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BJORKEN SUM RULE AND DEEP INELASTIC SCATTERING ON POLARIZED NUCLEI

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1. In spite of that the structure functions (SF) of deep inelastic scattering (DIS) are studied extentively, this problem continues to attract considerable attention nowadays motivated, in particular, by recent EMC experimental data[1] of DIS on polarized protons. It has been found there that the fraction of proton spin carried by quarks is close to zero ("spin crisis"). This discovery stimulated a number of works devoted to the spin content of nucleon[2,3,4]. Since the situation is still unclear, there is a great need of further experimental information on the spin, isospin and the flavour structures of nucleons. Consequently, the polarized lepton DIS on polarized nuclei should be a source of unique data. This especially concerns neutrons in view of the obvious difficulties with the neutron targets. On the other hand, this type of experiments would be important for the study of the nuclear structure and models using in description of DIS on nuclei such as a convolution model[5,6].

In the present letter we discuss (i) the problems of extraction of the first moments of the nucleon SF from the nuclear SF in polarized DIS processes; and (ii) the connection of the nuclear SF with the conventional nuclei data (weak decay, nuclear magnetic moments etc.).

Accordingly, we consider below the spin-dependent SF $G_1^T(x,Q^2)$ for different targets T (nucleons and nuclei). Also the Bjorken sum rule (BSR)[7] is discussed, which is one of the basic relations used in exploration of the "spin crisis"[2,3,4] (besides, it allows one to relate the SF $G_1^T(x,Q^2)$ to the quantities used in the standard nuclear physics): [1]

$$\int_0^1 \frac{dx}{x}(G_1^p(x,Q^2) - G_1^n(x,Q^2)) = \frac{1}{6}\cdot\frac{g_A}{g_V}\cdot(1 - \frac{\alpha_s}{\pi}), \tag{1}$$

where $G_1^{p,n}$ is the spin-dependent SF of a proton and a neutron (denoted through p and n, resp.). This relation can be applied to a nuclear target A in which case we substitute $(g_A/g_V)_A$ for g_A/g_V in the r.h.s. of eq. (1). The direct experimental specification of the spin-dependent SF of nucleons and verification of the basic relations such as BSR, are important in view of the contradictions between the QCD conclusions and proton data combined with indirect data of the quark spin structure of nucleons[3].

In the parton picture, the SF $G_1^T(x)$ is proportional to the probabilities of finding, in the target T, a constituent with the carried momentum fraction x and with the spin parallel and antiparallel to that of the target:

$$G_1^T(x) = \sum_a e_a^2 x[q_{a\uparrow/T\uparrow}(x) - q_{a\downarrow/T\uparrow}(x)], \tag{2}$$

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[1] Note that BSR has been obtained before the QCD creation so it did not contain perturbative corrections such as $(1-\alpha_s/\pi)$. (Here $\alpha_s(Q^2)$ is the QCD parameter, $\alpha_s \approx 0.27$ at $Q^2 = 15$ GeV^2).

where $T = p, n, A...$; a runs over sorts of partons.

Applying the convolution model to the nuclei, we get for $G_1^A(x)$ [5,6]:

$$G_1^A(x) = \sum_t \int_0^A dy_A G_1^t(x/y_A)[f_{t\uparrow/A\uparrow}(y_A) - f_{t\downarrow/A\uparrow}(y_A)], \qquad (3)$$

where t is a virtual hadron constituent of the nucleus A with SF $G_1^t(x)$ (2). In the present work we shall be interested only in the nucleon structure of nuclei, therefore in what follows $t = p, n$. Besides, we neglected the nucleon off-mass effects such as the EMC-effect since in the case of the lightest nuclei the off-mass corrections to the typical integral values [such as (1)] are insignificant [8]. Then, the integration of eq. (3) gives:

$$\int_0^1 \frac{dx}{x} G_1^A(x) = \sum_{t=p,n} \int_0^1 \frac{dx}{x} G_1^t(x) \cdot [P_{t\uparrow/A\uparrow} - P_{t\downarrow/A\uparrow}], \qquad (4)$$

where $P_{t\uparrow/A\uparrow}$ resp. $P_{t\downarrow/A\uparrow}$ is the probability to find, in the ground state of the nucleus A, a constituent t with spin parallel, resp., antiparallel to the target one. Within the framework of the conventional nuclear physics, the quantity $[P_{t\uparrow/A\uparrow} - P_{t\downarrow/A\uparrow}]$ can be calculated as a matrix element of the operator $\sum_i \sigma_z^t(i)$ (the sum is carried over the type t nucleons) over the nucleus wave function (WF) $\mid A\rangle$:

$$\int_0^1 \frac{dx}{x} G_1^A(x) = [\int_0^1 \frac{dx}{x} G_1^p(x)] \cdot S^{pA} + [\int_0^1 \frac{dx}{x} G_1^n(x)] \cdot S^{nA}, \qquad (5)$$

where $S^{pA(nA)} = \langle A \mid \sum_{p(n)} \sigma_z^{p(n)} \mid A\rangle$ are the nuclear spin structure factors. Equation (5) is a straightforward consequence of the convolution approach usually applying to the investigations of the DIS on nuclei. However, to interpret this relation, one should be extremely careful in view of (i) the eventual changes of the nucleon properties in nucleus, and in view of that (ii) the simplified consideration of the nuclear structure may lead to incorrect results. It is appropriate now to remember the lessons of the EMC-effect which give rise to a number of explanations using both the nucleon properties modification (Q^2−rescaling, "swelling"...) and the nuclear structure effects (binding effects, mesons...).

The authors of ref. [5] applied BSR (1) and relations like (5) to determine g_A/g_V for bound nucleons using the SF G_1^A and $G_1^{A^*}$ of the mirror nuclei i.e., of A and A^*, where the numbers of protons and neutrons are

reversed, with $J = 1/2$. For the mirror nuclei we have $S^{pA} = S^{nA^*}$, $S^{nA} = S^{pA^*}$ so that:

$$\int_0^1 \frac{dx}{x}(G_1^A(x) - G_1^{A^*}(x)) = \frac{1}{6} \cdot \left(\frac{g_A}{g_V}\right)_{AA^*} \cdot (1 - \frac{\alpha_s}{\pi}) =$$
$$= \frac{1}{6} \cdot \left(\frac{g_A}{g_V}\right)_{bd.\ nucl.} \cdot (1 - \frac{\alpha_s}{\pi}) \cdot [S^{pA} - S^{nA}]. \qquad (6)$$

Following to logic of the convolution approach we must assume that (see ref. [5]): $(g_A/g_V)_{bd.\ nucl.} = g_A/g_V$. In the context of our consideration this implies the coincidence of the first moments of the free and bound nucleon SF $G_1(x)$. However, the ratio g_A/g_V is known to be "renormalized" in nuclei to the effective value $(g_A/g_V)_{bd.\ nucl.}$[9] which can be determined from the β-decay and Gamov-Teller transitions of nuclei. This discrepancy arises from the limitation of the convolution approach which operates only with the nucleon degrees of freedom, whereas the meson and other non-nucleon constituents (Δ-isobars, multiquarks...) also contribute to the nuclear SF. If only nucleons are concerned, one should clearly use the effective constant; the only exception is probably the case of lightest nuclei where the non-nucleon contributions seem to be insignificant.

Next, $[S^{pA} - S^{nA}]$ is the matrix element of the operator $\sigma_3\tau_3$ over the nucleus A WF. Just this matrix element appears in the nuclear β-decay processes. The weak decay experiments with the mirror nuclei[9] give the empirical value $(g_A/g_V)_{AA^*}$ which allows one to determine the renormalized ratio g_A/g_V for bound nucleons:

$$\left(\frac{g_A}{g_V}\right)_{AA^*} = \left(\frac{g_A}{g_V}\right)_{bd.\ nucl.} \langle A \mid \sum_{i=1}^A \sigma_3^i \tau_3^i \mid A\rangle \equiv \left(\frac{g_A}{g_V}\right)_{bd.\ nucl.} \langle\sigma_3\tau_3\rangle. \qquad (7)$$

Equations (6) and (7) illustrate the relationship between DIS and weak processes. Using this relationship, the experimental β-decay information and computing the nuclear spin structure factors, we are able to obtain the neutron SF and the ratio g_A/g_V from the DIS on nuclei.

2. *The deuteron* (D). It seems evidently that the nucleon properties in a deuteron least of all deviate from free ones. Therefore it is just the deuteron that is to be considered as a more convenient source of the polarized neutron data like the unpolarized case[10]. The main problem in the extraction of the "nearly free" neutron SF from the deuteron one is the determination and exclusion of the nucleon (unpolarized or polarized) Fermi-smearing in the deuteron. If we bear in mind to operate with the integral values (1) and (5), the deuteron structure influence comes to

factors $S^{pA,nA}$. [2]

Then the direct calculation in (5) with the realistic deuteron WF yields:

$$\int_0^1 \frac{dx}{x} G_1^D(x) = \int_0^1 \frac{dx}{x} [G_1^p(x) + G_1^n(x)] \cdot S^D, \quad (8)$$

$$S^{pD} = S^{nD} = S^D = 1 - \frac{3}{2} P_D, \quad (9)$$

where P_D is the D-wave weight in the deuteron. Equations (8) and (9) give the way of obtaining the first moments of the polarized neutron SF from the relevant SF of the deuteron and proton. Now, neglecting the change of nucleon properties in the deuteron, it is easy to find the desired expression for $(g_A/g_V)_{pD} \cong g_A/g_V$:

$$\int_0^1 \frac{dx}{x} (2G_1^p(x) - (S^D)^{-1} G_1^D(x)) = \frac{1}{6} \cdot \frac{g_A}{g_V} \cdot (1 - \frac{\alpha_s}{\pi}). \quad (10)$$

Equation (10) is an useful relation for the experimental check of the validity of BSR or, if BSR holds valid, for verification of the perturbative QCD (for more detailed discussions of this problem, see, for instance, ref.[3]). Substituting the realistic value of $P_D = 0.0425$ [13] into (9) we find $S^D \cong 0.936$ which essentially deviates from unity due to the orbital motion of nucleons inside the deuteron (D-wave admixture in the deuteron WF). Taking account of this structure factor S^D in eq. (10) leads to an essential correction in final results in comparison with the case when the deuteron is presented as a simple sum of two nucleons at rest ($S^D = 1$). Using the experimental values for g_A/g_V (1.259 $\pm$ 0.004 from ref.[14]) and for $I_p \equiv \int G_1^p(x)dx/x$ (0.114 $\pm$ 0.012(stat.) $\pm$ 0.026(syst.) from EMC-experiments[1]) we can estimate the role of the structure factor S^D in the experimental determination of the BSR in DIS on polarized protons and deuterons:

$$\int_0^1 \frac{dx}{x} \Big[2G_1^p(x) - G_1^D(x)\Big] =$$

$$= \int_0^1 \frac{dx}{x} \Big[2G_1^p(x) - S^p \cdot G_1^p(x) - S^n \cdot G_1^n(x)\Big] = \quad (11)$$

$$= \frac{1}{6} \frac{g_A}{g_V} \cdot \Big(1 - \frac{\alpha_s}{\pi}\Big) \Big(1 - \frac{3}{2} P_D\Big) + \frac{3}{2} P_D \cdot 2I_p$$

and from eq.(11) we get that its r.h.s. deviate from (10) by ~1 –2%. Taking into account of such an apparently small deviation is important because of the sensitivity of further analysis of BSR and spin contents of the nucleon.

3. *Helium-3 and Tritium* (${}^3H - {}^3He$) *mirror pair.* As it is shown above, the difference of the mirror nuclear SF is connected with the ratio $(g_A/g_V)_{bd.\ nucl.}$ and nuclear spin structure factors $S^{pA(nA)}$ by the unsophisticated relation (6). The ${}^3H - {}^3He$ is the simplest example of a mirror nucleus pair with the well-known WF (see, for instance, refs.[15, 16]). The general form of such a WF is:

$$\Psi_A^{JM_JTM_T} = \sum_{L,m,\mu} R_{Lm}^{\nu}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \cdot (LmS\mu \mid JM_J) \cdot \mid S\mu, TM_T, \nu\rangle, \quad (12)$$

where J, T, M_J, M_T are the total momentum and total isospin momentum and their projections corresponding to the considered nucleus; S and L spin and angular momenta; index ν enumerates the states with different spin-isospin symmetries; $\mid S\mu, TM_T, \nu\rangle$ is the spin-isospin state with symmetry ν. The basic states in WF (12) are the completely symmetric S-state and mixed symmetry S'-state with $L = 0$ and, also, D-state of the mixed symmetry with $L = 2$. The WF (12) allows one to express the nuclear factors (5)-(6) in the explicit form:

$$S^{n^3He} = S^{p^3H} = P_S + \frac{1}{3} P'_S - \frac{1}{3} P_D, \quad S^{p^3He} = S^{n^3H} = \frac{2}{3} P'_S - \frac{2}{3} P_D, \quad (13)$$

$$S^{p^3H} - S^{n^3H} = P_S - \frac{1}{3} P'_S + \frac{1}{3} P_D, \quad (14)$$

where P_S, $P_{S'}$ and P_D are weights of the S-, S'- and D-wave in ${}^3He$ (or ${}^3H$) respectively. Using the typical values $P_S = 0.897$, $P_{S'} = 0.017$ and $P_D = 0.086$ we find:

$$S^{p^3H} - S^{n^3H} = 0.920, \quad (15)$$

[2] Deep inelastic scattering from a polarized deuteron was studied in refs. [11, 12]. The spin-dependent momentum distribution of nucleons in the deuteron was determined. However, the problem of the possibility of the extraction of the neutron SF and thereby of the ratio g_A/g_V was not considered.

which is essentially different from unity given by the standard shell model[5].

Using the experimental value[9] of $(g_A/g_V)_{A=3} \cong 1.2055$ (7) and estimate (15), we get:

$$\left(\frac{g_A}{g_V}\right)_{bd.\ nucl.} \approx 1.310, \qquad (16)$$

instead of 1.259 for the free nucleons. Since the ratio g_A/g_V is renormalized in the $A = 3$ nuclei about 4%, these nuclei are irrelevant sources of the free neutron SF data. However, the possibility to control the BSR remains. Substituting (15), (16) into (6) we find:

$$\int_0^1 \frac{dx}{x}(G_1^{^3H}(x) - G_1^{^3He}(x)) =$$

$$= \frac{1}{6}\cdot\left(\frac{g_A}{g_V}\right)_{bd.\ nucl.}(1 - \frac{\alpha_s}{\pi})\cdot(S^{p^3H} - S^{n^3H}) \approx 0.1836, \qquad (17)$$

which differs slightly ($\sim$ 5%) from the prediction of the usual convolution model using the nuclear structure factor (15) (not unity!):

$$\int_0^1 \frac{dx}{x}(G_1^{^3H}(x) - G_1^{^3He}(x)) =$$

$$= \frac{1}{6}\cdot\frac{g_A}{g_V}(1 - \frac{\alpha_s}{\pi})\cdot(S^{p^3H} - S^{n^3H}) \approx 0.1762. \qquad (18)$$

4. *Concluding remarks*

1. Our analysis shows that in the polarized DIS on nuclei one meet the reefs which are the same as in the unpolarized case (EMC-effect). Namely, the conventional convolution model must be corrected by taken into account that (i) the nucleon properties in a nucleus eventually change and (ii) the simplified consideration of the nuclear structure may lead to the incorrect results. Above, we considered the influence of nuclear spin-orbital structure on the nuclear SF. Then, using the "renormalized" constant g_A/g_V we also effectively took into account other effects such as the nucleon conversion in the nucleus, the non-nucleon degrees of freedom (Δ-isobars, multiquark states,...), etc.

2. To understand the role of the meson exchange currents let us consider a pedagogical example. The averaging operator in (5) is very similar to the conventional nuclear operator of the magnetic moment:

$$\hat{\mu}_z = \mu_p \cdot \sum_p \sigma_z^p + \mu_n \cdot \sum_n \sigma_z^n + \sum_p l_z^p. \qquad (19)$$

It is known that the calculation of the magnetic moments of nuclei is unsatisfactory if only the nucleon degrees of freedom are taken into account. For instance, in the ^3He and ^3H cases the discrepancy is nearly 20%:

$$\mu(^3He)_{calc.} \cong -1.743 \ ; \quad \mu(^3He)_{exp.} \cong -2.128;$$
$$\mu(^3H)_{calc.} \cong 2.558 \ ; \quad \mu(^3H)_{exp.} \cong 2.979.$$

The deviation of $\mu_{calc.}$ calculated with P_S, $P_{S'}$ and P_D given above from the experimental value $\mu_{exp.}$ is explained by the meson exchange corrections [15,17]. Note that in the deuteron case the analogous discrepancy is insignificant ($\sim$ 0.2%). It is an evident suggestion to the small contribution of meson corrections in deuteron and, therefore, the renormalzation effects are negligible.

The inclusion of the meson exchange currents into the analysis of deep inelastic scattering on nuclei was explored, in the unpolarized cases, in refs.[8]. For the matrix elements of axial-vector currents the meson-exchange corrections were investigated in refs.[18] in nuclear weak processes.

3. The established relations among the nuclear structure, the renormalized ratio g_A/g_V and the nuclear SF allow one to predict the $(g_A/g_V)_A$ values in (1). In particular, according to the experimental values of $(g_A/g_V)_A$[9], the BSR for nuclei must have a nontrivial A-dependence. On the other hand, one may use relation (6) to extract the renormalized ratio g_A/g_V from the DIS experimental data.

4. In our opinion, the extraction of the ratio g_A/g_V from the combined proton and deuteron SF is not worse than the extraction from mirror nucleus data. In both cases the calculation procedure is model-dependent since, inevitably we must deal with the model of the nuclear WF determining the nuclear factors $S^{pA,nA}$. Besides, the WF of the "exactly solvable" nuclear model, the deuteron, is the best understood object of the nuclear physics, and the renormalization effects are the smallest possible.

5. Note that the calculation of the x-behavior of the SF G_1^A is a more complicated problem than the calculation of its first moment. This

difficulty is especially relevant for heavy nuclei because the corresponding WF become extremely complicated and in view of the noticeable off-mass effects[19].

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