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ON REALIZATION OF THE SPACE OF DYNAMIC SU(3) GROUP SYMMETRY IN V_4

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In previous publications [1,2] we considered various aspects of the realization of symmetry groups in the four-dimensional space-time V_4. The present work deals with embedding of the space of the dynamic SU(3) symmetry group in the V_4 space, i.e. with construction of the fibre bundle E, which locally represents a product $E \sim F \times V_4$, where the fundamental representation of the SU(3) group is realized within the fibre F. A formulation is given of the coloured quark SU(3) theory of weak interaction.

The Poincare group $\mathcal{P}$ represents the symmetry group of the space-time V_4. This group is a kinematical group, i.e. the dynamic nature of the origin of spin has not yet been established. $x_\mu x^\mu$ is an invariant in the V_4 space. Therefore the effective dimensionality of the V_4 space is 3, instead of 4 (we choose the compact part V_4).

If one assumes (as in ref.2) that the symmetry group of the V_4 space is the Poincare group $\mathcal{P}$, which is not simple, $\mathcal{P} \to \mathcal{P} \to \overline{SU(2)} \times SU(2) \times T^4$, and the conventional definition is taken into account of the homomorphism between the SO(3) and SU(2) groups, then in V_4 only such groups can be realized that are characterized by three parameters (SO(3), SU(2)).

On the other hand, no relationship usually is assumed, in the general case, between the dimensionality of the basis space and that of the fibre [3] (or between the number of parameters in the basis and in the fibre).

However, when physical processes proceeding in the space-time are considered, it is necessary to relate the dimensionalities (or numbers of parameters) of the basis and the fibre. This is due to the fields related to dynamic symmetry groups being material. Now, these fields, being material too, must be related to the characteristics of the space itself. Here two extreme versions may be encountered: 1) in V_4 there exists a vector x_μ characterizing coordinates of the V_4 space, and then in this space a single vector can be defined that is associated with the said vector (i.e. in V_4 one can define a U(1) theory [1]); 2) in V_4, which is characterized by an effective dimensionality equal to three, the space of the symmetry group can be embedded in two ways: a) in accordance with the number of parameters (n = 3) of this space - this will be the G = SU(2) group [2], b) in accordance with the dimensionality of the space - such will be the G = SO(3), SP(3), SU(3) groups.

In the latter case, of the three groups SO(3), SP(3), SU(3) the most preferable, from a physical point of view, is the SU(3) group, the elements of which are unitary and unimodular (the usual conditions required of physical theories). For realization of the SO(3) and SP(3) groups we must impose additional conditions, besides unitarity and unimodularity, for the foundation of which additional physical restrictions are required.

Thus, we have arrived at the conclusion that the space of the SU(3) symmetry group can be realized in V_4. But which of the SU(3) representations must be realized in V_4? The obvious answer is that in V_4 only the fundamental representation of the SU(3) group is realized. This means that F (a fibre) is a space of the fundamental representation of the SU(3) group (the basis is V_4), while E represents an adjoint fibre bundle.

The covariant derivative of an arbitrary geometrical object in E has the form [4]

$$Y^J_{;\mu} = Y^J_{\mu} + \Gamma^a_\mu \, \phi^J_a (Y) \,.$$

For linear representations of SU(3) we obtain $Y^J \to \psi(x)$, a spinor, $\Gamma^a_\mu \to A^a_\mu$, a vector, T_a are generators of the SU(3) group,

$$\psi_{;\mu} = -\frac{\partial \psi}{\partial x^\mu} + T_a A^a_\mu \, \psi \, ; \quad a = 1 \div 8 .$$

The curvature tensor of the fibre bundle E is expressed through the connectedness coefficients Γ^a_μ as follows:

$$R^a_{\mu\nu} = \partial_{[\mu}\Gamma^a_{\nu]} - \frac{1}{2} f^a_{fc}\, \Gamma^b_{[\mu}\Gamma^c_{\nu]}$$

(f^a_{bc} is the structure constant of the SU(3) group); then, from $\Gamma^a_\mu \to A^a_\mu$ follows

$$F^a_{\mu\nu} = \partial_{[\mu} A^a_{\nu]} - \frac{1}{2} f^a_{bc}\, A^b_{[\mu} A^c_{\nu]} \, .$$

The expression

$$\Lambda = \bar{Y}^J D_\mu Y_J - \frac{1}{4} R^a_{\mu\nu} R^{\mu\nu}_a \qquad (1)$$

is a geometrical invariant, which, as $Y^J \to \psi$ and $\Gamma^a_\mu \to A^a_\mu$, acquires the form

$$\mathcal{L}(x) = \bar{\psi} D_\mu \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\ \mu\nu} \, . \qquad (2)$$

$\mathcal{L}(x)$ is the Lagrangian of the Yang-Mills fields ψ and A^a_μ [5]. That in embedding the space of the symmetry group G = SU(3) in V_4 one must take as a basis the spatial, instead of the parametric, embedding is seen from the fact that the appearance of external fields can be reduced to motion along curved trajectories. This means that if we consider a plane three-dimensional space, it can be parametrized by three parameters, while a curved three-dimensional space (given by the fields A^a_μ) must be parametrized by eight parameters (in the general case).

At present, the commonly accepted theory of strong interaction is QCD [5], i.e. dynamic SU(3) theory with the Lagrangian (2), in which coloured quarks interact by means of eight gluons.

Just like coloured SU(3) interaction is introduced for strongly interacting particles, one can introduce coloured $SU_c(3)$ interaction for weakly interacting particles (since gluons do not take part in weak interactions, the space in which $SU_c(3)$ is embedded, is also V_4). Then $\binom{\alpha_\rho}{\beta_\rho}$, the weak isospin doublet, $I_W = 1/2$, of coloured quarks interacts by means of S coloured gluons, g_W. The following are colourless states:

$$\begin{array}{lll}
\rho = 1, 2, 3 & & \ell \to \epsilon^{\rho\sigma\delta}\, \alpha_\rho \alpha_\sigma \beta_\delta \\
\quad Q \quad \ell & & \nu \to \epsilon^{\rho\sigma\delta}\, \alpha_\rho \beta_\sigma \beta_\delta \\
\alpha_\rho = \ \ 2/3 \quad 1/3 & & \Delta'^{++} \to \epsilon^{\rho\sigma\delta}\, \alpha_\rho \alpha_\rho \alpha_\delta \\
\beta_\rho = -1/3 \quad 1/3 & & \Delta'^{+} \to \epsilon^{\rho\sigma\delta}\, \alpha_\rho \alpha_\sigma \beta_\delta \\
 & & \Delta'^{o} \to \epsilon^{\rho\sigma\delta}\, \alpha_\rho \beta_\sigma \beta_\delta \\
 & & \Delta'^{-} \to \epsilon^{\rho\sigma\delta}\, \beta_\rho \beta_\sigma \beta_\delta
\end{array} \qquad (3)$$

$$W^{+,-,o} \to \bar{\beta}^\rho \alpha_\rho \, , \quad \bar{\alpha}^\rho \beta_\rho \, , \quad \frac{1}{\sqrt{2}} (\bar{\alpha}^\rho \alpha_\rho - \bar{\beta}^\rho \beta_\rho) \, ,$$

$$B \to \frac{1}{\sqrt{2}} (\bar{\alpha}^\rho \alpha_\rho + \bar{\beta}^\rho \alpha_\rho) \, .$$

Within this approach two versions of the mixing of W° and B are feasible, which are considered in ref.[6].

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О реализации пространства динамической группы симметрии SU(3) в V_4

Данная работа посвящена рассмотрению вложения пространства динамической группы симметрии SU(3) в пространство V_4, т.е. построению расслоенного пространства E, которое локально является произведением $E \sim F \times V_4$, где в слое F реализуется фундаментальное представление группы SU(3). Формулируется кварковая цветная $\mathrm{SU}_c(3)$ теория слабого взаимодействия.

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The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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