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SPIN CRISIS<br>AND NONPERTURBATIVE RCD DYNAMICS

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[^0]In the last time, the EMC results of measuring the first moment of the structure functions $g_{1}^{p}(x)^{[1]}$

$$
\begin{equation*}
G_{1}^{p}=\int_{0}^{1} d x g_{1}^{p}(x)=0.126 \pm 0.010 \pm 0.015 \tag{1}
\end{equation*}
$$

are intensively discussed.
By using the operator expansion (OPE) it was obtained that

$$
\begin{equation*}
\Delta \Sigma \equiv \Delta u+\Delta d+\Delta s=0.12 \pm 0.20 \tag{2}
\end{equation*}
$$

where $\Delta \Sigma$ is the difference of quark proton densities with different helicity. This result turned out to be quite unexpected (see f. i. the discussion in ${ }^{[2]}$ ) because within the quark model where the whole proton spin is due to quarks one should have

$$
\Delta \Sigma=0.7 \div 1
$$

To solve this discrepancy, it was supposed ${ }^{[3]}$ that in the EMC experiment the value

$$
\begin{equation*}
\Delta q^{\prime}=\Delta \Sigma-N_{f} \frac{\alpha_{s}}{2 \pi} \Delta G \tag{3}
\end{equation*}
$$

was measured, where $N_{f}$ is the number of quark flavour, and $\Delta G$ is the helicity carried by gluons. Appearence of the second term in (3) is related to the anomaly in the axial current divergence ${ }^{[4]}$

$$
\begin{equation*}
\partial_{\mu} j_{\mu}^{05}=\partial_{\mu} K_{\mu} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
K_{\mu}=N_{f} \frac{\alpha_{s}}{2 \pi} \epsilon_{\nu \mu \sigma \rho} A_{\mu}^{a}\left(\partial_{\sigma} A_{\rho}^{a}-\frac{g}{3} f_{a b c} A_{\sigma}^{b} A_{\rho}^{c}\right) \tag{5}
\end{equation*}
$$

However, some later based on the general principles of guage invariance and analyticity it was shown that there was no any, calculated in QCD perturbative theory, anomalous contribution to $\Delta q^{\prime[5]}$. As concerned the derivation of (3) in ${ }^{[3]}$, the factorization procedure was incorrectly applied.

In works ${ }^{[6]}$, it was pointed out that the second term in (3) may be due to the change of the topological properties of vacuum in a polarized proton because the divergency of current $K_{\mu}$

$$
\begin{equation*}
\partial_{\mu} K_{\mu} \equiv Q \tag{6}
\end{equation*}
$$

is expressed through the topological charge density $Q$.

It was shown that ${ }^{[6]}$

$$
\begin{align*}
\Delta q^{\prime} & =\Delta \Sigma-2 N_{f}<p\left|n_{+}-n_{-}\right| p>\approx \\
& \approx \Delta \Sigma-2 N_{f} n_{0} V_{4} \sin \theta_{v}^{p}, \tag{7}
\end{align*}
$$

where $n_{ \pm}$is the instanton (antiinstanton) density in a polarized proton, $n_{0}$ is the equilibrium density of instantons, $V_{4}$ is the four-dimensional volume of a proton, $\theta_{v}^{p}$ is the vacuum angle determing CP breaking inside a polarized proton. Equation (7) follows from the fact that quarks in the instanton field have zero modes of different helicities ${ }^{[7]}$.

We should note here that the anomaly in (4) is related with the $U_{A}(1)$-problem: the problem of anomalous large mass of a $\eta^{\prime}-$ meson ${ }^{[8]}$. Thus, in ${ }^{[8]}$ it was shown that the $\eta^{\prime}-$-meson mass is also connected with the fluctuations of the topological charge

$$
\begin{equation*}
m_{\eta^{\prime}}^{2}=\frac{\langle 0| T(Q Q)|0\rangle}{f_{\eta^{\prime}}^{2}} \tag{8}
\end{equation*}
$$

The existence of this relation initiated the works ${ }^{[9,10]}$ where the ghost pole contribution was considered which is due to the nonvanishing matrix element $<0|T(Q Q)| 0>$ in axial proton formfactor.

Namely, defining the matrix elements

$$
\begin{array}{r}
\left\langle p^{\prime}\right| j_{\nu}^{5} \mid p>=\bar{u}\left(p^{\prime}\right)\left[\gamma_{\nu} \gamma^{5} G_{1}\left(q^{2}\right)+q_{\nu} \gamma^{5} G_{2}\left(q^{2}\right)\right] u(p)  \tag{9}\\
<p^{\prime}\left|K_{\nu}\right| p>=\bar{u}\left(p^{\prime}\right)\left[\gamma_{\nu} \gamma^{5} \tilde{G}_{1}\left(q^{2}\right)+q_{\nu} \gamma^{5} \bar{G}_{2}\left(q^{2}\right)\right] u(p)
\end{array}
$$

one can obtain from (4) and (5)

$$
\begin{equation*}
2 M G_{1}\left(q^{2}\right)+q^{2} G_{2}\left(q^{2}\right)=2 m \tilde{G}_{1}\left(q^{2}\right)+q^{2} \tilde{G}_{2}\left(q^{2}\right) . \tag{10}
\end{equation*}
$$

By using the result ${ }^{[9]}$

$$
\begin{equation*}
\lim _{q^{2} \rightarrow 0} q^{2} \tilde{G}_{2}\left(q^{2}\right)=\sqrt{N_{f}} f_{\eta^{\prime}} g_{\eta^{\prime} N N} \tag{11}
\end{equation*}
$$

from (10) we receive the quark contribution to the proton spin [10]

$$
\begin{equation*}
\Delta \Sigma=G_{1}(0)-\tilde{G}_{1}(0)=\frac{\sqrt{N_{f}} f_{\eta^{\prime}}}{2 M} g_{\eta^{\prime} N N} \tag{12}
\end{equation*}
$$

where from (7)

$$
\begin{equation*}
\tilde{G}_{1}(0)=2 N_{f} n_{0} V_{4} \sin \theta_{v}^{p} \tag{13}
\end{equation*}
$$

Then,

$$
\begin{equation*}
\Delta \Sigma \approx 1.14 \pm 0.2 \tag{14}
\end{equation*}
$$

and thus the discrepancy between the composed quark model and the EMC result (2) is resolved.

However, the question about the value of the $r$. h. side of (2) remains open. In this connection, we should note that by using (6) equation (10) can be continued as follows

$$
\begin{align*}
2 M G_{1}\left(q^{2}\right)+q^{2} G_{2}\left(q^{2}\right) & =2 M \tilde{G}_{1}\left(q^{2}\right)+q^{2} \tilde{G}_{2}\left(q^{2}\right)= \\
& =\left\langle p^{\prime}\right| Q|p\rangle \tag{15}
\end{align*}
$$

The matrix element of total topological charge over the proton state may be estimated analogously ( 7 ) (see ${ }^{[6]}$ )

$$
\begin{equation*}
<p|Q| p>\approx 2 M 2 N_{f} n_{0} V_{4} \sin \theta_{v} \tag{16}
\end{equation*}
$$

where $\theta_{v}$ is the QCD vacuum angle. Substituting the value of the equilibrium density of instantons in the QCD vacuum $n_{0} \simeq 810^{-4} G e v^{4}[11]$ and $V_{4} \simeq 1 \mathrm{fm}^{3} / M_{p}$ we obtain

$$
\begin{equation*}
G_{1}(0) \approx 2 N_{f} \sin \theta_{v} \tag{17}
\end{equation*}
$$

This is the central point of our work.
It is experimentally known that $\left|\theta_{v}\right| \leq 10^{-9}$ and thus

$$
\begin{equation*}
\text { . } \quad G_{1}(0) \leq 6 \cdot 10^{-9} \tag{18}
\end{equation*}
$$

We should stress the analogy between the results (12),(13),(17) and the appearance mechanism of a baryon charge in hybrid chiral bag models with external mesonic fields ${ }^{[12]}$. In these models, the baryon number is expressed as a sum of the charge of quarks inside a baryon and the charge induced by external mesonic fields. Then, though the charge is redistributed between internal and external regions of the bag-baryon, the total charge remains equal to one.

Analogously, equation (18) means that the zero topological proton charge is summed from intercompensating contributions of the charge inside a polarized proton (13) and the charge induced by the external $\eta^{\prime}-$ mesonic field (12).

Now, let us estimate different contributions to the proton spin. The result (12) was obtained in the chiral limit at $m_{u}=m_{d}=m_{s}=0$. Here, we estimate mass corrections. From equation (4) with $m_{u} \neq m_{d} \neq m_{s} \neq 0$.

$$
\partial_{\mu} j_{\mu}^{05}=\partial_{\mu} K^{\mu}+2 i \sum_{i} m_{i} \bar{q}_{i} \gamma_{5} q_{i}
$$

we get

$$
\begin{equation*}
\Delta \Sigma=f_{\eta^{\prime}} \frac{\sqrt{N_{f}}}{2 M} g_{\eta^{\prime} N N}-\frac{1}{2}\left(3-4 \frac{D}{F+D}\right) g_{A} \tag{19}
\end{equation*}
$$

where the equation

$$
\partial_{\mu} j_{\mu}^{85}=2 i\left(m_{u} \bar{u} \gamma_{5} u+m_{d} \bar{d} \gamma_{5} d-2 m_{s} \bar{s} \gamma_{5} s\right)
$$

and the supposition of equality of the axial densities

$$
<p\left|\bar{u} \gamma_{5} u\right| p>\approx<p\left|\bar{d} \gamma_{5} d\right| p>\approx<p\left|\bar{s} \gamma_{5} s\right| p>
$$

are used. The latter supposition is evidently correct within the models where a sea is caused by chiral invariant four-fermion Lagrangians such as the Lagrangian induced by instantons ${ }^{13]}$. There, it should be

$$
\begin{equation*}
\frac{2<p\left|\bar{s} \gamma_{5} s\right| p>}{<p\left|\bar{u} \gamma_{5} u+\bar{d} \gamma_{5} d\right| p>} \approx \frac{2<p|\bar{s} s| p>}{<p|\bar{u} u+\bar{d} d| p>} \tag{20}
\end{equation*}
$$

Then, by using the experimental value for the pion-nucleon $\sigma_{\pi N}$-term (for a review see ${ }^{[14]}$ ), we have

$$
\frac{2<p\left|\bar{s} \gamma_{5} s\right| p>}{<p\left|\bar{u} \gamma_{5} u+\bar{d} \gamma_{5} d\right| p>} \approx 0.47
$$

From (19) by using the values $f_{\eta^{\prime}}=1.26 f_{\pi} ; g_{\eta^{\prime} N N}=7.5 \pm 1.5^{[15]}$ and $F=0.47 \pm 0.04, D=0.81 \pm 0.03^{[2]}, g_{A}=1.24^{[16]}$ we get

$$
\begin{equation*}
\Delta \Sigma=0.8 \pm 0.2 \tag{21}
\end{equation*}
$$

that somewhat differs from the result in ${ }^{[10]}$. From (12) the value of a strange sea contribution becomes

$$
\Delta s=\frac{1}{3} f_{\eta^{\prime}} \frac{\sqrt{N_{f}}}{2 M} g_{\eta^{\prime} N N}-\frac{1}{2}\left(3-4 \frac{D}{F+D}\right) g_{A}=0.0 \pm 0.1
$$

Thus, by using (18), we find the strange sea value measured in the $\nu p$ elastic scattering ${ }^{[10]}$

$$
\Delta \tilde{s}=\Delta s-\frac{1}{3} \Delta \Sigma=-0.26 \pm 0.2
$$

This should be compared with the experiment ${ }^{[17]}$

$$
\Delta \tilde{s}=-0.15 \pm 0.09
$$

So, we have shown that the EMC result means equality to zero of the QCD vacuum angle $\theta_{v}$. The equality to zero of the topological charge of a proton is explained by the compensation of the QCD vacuum polarization inside a proton by the topological charge induced by the external $\eta^{\prime}$-mesonic field.

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Дорохов А.Е., Кочелев Н.И.
E2-89-858
Спиновый кризис и непертурбативная динамика КХД

Получено соотношение между вакуумным углом КХД $\theta_{\text {マ }}$ и первым моментом синглетной части структурной функции протона $\mathrm{g}_{1}^{\mathrm{p}}(\mathrm{x}): \mathrm{G}_{1}(0) \approx 2 \mathrm{~N}_{\mathrm{p}} \sin \theta_{\mathrm{v}}$.

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``` Spin Crisis and Nonperturbative QCD Dynamics

The relation between the \(Q C D\) vacuum angle value \(\theta_{v}\) and the first moment of the singlet part of the proton structure function \(g_{1}^{p}(x)\) is obtained: \(G_{1}(0) \approx 2 N_{f} \sin \theta_{v}\).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.```


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