha)^n G_k^n f(x) \qquad (II.7)$$

converge, G_k being any of the infinitesimal operators.
These properties assure that infinitesimal and finite group elements can be used on an equal footing.

It is clear that the representation (II.4-6) is reducible. We shall restrict ourselves to its irreducible parts proceeding in the following manner. We consider two fixed physical states ϕ_α, ϕ_β, and define the irreducible set of functions $f_{\alpha\beta}(x;(a,\Lambda))$ of x by

$$f_{\alpha\beta}(x;(a,\Lambda)) \equiv \left(\phi_\beta,\ s(x\Lambda+a)\phi_\alpha\right), \qquad \text{(II.8)}$$

where all functions are enumerated when all elements (a,Λ) of the Poincaré group are enumerated. It is obvious that an irreducible representation of the Poincaré group can be defined on these functions, the representation being given by the same formulas (II.4-6) as previously. Let us remark that all information about this irreducible representation is involved in the two properties described above and in the infinitesimal relations (II.5,6) specifically applied to the function

$$f_{\alpha\beta}(x) = \left(\phi_\beta,\ s(x)\phi_\alpha\right). \qquad \text{(II.9)}$$

It is an important consequence of relativistic covariance that the same representation space as (II.8) arises from the functions

$$f'_{\alpha\beta}(x;(a,\Lambda)) \equiv \left(\phi'_\beta, s(x\Lambda+a)\phi'_\alpha\right), \qquad \text{(II.10)}$$

where $\phi'_\beta = U(a',\Lambda')\phi_\beta$, $\phi'_\alpha = U(a',\Lambda')\phi_\alpha$, and (a',Λ') is fixed. The relation between the functions (II.8) and (II.10) is as follows: (II.11)

$$f'_{\alpha\beta}(x;(a,\Lambda)) = T(a',\Lambda')f_{\alpha\beta}(x;(a',\Lambda')^{-1}(a,\Lambda)(a',\Lambda')),$$

for every (a,Λ). The corresponding representations of the Poincaré group can also be simply related:

$$T'(a_1, \Lambda_1)\, f'_{\alpha\beta}(x;(a,\Lambda)) = \qquad (II.12)$$

$$= T(a', \Lambda')\, T\left((a', \Lambda')^{-1}(a_1, \Lambda_1)(a', \Lambda')\right) \times$$

$$\times\, f_{\alpha\beta}(x;(a', \Lambda')^{-1}(a,\Lambda)(a', \Lambda'))\ ,$$

for every (a,Λ) and (a_1, Λ_1). Especially, if we choose a z-boost $e^{-i\xi N_3}$ for (a', Λ'), the primed functions $f'_{\alpha\beta}$ and operators T' give the description of the physical system, in comparison with the unprimed ones $f_{\alpha\beta}$ and T, in a moving reference frame. In the limit $\xi \to \infty$ we obtain the description in the IMF. In order to be able to specify the symmetry properties of the theory in the IMF one must discuss the following problems:

Problem I

We are to describe the group arising from the limit

$$\lim_{\xi\to\infty} \left[e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3}\right] \equiv (a,\Lambda)_\infty \quad ; \qquad (II.13)$$

Problem II

We must specify all the functions

$$\lim_{\xi\to\infty} T(e^{-i\xi N_3})T(e^{+i\xi N_3}(a,\Lambda)e^{-i\xi N_3})\, f_{\alpha\beta}(x) \equiv \qquad (II.14)$$

$$\equiv f^{\infty}_{\alpha\beta}(x;(a,\Lambda)_\infty)\ ;$$

Problem III

Finally, we must interpret the functions $f^{\infty}_{\alpha\beta}(x;(a,\Lambda)_\infty)$ as matrix elements of an IMF current $s^\infty(x)$ between states ϕ^∞ all having well-defined transformation properties with respect to the group $(a,\Lambda)_\infty$.

Problem I is, actually, a contraction problem for the Poincaré

group. It has several solutions, the limit gives either the Poincaré group itself or one of its subgroups /4/. (Strictly speaking, these groups are isomorphic to the original Poincaré group or its subgroups.) Contraction into the Poincaré group has been described in ref. 5 , and into some of its subgroups by the present author in ref. 1. No à priori reason can be given for choosing one or another solution of the contraction problem. Only some physical hints may inspire one to make a definit choice.
In this paper we look for such solutions of Problem I that (II.13) leads to contraction of the Poincaré group into some of its subgroups. This choice is motivated by refs. 2 and 3 . These subgroups will be Galilei and Poincaré subgroups which transform two "space" coordinates and a non-relativistic or relativistic "time" coordinate, respectively. (In what follows we use the terminology of ref.1. and call 3-Poincaré group the one described by the formulas (II.4-6), its contractions will be called 2-Galilei and 2-Poincaré groups, respectively.) After coming to this decision on Problem I it is clear that the four-dimensional homogeneous space x^{μ} of the 3-Poincaré group is to be reduced to some three-dimensional one. One may hope to achieve this by integrating the functions $f'_{\alpha\beta}(x;(a,\Lambda))$ over one of its variables and may reformulate Problems II and III in terms of these integrated functions. For a convenient choice of the integration variable we change t and z by τ and $\mathfrak{z}$:

$$\zeta = \frac{1}{2}(z-t), \qquad \tau = (t+z), \tag{II.15}$$

and, for the functions $f'_{\alpha\beta}\ (t,\underline{x}_\perp,z;(a,\Lambda))$ we use the notation $g'_{\alpha\beta}\ (\tau,\underline{x}_\perp,\zeta;(a,\Lambda)) \equiv g'_{\alpha\beta}\ (x;(a,\Lambda))$. We shall be interested in the functions

$$g'_{\alpha\beta}\ (\tau,\underline{x}_\perp;(a,\Lambda)) \equiv \int_{-\infty}^{\infty} g'_{\alpha\beta}\ (x;(a,\Lambda))\,d\zeta \tag{II.16}$$

in the IMF, that is, for $\xi\to\infty$. We must specify the functions

$$g^{\infty}_{\alpha\beta}\ (\tau,\underline{x}_\perp;(a,\Lambda)_\infty) \equiv \lim_{\xi\to\infty}\int_{-\infty}^{\infty} T(e^{-i\xi N_3})T(e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3})\,g_{\alpha\beta}(x)\,d\zeta \tag{II.17}$$

and deduce the transformation rules in the IMF for the "transverse current"

$$\int_{-\infty}^{\infty} s(x)\,d\zeta\ .$$

Before concluding this section we make an important remark concerning the solutions of Problem II. In general, the solution of Problem II yields different representation spaces and therefore different representations of the group $(a,\Lambda)_\infty$, if $f_{\alpha\beta}\ (x)$ is changed to some $f_{\tilde{\alpha}\tilde{\beta}}\ (x)$, $f_{\tilde{\alpha}\tilde{\beta}}\ (x)$ being the matrix element of $s(x)$ between the states $U(\tilde{a},\tilde{\Lambda})\phi_\alpha$, $U(\tilde{a},\tilde{\Lambda})\phi_\beta$, with $(\tilde{a},\tilde{\Lambda})$ fixed. Problem II for the function $f_{\tilde{\alpha}\tilde{\beta}}\ (x)$ would mean the calculation of

$$\lim_{\xi\to\infty} T(e^{-i\xi N_3})T(e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3})T(\tilde{a},\tilde{\Lambda})f_{\alpha\beta}\ (x). \tag{II.18}$$

Since $(\tilde{a},\tilde{\Lambda})$ is a fixed element of the ζ-Poincaré group, in the limit $\xi\to\infty$ it becomes, in general, a foreign object from the point of view of the contracted group $(a,\Lambda)_\infty$.

[illegible]ally, for the reader's convenience, we write down here the action of the ζ-Poincaré generators on the functions $g_{\alpha\beta}(x)$:

$$
\begin{aligned}
(M_1-N_2)g(x) &= i(\tau\frac{\partial}{\partial y} - y\frac{\partial}{\partial \zeta})\ g(\tau, \underline{x}_\perp, \zeta), \\
(M_1+N_2)g(x) &= 2i(\zeta\frac{\partial}{\partial y} - y\frac{\partial}{\partial \tau})\ g(x), \\
(M_2-N_1)g(x) &= 2i(x\frac{\partial}{\partial \tau} - \zeta\frac{\partial}{\partial x})\ g(x), \\
(M_2+N_1)g(x) &= i(x\frac{\partial}{\partial \zeta} - \tau\frac{\partial}{\partial x})\ g(x), \\
M_3\ g(x) &= i(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y})\ g(x), \\
N_3\ g(x) &= i(\zeta\frac{\partial}{\partial \zeta} - \tau\frac{\partial}{\partial \tau})\ g(x), \\
(P_0+P_3)g(x) &= -2i\ \frac{\partial}{\partial \tau}\ g(x), \\
(P_0-P_3)g(x) &= i\ \frac{\partial}{\partial \zeta}\ g(x), \\
\underline{P}_\perp\ g(x) &= -i\ \frac{\partial}{\partial \underline{x}_\perp}\ g(x).
\end{aligned}
\qquad (II.19)
$$

III. Contractions of the ζ-Poincaré group

In order to make this paper self-contained as much as possible we give a short summary of those contractions of the ζ-Poincaré group which we are interested in. (For more details see also ref.1)

The two-dimensional Galilean description in the IMF seems from the following connection between the generators of space-time transformations in the limiting and ordinary reference frames, O and O', respectively /1,2/:

$$S_1 = \lambda \lim_{\xi\to\infty} \left\{ e^{-\xi} U(\xi)(M_2'+N_1')U^{-1}(\xi) \right\} , \qquad (III.1a)$$

$$S_2 = -\lambda \lim_{\xi\to\infty} \left\{ e^{-\xi}\, U(\xi)(M_1' - N_2')U^{-1}(\xi) \right\} , \qquad \text{(III.1b)}$$

$$M_3 = \lim \left\{ U(\xi) M_3' U^{-1}(\xi) \right\} , \qquad \text{(III.1c)}$$

$$H^G = \frac{1}{2\lambda} \lim \left\{ e^{\xi}\, U(\xi)(P_0' + P_3')U^{-1}(\xi) \right\} , \qquad \text{(III.1d)}$$

$$P_i = \lim \left\{ U(\xi) P_i' U^{-1}(\xi) \right\} , \qquad i=1,2 \qquad \text{(III.1e)}$$

$$\mu_0 = \lambda \lim \left\{ e^{-\xi}\, U(\xi)(P_0' - P_3')U^{-1}(\xi) \right\} , \qquad \text{(III.1f)}$$

$$\lim \left\{ e^{-\xi}\, U(\xi) N_3' U^{-1}(\xi) \right\} = \lim \left\{ U(\xi)(M_1' + N_2')U^{-1}(\xi) \right\} =$$
$$= \lim \left\{ U(\xi)(M_2' - N_1')U^{-1}(\xi) \right\} = 0 . \qquad \text{(III.1g)}$$

The symbol $U(\xi)$ denotes a z-boost, $U(\xi) = e^{i\xi N'}$, λ is an arbitrary positive number. These relations give a mapping of the 3-Poincaré algebra

$$[M_i' , M_j'] = -[N_i' , N_j'] = i\,\varepsilon_{ijk} M_k' ,$$
$$[M_i' , N_j'] = i\,\varepsilon_{ijk} N_k' , \qquad \text{(III.2)}$$
$$[M_{\mu\nu}' , P_\rho'] = i(g_{\nu\rho} P_\mu' - g_{\mu\rho} P_\nu')$$

onto the 2-Galilei algebra, its elements being S_i , P_i , (i=1,2), M_3 , H^G and μ_0 :

$$[S_i , S_j] = 0 , \qquad [M_3 , S_i] = i\,\varepsilon_{ij} S_j ,$$
$$[P_i , P_j] = 0 , \qquad [H^G , P_i] = 0 ,$$
$$[S_i , H^G] = iP_i , \qquad [S_i , P_j] = i\mu_0 \delta_{ij} , \qquad \text{(III.3)}$$
$$[M_3 , H^G] = 0 , \qquad [M_3 , P_i] = i\,\varepsilon_{ij} P_j ,$$
$$[S_i , \mu_0] = [M_3 , \mu_0] = [P_i , \mu_0] = [H^G , \mu_0] = 0 .$$

The mapping (III.1) can also be expressed as follows:

$$\begin{bmatrix} H^{c} \\ P_1 \\ P_2 \\ \mu_0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2\lambda} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} P_0' + P_3' \\ P_1' \\ P_2' \\ P_0' - P_3' \end{bmatrix} , \qquad \text{(III.4a)}$$

$$\begin{bmatrix} S_1 \\ S_2 \\ M_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \lim_{\xi\to\infty} \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\xi} & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-\xi} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\xi} \end{bmatrix} \begin{bmatrix} M_2' + N_1' \\ M_1' - N_2' \\ M_3' \\ N_3' \\ M_2' - N_1' \\ M_1' + N_2' \end{bmatrix} . \qquad \text{(III.4b)}$$

This is obviously a Wigner-Inönü contraction of the 3-Poincaré algebra /4/. Now the question arises if other Wigner-Inönü contractions of the 3-Poincaré algebra may also be of interest. As it was shown in ref. 1 it seems natural to consider, for example, the following contraction:

$$\begin{bmatrix} K_1 \\ K_2 \\ M_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \lim_{\xi\to\infty} \begin{bmatrix} \lambda & 0 & 0 & 0 & -\frac{1}{4\lambda c^2} & 0 \\ 0 & -\lambda & 0 & 0 & 0 & \frac{1}{4\lambda c^2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\xi} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\xi} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\xi} \end{bmatrix} \begin{bmatrix} M_2' + N_1' \\ M_1' - N_2' \\ M_3' \\ N_3' \\ M_2' - N_1' \\ M_1' + N_2' \end{bmatrix} , \qquad \text{(III.5a)}$$

$$\begin{bmatrix} H^P \\ P \\ P \\ \mu \end{bmatrix} = \begin{bmatrix} \frac{1}{4\lambda} & 0 & 0 & \lambda c^2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-1}{4\lambda c^2} & 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} P_0' + P_3' \\ P_1' \\ P_2' \\ P_0' - P_3' \end{bmatrix} \quad . \qquad \text{(III.5b)}$$

The "infinite momentum limit" character of this mapping becomes more obvious if one rewrites (III.5) in terms of z-boosts:

$$K_1 = \lim_{\xi \to \infty} \left\{ U(\xi) \left[\lambda e^{-\xi}(M_2' + N_1') - \frac{1}{4\lambda c^2} e^{\xi}(M_2' - N_1') \right] U^{-1}(\xi) \right\} \quad ; \qquad \text{(III.6a)}$$

$$K_2 = \lim \left\{ U(\xi) \left[-\lambda e^{-\xi}(M_1' - N_2') + \frac{1}{4\lambda c^2} e^{\xi}(M_1' + N_2') \right] U^{-1}(\xi) \right\} \quad ; \qquad \text{(III.6b)}$$

$$H^P = \lim \left\{ U(\xi) \left[\frac{1}{4\lambda} e^{\xi}(P_0' + P_3') + \lambda c^2 e^{-\xi}(P_0' - P_3') \right] U^{-1}(\xi) \right\} \quad ; \qquad \text{(III.6c)}$$

$$\mu = \lim \left\{ U(\xi) \left[-\frac{1}{4\lambda c^2} e^{\xi}(P_0' + P_3') + \lambda e^{-\xi}(P_0' - P_3') \right] U^{-1}(\xi) \right\} \quad ; \qquad \text{(III.6d)}$$

In (III.5,6) the letters λ, c denote arbitrary positive numbers. That this contraction scheme is natural to be considered is obvious from the following commutators:

$$\begin{aligned} &[K_1 , K_2] = -i \frac{1}{c^2} M_3 , && [M_3 , K_i] = i \varepsilon_{ij} K_j , \quad i=1,2 \\ &[H^P , P_i] = 0 , && [P_i , P_j] = 0 , \\ &[M_3 , H^P] = 0 , && [M_3 , P_i] = i \varepsilon_{ij} P_j , \qquad \text{(III.7)} \\ &[K_i , H^P] = i P_i , && [K_i , P_j] = i \frac{1}{c^2} \delta_{ij} H^P , \\ &[\mu , H^P] = [\mu , P_i] = [\mu , M_3] = [\mu , K_i] = 0 . \end{aligned}$$

This is a 2-Poincaré algebra, its elements being the generators of inhomogeneous Lorentz transformations of two spacelike and one timelike coordinates. The parameter c plays the same mathematical role as light velocity does in the usual 3+1 dimensional algebra.

When c goes to infinity the algebra (III.7) contracts into the non-relativistic one (III.3).

In contrast with the parameter c we have an arbitrary positive number λ in both cases of contractions which does not appear at all in the commutators (III.3), (III.7). Obviously, for the different values of λ the mappings (III.4) and (III.5) yield different (but isomorphic) 2-Galilei and 2-Poincaré subalgebras of the 3-Poincaré algebra, respectively. We are going to assume that 3-Poincaré covariant theories become 2-Poincaré covariant ones in the IMF with some given value of the parameter c. (Its value is to be determined phenomenologically. For the 2-Galilei covariant case $c=\infty$.) It seems natural to postulate that all contractions corresponding to the various values of λ are physically equivalent, that is, none of the predictions of the theory in the IMF may depend on λ.

For a comparison with Susskind's treatment of the 2-Galilei symmetry in the IMF we mention that he seems to choose $\lambda=1$, but preserves the N_3 generator /2,3/. Thus the symmetry group in the IMF becomes a 2-Galilei group extended with dilatations. Since the dilatations correspond to changing the value of λ , dilatation invariance of the theory corresponds just to the postulate we formulated above. Our formulation has the advantage that it can be generalized without difficulty to the 2-Poincaré case, while the N_3 generator cannot be joined to the 2-Poincaré generators to form a closed algebra.

There are, obviously, further ambiguities in choosing the matrices on the right hand side of (III.4) and (III.5). Let us denote by A any of the matrices in (III.4a,b) (or (III.5a,b)). The algebra (III.3) (or (III.7)) remains unchanged if $A_1 A A_2$ is substituted for A, where A_2 is any 3-Poincaré transformation of the generators $M'_{\mu\nu}$, P'_{μ} , and A_1 is any 2-Galilei (or 2-Poincaré) transformation of the generators S_i , M_3 , H^G , P_i (or K_i , M_3 , H^P , P_i). The requirement that the IMF theory should be independent of the choice of A_1 and A_2 is, in the case of A_2, fulfilled by the 3-Poincaré covariance of the theory before transforming into the IMF. In the case of A_1 this requirement leads to the covariance of the theory with respect to the 2-Galilei (or 2-Poincaré) group in the IMF.

IV. The representation spaces in the IMF

In this section we deal with the solution of Problem II, and construct those function spaces which may serve as representation spaces for the contracted groups. This task, in its original form (II.14) means the calculation of such limits:

$$\lim_{\xi\to\infty} T(e^{-i\xi M_3}) g_{\alpha\beta}(\tau, \underline{x}_\perp, \zeta) = \lim_{\xi\to\infty} g_{\alpha\beta}(\tau e^{-\xi}, \underline{x}_\perp, \zeta e^{\xi}), \qquad \text{(IV.1)}$$

and we need much more detailed properties of the current matrix elements as we have used so far. To overcome this one may use

Susskind's proposal [2] for integrating over the variable ζ and calculating by means of the rule

$$\int_{-\infty}^{\infty} \lim g_{\alpha\beta}(\tau e^{-\xi}, \underline{x}_\perp, \zeta e^{\xi})\,d\zeta = \lim e^{-\xi}\int_{-\infty}^{\infty} g_{\alpha\beta}^{\bullet}(\tau e^{-\xi}, \underline{x}_\perp, \zeta)\,d \quad , \qquad (IV.2)$$

but then one faces the problem that the factor $e^{-\xi}$ makes zero the functions we are looking for. One may use certain "physical" arguments [2] to eliminate the factor $e^{-\xi}$ from (IV.2) and may conclude that in the IMF the correspondence

$$\int_{-\infty}^{\infty} g_{\alpha\beta}(\tau, \underline{x}_\perp, \zeta)\,d\zeta \Longrightarrow \int_{-\infty}^{\infty} g_{\alpha\beta}(0, \underline{x}_\perp, \zeta)\,d\zeta \qquad (IV.3)$$

is valid. One must notice, however, that (IV.3) is part of the mapping

$$\int_{-\infty}^{\infty} g_{\alpha\beta}(x;(a,\Lambda))\,d\zeta \Longrightarrow \int_{-\infty}^{\infty} g_{\alpha\beta}^{\infty}(x;(a,\Lambda)_{\infty})\,d\zeta \qquad (IV.3)$$

we are to specify when we solve Problem II. It is this mapping what really determines the content of the theory in the IMF.

As a first step to specify the mapping (IV.3) we deal with integrals of the following type:

$$\int G g_{\alpha\beta}(x)\,d\zeta \ , \qquad (IV.4)$$

where G is an arbitrary polynomial of the 3-Poincaré generators (II.19). In practice, (IV.4) means such expressions:

$$\int D\left(\tau, \underline{x}_\perp; \frac{\partial}{\partial\tau}, \frac{\partial}{\partial\underline{x}_\perp}\right)\zeta^k \frac{\partial^l}{\partial\zeta^l} g_{\alpha\beta}(\tau, \underline{x}_\perp, \zeta)\,d\zeta \ , \qquad (IV.5)$$

where k,l = 0, 1, 2, ..., and D is some polynomial of its arguments. To obtain useful formulas we must assume that derivatives with respect to τ, $\underline{x}_\perp$ are interchangeable with the integral in

(IV.5). This means that the integral

$$\int \zeta^k \frac{\partial^l}{\partial \zeta^l} g_{\alpha\beta}(x)\, d\zeta \qquad \text{(IV.6)}$$

must exist for every k,l= 0, 1, 2, ... Then it follows that

$$\left| \zeta^k \frac{\partial^l}{\partial \zeta^l} g_{\alpha\beta}(x) \right| \longrightarrow 0 \,, \quad \text{if } |\zeta| \to \infty \,. \qquad \text{(IV.7)}$$

In general, this condition does not fulfill even if the function $g_{\alpha\beta}(x)$ is a matrix element of the current s(x) between normalizable states ϕ_α, ϕ_β. Since the variable ζ was arbitrarily chosen as integration variable the strong asymptotic behaviour (IV.7) must be required also for the dependences on τ and $\underline{x}_\perp$. But this class of functions is mapped onto itself by Fourier transformation, therefore, if

$$\hat{g}_{\alpha\beta}(q) = \int_{-\infty}^{\infty} g_{\alpha\beta}(x)\, e^{iqx}\, d^4x \,, \qquad \text{(IV.8)}$$

then it follows, for example, that

$$(q^2)^n \; \hat{g}_{\alpha\beta}(q) \to 0 \qquad \text{(IV.9)}$$

for any n=0, 1, 2, ..., if $|q^2| \to \infty$. If, especially, $g_{\alpha\beta}(x)$ is a matrix element of s(x) between normalizable superpositions of momentum eigenstates, the condition (IV.9) means, in general, that the form factor

$$F((k-k')^2) = \langle k' | s(0) | k \rangle$$

decreases faster than any inverse power of $(k-k')^2$, if $|(k-k')^2| \to \infty$. If do not want such an unduly restricted theory, we must accept that the integrals (IV.5) diverge, and we must decide on the meaning we are going to attribute to them. This is, in fact, a reformulation of the problem of the mapping (IV.3).

With special attention to the purpose of describing functions in the IMF we define (IV.5) as follows:

$$\int_{-\infty}^{\infty} D(\tau, \underline{x}_\perp; \frac{\partial}{\partial \tau}, \frac{\partial}{\partial \underline{x}_\perp}) \zeta^k \frac{\partial^l}{\partial \zeta^l} g_{\alpha\beta}(x) d\zeta \equiv \qquad \text{(IV.10)}$$

$$\equiv D(\tau, \underline{x}_\perp; \frac{\partial}{\partial \tau}, \frac{\partial}{\partial \underline{x}_\perp}) g_{\alpha\beta}(\tau, \underline{x}_\perp),$$

where

$$g_{\alpha\beta}(\tau, \underline{x}_\perp) \equiv \int_{-\infty}^{\infty} g_{\alpha\beta}(\tau, \underline{x}_\perp, \zeta) d\zeta \qquad \text{(IV.11)}$$

denotes the canonical distribution theoretic value of the integral /6/.

First of all, it follows from the definition (IV.10) that

$$\lim_{\xi\to\infty} \int T(e^{-i\xi N_3}) g_{\alpha\beta}(\tau, \underline{x}_\perp, \zeta) d\zeta = g_{\alpha\beta}(0, \underline{x}_\perp). \qquad \text{(IV.12)}$$

This is the function in the IMF which corresponds to the unit element of the group $(a,\Lambda)_\infty$, independently of the actual group contraction scheme we want to choose. Let us notice that in (IV.12) we arrived at a function of two variables only.

In order to construct the other $g^\infty_{\alpha\beta}$ functions one must calculate the action of the generators of the group $(a,\Lambda)_\infty$ on $g_{\alpha\beta}(0, \underline{x}_\perp)$. In the 2-Galilei case we proceed by using (II.17), (II.19), (III.1), (III.10) and obtain:

$$\begin{aligned} S_i\, g_{\alpha\beta}(0,\underline{x}_\perp) &= 0, \qquad i=1,2, \\ M_3\, g_{\alpha\beta}(0,\underline{x}_\perp) &= i(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y})\, g_{\alpha\beta}(0,\underline{x}), \\ \underline{P}_\perp\, g_{\alpha\beta}(0,\underline{x}_\perp) &= -i\frac{\partial}{\partial \underline{x}_\perp} g_{\alpha\beta}(0,\underline{x}), \\ H^G\, g_{\alpha\beta}(0,\underline{x}_\perp) &= -\frac{i}{\lambda}\frac{\partial}{\partial \tau} g_{\alpha\beta}(0,\underline{x}_\perp). \end{aligned} \qquad \text{(IV.11)}$$

From these relations one can easily reconstruct all the functions $g^{\infty}_{\alpha\beta}(\tau, \underline{x}_{\perp}; (a,\Lambda)_{\infty})$:

$$g^{\infty}_{\alpha\beta}(\tau, \underline{x}_{\perp}; (a,\Lambda)_{\infty}) \equiv g^{G}_{\alpha\beta}(0, \underline{x}_{\perp}; (\ell, L)) \equiv g_{\alpha\beta}(\tfrac{1}{\lambda}\ell_t, \underline{x}_{\perp}R + \underline{\ell}_{\perp}), \tag{IV.12}$$

where (ℓ, L) denotes a general, six-parameter element of the 2-Galilei group. Its homogeneous part $L \equiv (R, \underline{v}_{\perp})$ involves rotations R and Galilei transformations in the two-dimensional plane $\underline{x}_{\perp}=(x,y)$. Its inhomogeneous part $\ell = (\ell_t, \underline{\ell}_{\perp})$ corresponds to "time" and space translations. Altogether the transformation rule:

$$(t_0, \underline{x}_{\perp})(\ell, L) = (t_0+\ell_t, \underline{x}_{\perp}R + t_0\underline{v}_{\perp} + \underline{\ell}_{\perp}), \qquad R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{IV.13}$$

The functions (IV.12) depend on the variables $\underline{x}_{\perp}=(x,y)$ only, and look like the functions

$$f^{G}_{\alpha\beta}(t_0, \underline{x}_{\perp}; (\ell, L)) \equiv e^{it_0 H^{G}} g^{G}_{\alpha\beta}(0, \underline{x}_{\perp}; (\ell, L)) \tag{IV.14}$$

of the variables $(t_0, \underline{x}_{\perp})$ for zero value of the non-relativistic "time" t_0. It follows from the construction of the functions (IV.14) that a scalar representation of the 2-Galilei group can be defined on these functions by the rule:

$$T^{G}(\ell', L') f^{G}_{\alpha\beta}(t_0, \underline{x}_{\perp}; (\ell, L)) = f^{G}_{\alpha\beta}(t_0, \underline{x}_{\perp}; (\ell', L')(\ell, L)) =$$
$$= f^{G}_{\alpha\beta}(t_0+\ell'_t, \underline{x}_{\perp}R' + t_0\underline{v}'_{\perp} + \underline{\ell}'_{\perp}; (\ell, L)) . \tag{IV.15}$$

This procedure can be repeated also when $(a,\Lambda)_{\infty}$ is the 2-Poincaré group. Equations (IV.11) remain unchanged in the case of M_3 and $\underline{P}_{\perp}$, the action of the generators $\underline{K}_{\perp}=(K_1, K_2)$, H^{P} on $g_{\alpha\beta}(0, \underline{x}_{\perp})$

is as follows:

$$\underline{K}_\perp \, g_{\alpha\beta} (0, \underline{x}_\perp) = -i \, \frac{1}{2\lambda c^2} \underline{x}_\perp \frac{\partial}{\partial \tau} \, g_{\alpha\beta} (0, \underline{x}_\perp) \; ,$$
$$H^P \, g_{\alpha\beta} (0, \underline{x}_\perp) = -i \, \frac{1}{2\lambda} \frac{\partial}{\partial \tau} \, g_{\alpha\beta} (0, \underline{x}_\perp) \; . \qquad \text{(IV.16)}$$

Instead of eq. (IV.12) now the functions

$$g^{\infty}_{\alpha\beta} \, (\tau, \, \underline{x}_\perp ; (a, \Lambda)_\infty) \equiv g^P_{\alpha\beta} \, (0, \, \underline{x}_\perp ; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) \equiv g_{\alpha\beta} (\overset{\circ}{\underset{\sim}{x}} \underset{\sim}{\Lambda} + \hat{\underset{\sim}{a}}) \qquad \text{(IV.17)}$$

appear, where $(\underset{\sim}{a}, \underset{\sim}{\Lambda})$ denotes a general element of the 2-Poincaré group, $\underset{\sim}{\Lambda}$ being a 3x3 matrix for the homogeneous 2-Lorentz transformations, the three-vector $\underset{\sim}{a} = (a_0, \, \underline{a}_\perp)$ refers to the translations of the 1+2 dimensional Minkowski space-time $\underset{\sim}{x} = (t, \, \underline{x}_\perp)$. In (IV.17) also the following notations are used:

$\overset{\circ}{\underset{\sim}{x}} = (0, \, \underline{x}_\perp)$, $\hat{\underset{\sim}{a}} = (\frac{1}{2\lambda} a_0 , \, \underline{a}_\perp)$. Again, the two-variable functions (IV.17) can be provided with "time dependence" by means of the definition

$$f^P_{\alpha\beta} \, (t, \, \underline{x}_\perp ; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) \equiv e^{itH^P} \, g^P_{\alpha\beta} \, (0, \underline{x}_\perp ; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) . \qquad \text{(IV.18)}$$

On the functions (IV.18) it is easy to define a scalar representation of the 2-Poincaré group:

$$T^P \, (\underset{\sim}{a}', \underset{\sim}{\Lambda}') \, f^P_{\alpha\beta} \, (\underset{\sim}{x}; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) = f^P_{\alpha\beta} \, (\underset{\sim}{x}; (\underset{\sim}{a}', \underset{\sim}{\Lambda}')(\underset{\sim}{a}, \underset{\sim}{\Lambda})) =$$
$$= f^P_{\alpha\beta} \, (\underset{\sim}{x} \underset{\sim}{\Lambda}' + \underset{\sim}{a}' ; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) . \qquad \text{(IV.19)}$$

Only the last point, Problem III , of our programme remains, namely, to express the equations (IV.15) and (IV.19) as transformation rules of the transverse current $\int s(\tau, \underline{x}_\perp, \zeta) d\zeta$ in the IMF. In the 2-Galilei case we interpret the functions

$f^{G}_{\alpha\beta}\ (t_0,\ \underline{x}_\perp;(\ell,L))$ as matrix elements of a 2-Galilei scalar current

$$S(t_0,\underline{x}_\perp) \equiv U_G^{-1}(e^{it_0 H^G}) \times \tag{IV.20}$$
$$\times \left\{ \lim_{\xi\to\infty} \int_{-\infty}^{\infty} U^{-1}(\xi)\, s(\tau,\ \underline{x}_\perp, \zeta)\, U(\xi)\, d\zeta \right\} U_G(e^{it_0 H^G})$$

between the states $U_G(\ell,L)\phi_\alpha$, $U_G(\ell,L)\phi_\beta$, where $U_G(\ell,L)$ stands for the operators representing the 2-Galilei subgroup, generated by (III.1), of the 3-Poincaré group on the physical states ϕ.

By similar definition the operator

$$s(t,\ \underline{x}_\perp) \equiv U_P^{-1}(e^{itH^P}) \times \tag{IV.21}$$
$$\times \left\{ \lim \int_{-\infty}^{\infty} U^{-1}(\xi)\, s(\tau,\ \underline{x}_\perp, \zeta)\, U(\xi)\, d\zeta \right\}\ U_P(e^{itH^P})$$

is a scalar operator with respect to the 2-Poincaré group. As we have shown, the integral in both cases assumes a careful definition.

Before concluding this section we shortly discuss vector currents $j_\mu(\tau,\ \underline{x}_\perp, \zeta)$ in the IMF. By repeating the arguments described in detail for the scalar current one obtains the following results.

1. If the 3-Poincaré group is contracted into the 2-Galilei one the following quantities have simple transformation properties in the IMF:

$$\underline{J}_\perp(t_0,\underline{x}_\perp) \equiv U_G^{-1}(e^{it_0H^G})\left\{\lim \int_{-\infty}^{\infty} U^{-1}(\xi)\, \underline{j}_\perp(\tau,\underline{x}_\perp,\zeta)\, d\zeta\right\} U_G(e^{it_0H^G}), \tag{IV.22}$$

$$\rho_\pm(t_0,\underline{x}_\perp) \equiv \frac{1}{2}\, U_G^{-1}(e^{it_0H^G}) \times$$
$$\times \left\{ \lim\ e^{\pm\xi} \int_{-\infty}^{\infty} U^{-1}(\xi)\left[j_0 \pm j_3\right] U(\xi)\, d\zeta \right\} U_G(e^{it_0H^G}). \tag{IV.23}$$

The transformation rules for $\underline{J}_\perp$, ρ_+ and ρ_- are those of the two-momentum, energy and mass densities, respectively, in a non-relativistic theory. (See also ref.2)

2. When the 3-Poincaré group is contracted into the 2-Poincaré one in the IMF a vector current $\underset{\sim}{J}(\underset{\sim}{x}) \equiv \left(J_o(\underset{\sim}{x}), \underline{J}_{\perp}(\underset{\sim}{x}) \right)$ and a scalar one $s(\underset{\sim}{x})$ can be found. They can be expressed by formulas similar to eq. (IV.21), only the quantities

$$\frac{1}{4\lambda} e^{\xi} (j_c + j_3) + \lambda c^2 e^{-\xi} (j_o - j_3) \ , \quad \underline{j}_{\perp} \qquad \text{(IV.24)}$$

and $\lambda e^{-\xi}(j_o - j_3) - \frac{1}{4\lambda c^2} e^{\xi} (j_o + j_3)$, (IV.25)

respectively, are to be substituted for $s(\tau, \underline{x}_{\perp}, \zeta)$.

Finally, we must remark that, naturally, the results obtained in this section for the current transformation properties depend very much on the solution we have given for Problem II , or, explicitly, on the definition (IV.10,11). Other prescriptions for the restriction of the 3-Poincaré matrix elements to some IMF functions can, of course, be proposed, but, in general, then one finds more complicated expressions instead of (IV.11) or (IV.16) which are of basic importance in the construction of all functions in the IMF. Also, the relations (IV.11) or (IV.16) made us able to recognize definit transformation properties, which would otherwise be complicated and of no use.

V. A simple application

In this concluding section we illustrate how the procedure described works in practice. The simplest possible objects to consider are the matrix elements of a scalar current between momentum eigenstates with zero spin:

$$g_{\alpha\beta}(\tau,\ \underline{x}_\perp;(a,\Lambda)) = \int_{-\infty}^{\infty} \langle p'|\, s(x\Lambda+a)\,|p\rangle\, d\zeta\ , \qquad (V.1)$$

the states ϕ_α and ϕ_β being labelled by the four-momenta $p_\mu = (p_+, \underline{p}_\perp, p_-)$, $p_\mu^2 = p_+ p_- - \underline{p}_\perp^2 = m^2$, and $p_\mu' = (p_+', \underline{p}_\perp', p_-')$, $p_\mu'^2 = m^2$, respectively. In the ordinary reference frame and for $(a,\Lambda)=(0,1)$ the function (V.1) can be written as

$$g_{\alpha\beta}(\tau,\underline{x}_\perp) = 2\pi F\left((p-p')^2\right) e^{i\frac{1}{2}\tau(p_+-p_+')+i(\underline{p}_\perp-\underline{p}_\perp')\underline{x}_\perp}\ \delta(p_- - p_-') . \qquad (V.2)$$

As usual, the dependence of F on m is not denoted. Now we examine the functions (V.1) in the IMF in that case when the 2-Poincaré scheme of contraction is used. According to the conclusions of the previous section the counterpart $f_{\alpha\beta}(\underset{\sim}{x})$ of $g_{\alpha\beta}(\tau,\underline{x}_\perp)$ in the IMF appears as the matrix element of a 2-Poincaré scalar current $s(\underset{\sim}{x})$ between some states $|\underset{\sim}{k},\mu\rangle$ and $|\underset{\sim}{k}',\mu'\rangle$ corresponding to $|p\rangle$ and $|p'\rangle$, respectively. A state $|\underset{\sim}{k},\mu\rangle$ is a momentum eigenstate in an irreducible 2-Poincaré representation space with "spin" zero and "mass" $\underset{\sim}{m}$, $\underset{\sim}{m}^2 c^2 = m^2 + \mu^2 c^2$, (V.3)

$$\mu = \lambda p_- - \frac{1}{4\lambda c^2} p_+ . \qquad (V.4)$$

Based upon the standard arguments we may write:

$$\langle \underset{\sim}{k}', \mu' | s(\underset{\sim}{x}) | \underset{\sim}{k}, \mu \rangle = \tilde{F}(\mu, \mu', (\underset{\sim}{k} - \underset{\sim}{k}')^2) e^{i(\underset{\sim}{k}-\underset{\sim}{k}')\underset{\sim}{x}} , \qquad (V.5)$$

where

$$(\underset{\sim}{k}-\underset{\sim}{k}')^2 = \frac{1}{c^2}(k_0 - k_0')^2 - (\underline{k}_\perp - \underline{k}_\perp')^2 \equiv (p-p')^2 + c^2(\mu - \mu')^2 , \qquad (V.6)$$

$$(\underset{\sim}{k}-\underset{\sim}{k}')\underset{\sim}{x} = (k_0 - k_0')t + (\underline{k}_\perp - \underline{k}_\perp')\underline{x}_\perp , \qquad (V.7)$$

$$k_0 = c\left[\underset{\sim}{m}^2 c^2 + \underline{k}_\perp^2\right]^{1/2} , \qquad k_0' = c\left[\underset{\sim}{m}'^2 c^2 + \underline{k}_\perp'^2\right]^{1/2} . \qquad (V.8)$$

Notice, firstly, that we did not provide the function $\tilde{F}$ with a dependence on λ. This is explained by the arguments of Sect.III. Secondly, one must notice that while in the ordinary reference frame one could express the matrix element (V.2) by means of an unknown function F of one variable only, now in (V.5) the function $\tilde{F}$ of three variables has appeared. Carrying out, however, on (V.1,2) the procedure we have described in the previous section for the reduction of 3-Poincaré covariant functions to 2-Poincaré covariant ones the dependence of $\tilde{F}$ on μ and μ' can be made explicit. What remains is again an unknown function of $(p - p')^2$.

First we consider the special case when $\underline{p}_\perp = 0$. From the general formulas (V.4) and (V.8), and from

$$k_0 = \frac{1}{4\lambda} p_+ + \lambda c^2 p_- \qquad (V.9)$$

it follows that

$$2\lambda c p_- = \mu c + \left[m^2 + \mu^2 c^2\right]^{1/2} . \qquad (V.10)$$

Once the value of the parameter λ is specified the quantity μ, or, the 2-Poincaré mass $\underset{\sim}{m}$, can be determined as function of p_-

and c. Furthermore, by identifying $\underline{k}'_\perp = \underline{p}'_\perp$, and using

$$\mu' = \lambda p'_- - \frac{1}{4\lambda c^2} p'_+ \,, \qquad (V.11)$$

the argument $p_- - p'_-$ of the Dirac delta in (V.2) gets easily expressed in terms of μ, μ' and $(p - p')^2$:

$$p_- - p'_- = \frac{1}{2\lambda}\left[(\mu - \mu')(1 + \frac{\mu c}{\sqrt{m^2 + \mu'^2 c^2}}) + \frac{(p - p')^2}{2c\sqrt{m^2 + \mu'^2 c^2}}\right] . \qquad (V.12)$$

Now we may write the equality:

$$\tilde{F}(\mu, \mu'; (\underline{k} - \underline{k}')^2) = 2\pi F((p-p')^2)\, \delta(p_- - p'_-) . \qquad (V.13)$$

Let us remark that in the limit $c \to \infty$ eqs. (V.3,12,13) reproduce the formulas of the 2-Galilei symmetric case familiar from ref.3. It is also worth mentioning that while in the 2-Galilei case a simple scaling property of $\tilde{F}$ follows from the "no λ-dependence" assumption /3/, in the 2-Poincaré case a rather complicated implicit relation can be extracted from eqs. (V.10,11,12,13) for the function $\tilde{F}$.

To complete the discussion of (V.2) in the IMF only the case $\underline{p}_\perp \neq 0$, $\underline{p}'_\perp \neq 0$ remained to be dealt with. (For simplicity, we still assume that $\underline{p}_\perp = (p_1, 0)$.) We make explicitly use of the freedom in choosing the matrices (III.5) up to arbitrary fixed 3-Lorentz transformation. For the matrix A_λ (see the discussion at the end of Sect.III) we choose the one corresponding to

$$e^{-i\alpha(M_2 + N_1)}$$

with

$$\alpha = -\frac{p_1}{p_-} .$$

It is not very hard to verify, that eqs. (V.12,13) remain unchanged, only the mapping between the momenta p_μ , p'_μ and $\underset{\sim}{k}$, $\underset{\sim}{k}'$ must be modified. The mapping in this case is as follows:

$$\underline{k}_\perp = 0 , \qquad \underline{k}'_\perp = (p'_1 + \alpha p'_- , p'_2) , \qquad \text{(V.14)}$$

$$\mu' = \lambda p'_- - \frac{1}{4\lambda c^2}(p'_+ + 2\alpha p'_1 + \alpha^2 p'_-) , \qquad \text{(V.15)}$$

and eqs. (V.8,10) survive.

In possession of these results one may already start with making models for the calculation of various physical processes. These models will contain the free parameter c which remained completely undetermined. What actually happens when transforming into the ILF is that we restrict the surface $(p - p')^2 =$ const. to its subsurfaces, to its intersection with the surface μ = const. To each given value of μ corresponds a two-parameter family of surfaces in our construction, the parameters being λ and c. On the basis of the assumption that the ILF world is of reduced dimensionality one can argue for the equivalence of subsurfaces with different λ , but nothing can be said about the value of c. It is possible that dynamics prefers the "relativistic" subsurfaces with some given c. In this case all such phenomena, like scaling, etc./3/, which agree with the result of calculations making use of the Galilean symmetry, must be considered as the "non-relativistic" limits of "relativistic" phenomena. There is no a priori reason to believe that this is n t the case. If this is, then an enlargement of the present experimental input figures, probably the enlargement of energy, must be accompanied by remarkable changes in the present experimental trends, by the breakdown of scaling, and so on.

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