ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-89-859

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THE PROTON QUARK SPIN AND THE $U_A(1)$ PROBLEM

Submitted to "Ядерная физика"

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1989

1. Recently, the European muon collaboration (EMC) has measured the spin-dependent nucleon structure function $g_1^p(x)$ in the region $x > 0.01$ at average momentum transfer squared $< Q^2 >= 10.7\ Gev^2$. Extrapolating these data to $0 \leq x < 0.01$, the EMC group obtained the estimation of the first moment of the structure function [1]

$$M_1^p \equiv \int_0^1 dx\ g_1^p(x) = 0.126 \pm 0.010 \pm 0.015\,. \tag{1}$$

In QCD OPE on the light-cone, within the leading twist approximation M_1^p is related to the matrix element of the flavour-singlet axial-vector current

$$j_{05}^\mu = \sum_{i=1}^{N_f} \bar{q}_i \gamma_\mu \gamma_5 q_i \tag{2}$$

over the polarized proton states $| p, s >$ (p is the proton momentum, s is its spin with $s^2 = -M^2,\ sp = 0$). The matrix element determines the $\Delta q'$ value of a proton spin carried by quarks and antiquarks

$$\Delta\Sigma s_\mu(p) = < p, s \mid \bar{q}\gamma_\mu\gamma_5 q \mid p, s >\,. \tag{3}$$

It is important to note that there is no gauge invariant gluonic operator with the same dimension and spin.

The result (1) unexpectedly turned out to be in strong disagreement with the predictions of the naive quark-parton model (Ellis-Jaffe sum rule [2]) where

$$M_1^p(EJ) = 0.19 \tag{4}$$

and absence of strangeness in a proton is assumed.

2.In [3]-[5], it was supposed that naive interpretation is incorrect because due to nonconservation of axial current the singlet part of M_1^p receives an additional (anomalous) contribution related with gluonic degrees of freedom. However, within QCD perturbation theory there is no any additional contribution [6,7]. The results obtained earlier in[4,5] appeared to be invalid because of incorrect use of the factorization procedure.

It is known that axial anomaly trully resolves the $U_A(1)$ problem only within the nonperturbative approach or through instantons[8] or in the current algebra[9]. Both the techniques were applied to estimate the axial anomaly contribution to the proton quark helicity[10]-[15].

3.In the first approach[10,11], the fact that a quark changes its chirality on an instanton is used. Further, the QCD vacuum is supposed to be an

instanton liquid[16,17] with the instanton (antiinstanton) density $n^+(n^-)$: $n^\pm \approx 0.8\ 10^{-8}\ Gev^4$ and effective instanton size $\rho_c \approx 2\ Gev^{-1}$. Then, the anomaly is a consequence of the polarization of instanton vacuum and gives the contribution to the polarised proton helicity[10,11]

$$\Delta q' = \Delta q - 2N_f < n^+ - n^- > V_p, \tag{5}$$

where V_p is the four dimensional "value" of a proton. The value $< n^+ - n^- >$ is nonzero and proportional to the topological susceptibility of QCD vacuum that is expressed through the $\eta - \eta'$ mass splitting[9].

Within this approach it is evident that there is no any computed in the QCD perturbative theory anomalous contribution to $\Delta q'$. In fact, this contribution is proportional to the topological charge of a polarized proton[10]

$$\Delta q' - \Delta q = -\frac{\alpha_s}{\pi} \int d\vec{x}\ tr\ \epsilon^{ijk}(A_i \partial_j A_k + \frac{2}{3} g A_i A_k A_k), \tag{6}$$

which is nonzero only on topological nontrivial instanton configurations. Absence of anomalous contribution in the QCD perturbation theory is proved by strong calculations in[18] based only on general considerations of guage invariance and analyticity.

4.In the second approach[12]-[15] based on the current algebra the anomaly is related directly to η'–meson parameters[9,14]. In[15], in the chiral limit a relation between proton spin carried by quarks and η' couplings has been derived. Here, we will estimate mass correction to this relation.

Let us rename $q^\mu \equiv \bar{q}\gamma_\mu\gamma_5 q,\ q^5 \equiv \bar{q}\gamma_5 q$ and consider the divergence relations

$$\begin{aligned} \partial_\mu(u^\mu + d^\mu - 2s^\mu) &= 2i(m_u u^5 + m_d d^5 - 2m_s s^5), \\ \partial_\mu(u^\mu + d^\mu + s^\mu) &= 2i(m_u u^5 + m_d d^5 + m_s s^5) + \\ &+ N_F \frac{\alpha_s}{8\pi} G\tilde{G}, \end{aligned} \tag{7}$$

where $\tilde{G}_A^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma} G_{\lambda\sigma}^A$. Generally, the matrix elements of different terms in (7) over proton states with small momentum transfer are expressed as

$$\begin{aligned} < p' \mid \partial^\nu u_\nu + \partial^\nu d_\nu + \partial^\nu s_\nu \mid p > &= [2MG_1(q^2) + q^2 G_2(q^2)]\bar{q}'\gamma^5 q, \\ < p' \mid \partial^\nu u_\nu + \partial^\nu d_\nu - 2\partial^\nu s_\nu \mid p > &= [2MF_1(q^2) + q^2 F_2(q^2)]\bar{q}'\gamma^5 q, \\ < p' \mid N_F \frac{\alpha_s}{8\pi} G\tilde{G} \mid p > &= [2M\tilde{G}_1(q^2) + q^2 \tilde{G}_2(q^2)]\bar{q}'\gamma^5 q. \end{aligned} \tag{8}$$

The solution of the $U_a(1)$ problem within the current algebra gives the following behaviour of the form-factors in the limit $q^2 \to 0$[9,14]

$$\lim_{q^2\to 0} q^2 G_2(q^2) = 0, \lim_{q^2\to 0} q^2 F_2(q^2) = 0, \lim_{q^2\to 0} q^2 \tilde{G}_2(q^2) \equiv R = \sqrt{N_F} f_{\eta'} g_{\eta' NN}. \qquad (9)$$

The pole of $\tilde{G}_2(q^2)$ caused by the Kogut-Susskind ghost appears due to the $U_A(1)$ symmetry breaking in nonperturbative QCD vacuum. As the contribution (5) the pole has the anomalous origin[9,19].

By using (9) in the limit $m_u = m_d = 0$, a linear combination of (8) yields

$$2M[2(G_1(0) - \tilde{G}_1(0)) + F_1(0)] = 2R. \qquad (10)$$

The values shown in (10) are expressed through spin composites of a proton

$$G_1(0) = \Delta u + \Delta d + \Delta s - \Delta\tilde{g}, F_1(0) = \Delta u + \Delta d - 2\Delta s, \tilde{G}_1(0) = -\Delta\tilde{g}, \qquad (11)$$

where $\Delta\tilde{g}$ has been estimated in (5).

Thus, the total amount of proton helicity carried by quarks is

$$\Delta u + \Delta d + \Delta s = \frac{\sqrt{N_F}}{2M} f_{\eta'} g_{\eta' NN} - \frac{1}{2}(3 - 4\frac{D}{F+D}) g_A, \qquad (12)$$

where g_A is the axial-vector nucleon charge, F and D are the axial parameters describing the $\beta-$decays of the baryon octet.

From eq. (8-12) for strange quark contribution we have

$$\Delta s = \frac{1}{3}\frac{\sqrt{N_F}}{2M} f_{\eta'} g_{\eta' NN} - \frac{1}{2}(3 - 4\frac{D}{F+D}) g_A. \qquad (13)$$

By using $f_{\eta'} = 1.26 f_\pi (f_\pi = 132\ Mev)$, $g_{\eta' NN} = 7.5 \pm\ 1.5$[20] and $F = 0.47 \pm\ 0.04, D = 0.81 \pm 0.03$[6] eqs. (12,13) give

$$\Delta\Sigma = 0.8 \pm\ 0.2,\ \Delta s = 0.0 \pm\ 0.1 \qquad (14)$$

that somewhat differ from the results[15], and from (5) we have

$$\Delta\tilde{g} \approx 0.7. \qquad (15)$$

From this we find the value of strange sea measured in νp elastic scattering

$$\Delta\tilde{s} = \Delta s - \frac{1}{3}\Delta\Sigma = -0.26 \pm\ 0.2,$$

that should be compared with experiment [21]

$$\Delta\tilde{s} = -0.15 \pm\ 0.09.$$

The negative contribution of a strange sea to the proton helicity qualitatively agrees with the instanton approach picture[11]. From the values obtained, one can see that with the accuracy of model uncertainties the results satisfy the EMC experiment.

5. So we have shown that the anomalous contribution to proton helicity is explained fully by the nontrivial QCD vacuum structure. We have estimated the gluon induced contribution $\Delta\tilde{g}$ within the instanton vacuum model and the total contribution of quark within current algebra. The result of EMC $\Delta\Sigma - \Delta\tilde{g} \approx 0$ is explained by equality to zero of the QCD vacuum angle[22].

The authors thank S. B. Gerasimov, A. V. Efremov, B. L. Ioffe, V. T. Kim, O. V. Teryaev for discussions of the concerned problems and especially P. N. Bogolubov for constant interest and support.

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Received by Publishing Department
on December 25, 1989.

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Спин кварков в протоне и $U_A(1)$ проблема

Дана интерпретация результатов ЕМС по измерению спиновой структурной функции $g_1^p(x)$ в рамках непертурбативного подхода. Показано, что аномальный вклад в спиральность нуклона, переносимую кварками, является проявлением поляризации инстантонного вакуума в поляризованном нуклоне. Вычислены массовые поправки к соотношению между полным спином кварков в нуклоне и константами связи η' -мезона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

Dorokhov A.E., Kochelev N.I. E2-89-859

The Proton Quark Spin and the $U_A(1)$ Problem

Within the nonperturbative approach interpretation of the EMC measurements of the spin-dependent structure function $g_1^p(x)$ are given. It is shown that the anomalous contributions of quarks to the proton helicity is caused by the polarization of instanton vacuum in polarized nucleon. Mass corrections to the relation between the total quark spin in nucleon and the η' - meson couplings are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1989