объединенный
институт
ядерных
исследований
дубна

E2-89-849

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N = 3 AND N = 4 SUPERCONFORMAL WZNW SIGMA MODELS IN SUPERSPACE

I. GENERAL FORMALISM AND N = 3 CASE

Submitted to "International Journal of Modern Physics A"

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1989

1. INTRODUCTION

Interest in studies in 2D conformal field theory [1,2,3] is permanently increasing. In the last years, this theory gained general recognition as the basis of (super)string models and the statistical mechanics systems at criticality.

Important examples of conformal field theory are provided by the Wess-Zumino-Novikov-Witten (WZNW) group manifolds sigma models and their supersymmetric extensions. With a special ratio of the coupling constants, the WZNW action exibits both (super)conformal and (super)Kac-Moody (KM) symmetries and gives a field-theoretical realization of the Sugawara construction. All the non-trivial rational conformal field theories are expected to follow from WZNW model by the GKO projection [4].

The classical and quantum properties of ordinary bosonic and $N = 1$ superconformal WZNW sigma models were exhaustively analyzed in [5,6] and [7] , using the standard techniques of conformal field theory. The first examples of $N = 3$ and $N = 4$ superconformally-invariant WZNW sigma models on group manifolds have been constructed in our papers [8,9]. These models involve, as essential blocks, the bosonic $O(3), SU(2)$ or $O(4)$ WZNW actions and admit a nice interpretation in terms of nonlinear realizations of 2D $N = 3$ and $N = 4$ superconformal symmetries ($\hat{O}(3), \hat{SU}(2)$ and $\hat{O}(4)$ KM symmetries of the relevant actions naturally come out as parts of $N = 3$ and $N = 4$ superconformal ones). A necessary ingredient of these models is the presence of additional free bosonic fields interpreted as Goldstone fields corresponding to broken 2D scale or $U(1)$ invariances [8,9] [1] . Later on, a wider class of such models was discovered, starting with a different geometric set-up [10,11]. The most essential general features of them are, first,that $N = 3$ supersymmetry of the action always implies $N = 4$ supersymmetry and, second, that the relevant bosonic group spaces in most cases involve $U(1)$ factors. A complete list of admissible bosonic target manifolds is given in [10]. The models we have constructed in [8,9] correspond to the manifolds $U(1) \times SU(2)$, $U(1) \times O(3)$ and $U(1) \times U(1) \times O(4)$. As has been observed in [12,13,14], $N = 4$ superconformal WZNW sigma models of this type actually reveal a new kind of $N = 4$ superconformal symmetry which was missed in the paper of Ademollo *et al.*, [15]. It contains an additional $U(1)$ generator giving origin to the whole $\hat{U}(1)$ KM symmetry. The latter is realized as shifts of one of free scalar fields present in the action.

The quantum structure of these higher N WZNW sigma models has been studied in [12,13,14]. The OPE's (operator product expansions) of the relevant currents have been constructed and the general expressions for central charges have been found. An interesting peculiarity of this superconformal theory is the appearance of the Feigin-Fuchs type terms in the currents. In our letter [14] several important cases were considered when a fermionization of the KM currents becomes possible. As has been pointed out in [16], the WZNW sigma model realizations of $U(1)$ extended $N = 4$ superconformal algebra (SCA)

[1] One may add to the sigma model action the Liouville term for 2D dilaton (accompanied by appropriate Yukawa couplings with fermions) without affecting the underlying superconformal symmetries [8,9].

bear an intimate relation to the theory of representations of Knizhnik's superalgebras [17,18]. In more detail the representation theory of this new $N = 4$ SCA was constructed in [19].

Until now, all these considerations were performed using the language of ordinary 2D fields. However, the most elegant formalism for handling supersymmetric theories is offered by superfields. The superfield methods, in particular the techniques of superfield operator product expansions (SOPE),have already shown their worth in $N = 1$ super WZNW models [7]. The general superfield calculus of higher N superconformal symmetries has been developed in [20,21], without referring to particular models. The purposes of the present paper are to specialize this general formalism to the $N = 3$ and $N = 4$ WZNW sigma models mentioned above and to construct the self-contained superfield description of the latter, both on classical and quantum levels. We confine our study to the simplest models of this type proposed in [9] because, for the time being, only for them the basic superfields are known. Besides, they are directly related to the structure of $N = 3$ and $N = 4$ SCA's, originating from nonlinear realizations of the latter. Nevertheless, it is likely that the remaining models of this kind can also be translated into the superfield language without serious difficulties.

The paper is divided into two parts. In this first part we begin (in Sect.2) by reviewing a general superfield formalism of superconformal theories for any N, with focusing on the cases $N = 3, 4$. As a new development, we give here the basic elements of superconformal superfield calculus for $U(1)$ extended $N = 4$ SCA. Further, in Sect.3,4 a superfield formulation of $N = 3$ WZNW sigma model is presented. We show how to define the $N = 3$ supercurrent through the basic primary superfields, both in classical and quantum cases, and construct the relevant SOPE's. We also find out the necessary presence of one more $N = 3$ superfield of the supercurrent type which, together with the standard $N = 3$ supercurrent, generate $U(1)$ extended $N = 4$ superconformal symmetry. Finally, we formulate the superfield fermionization rules corresponding to the cases considered in [14].

The second part of the paper will be devoted to the superfield description of $N = 4$ WZNW sigma models.

2. GENERALITIES OF N EXTENDED 2D SUPERCONFORMAL THEORIES IN SUPERSPACE

Here we sketch the superspace formalism of 2D superconformal theories for arbitrary N, with a special emphasis on $N = 3, 4$. We basically follow refs.[20,21].

Denote the coordinates of N extended 2D superspace by Z and $\bar{Z}$, where [2]

$$Z = (z, \theta^i), \quad \bar{Z} = (\bar{z}, \bar{\theta}^{\underline{i}}), \quad i, \underline{i} = 1, 2, ..., N \tag{2.1}$$

[2]We do not need Z and $\bar{Z}$ to be mutually conjugated.

and define spinor covariant derivatives D^i, $\overline{D}^{\underline{i}}$

$$D^i = \frac{\partial}{\partial\theta^i} - \theta^i\frac{\partial}{\partial z}, \quad \overline{D}^{\underline{i}} = \frac{\partial}{\partial\bar{\theta}^{\underline{i}}} - \bar{\theta}^{\underline{i}}\frac{\partial}{\partial\bar{z}} \tag{2.2}$$

$$\{D^i, D^j\} = -2\delta^{ij}\partial_z, \quad \{\overline{D}^{\underline{i}}, \overline{D}^{\underline{j}}\} = -2\delta^{\underline{ij}}\partial_{\bar{z}}, \quad \{D^i, \overline{D}^{\underline{j}}\} = 0 .$$

The basic entities of N extended superconformal theory are the analytic (antianalytic) superfunctions $f(Z)$ $(f(\bar{Z}))$ which are defined by the Grassmann analyticity condition

$$\overline{D}^{\underline{i}} f(Z, \bar{Z}) = 0 \Rightarrow f = f(Z) \tag{2.3}$$

(and analogously for $f(\bar{Z})$). One may decompose them in generalized Laurent series [3] which involve ordinary Laurent series in z and Taylor series in θ^i:

$$f(Z_1) = \sum_n \sum_{[i]} (-1)^{f_R} \frac{1}{n!}\frac{1}{R!} Z_{12}^n \theta_{12}^{[i]} D_2^{[i]} \partial_{z_2}^n f(Z_2) . \tag{2.4}$$

For the coefficients in (2.4), the following modified integral representation is valid [21]

$$(-1)^{f_R}\frac{1}{n!}\partial_{z_2}^n D_2^{[i]} f(Z_2) = \oint_{C_2}\frac{dz_1}{2\pi i}\int d^N\theta_1 \frac{\theta_{12}^{N-[i]}}{Z_{12}^{n+1}} f(Z_1) , \tag{2.5}$$

where the contour C_2 encircles the point z_2.

The possibility to deal with the superanalytic functions in 2D superconformal theories is related to the property that 2D superconformal transformations preserve analyticity

$$z \to \tilde{z} = z + \delta z, \quad \theta^i \to \tilde{\theta}^i = \theta^i + \delta\theta^i$$
$$\delta z = E(Z) - \frac{1}{2}\theta^i D^i E(Z), \quad \delta\theta^i = -\frac{1}{2}D^i E(Z) \tag{2.6}$$

$$\bar{z} \to \tilde{\bar{z}} = \bar{z} + \delta\bar{z}, \quad \bar{\theta}^{\underline{i}} \to \tilde{\bar{\theta}}^{\underline{i}} = \bar{\theta}^{\underline{i}} + \delta\bar{\theta}^{\underline{i}},$$
$$\delta\bar{z} = \bar{E}(\bar{Z}) - \frac{1}{2}\bar{\theta}^{\underline{i}}\overline{D}^{\underline{i}}\bar{E}(\bar{Z}), \quad \delta\bar{\theta}^{\underline{i}} = -\frac{1}{2}\overline{D}^{\underline{i}}\bar{E}(\bar{Z}) , \tag{2.7}$$

where $E(Z)$ and $\bar{E}(\bar{Z})$ are two arbitrary analytic and antianalytic superfunctions collecting parameters of superconformal transformations.

[3]Our notation is basically the same as in [21]

$$[i] = i_1 i_2 \ldots i_R, \ i_j \neq i_k, \ 0 \leq R \leq N, \ f_R = \frac{1}{2}R(R-1),$$

$$\theta^{[i]} = \theta^{i_1}\theta^{i_2}\ldots\theta^{i_R} \ \ etc., \ \ \theta^{N-[i]} = \frac{1}{(N-R)!}\epsilon^{j_1 \ldots j_{(N-R)} i_1 \ldots i_R}\theta^{j_1}\ldots\theta^{j_{(N-R)}},$$

$$\theta_{12}^i = \theta_1^i - \theta_2^i, \quad Z_{12} = z_1 - z_2 + \theta_1^i\theta_2^i .$$

Spinor derivatives are transformed homogeneously under (2.6), (2.7)

$$D^i = (D^i\tilde{\theta}^j)\widetilde{D^j}, \quad \overline{D}^i = (\overline{D}^i\tilde{\overline{\theta}}^j)\widetilde{\overline{D}^j}, \tag{2.8}$$

so Grassmann analyticity conditions (2.3) are covariant and one may restrict oneself to considering the analytic or antianalytic superfunctions. All this can be expressed as the statement that the 2D superconformal group is a direct product of two groups acting, respectively, on Z and $\bar{Z}$. Correspondingly, the supercurrents generating these two groups are analytic and antianalytic superfields. In what follows we shall consider, without loss of generality, the objects depending on Z.

The important quantities are primary superfields Φ^α_Δ which are postulated to have the following superconformal transformation law

$$\delta_E\Phi^\alpha_\Delta = -E(\partial_z\Phi^\alpha_\Delta) + \frac{1}{2}(D^iE)(D^i\Phi^\alpha_\Delta) - \Delta(\partial_zE)\Phi^\alpha_\Delta - \frac{1}{4}i(D^iD^jE)(T^{ij})^{\alpha\beta}\Phi^\beta_\Delta . \tag{2.9}$$

Here Δ is the conformal weight of Φ^α_Δ and α is an index of the representation of the group $O(N)$ which extends to the whole $\hat{O}(N)$ KM symmetry with the parameters contained in $E(Z)$ ($\hat{O}(N)$ acts also on Grassmann coordinates which transform as an $O(N)$ vector). The matrices $(T^{ij})^{\alpha\beta}$ are generators of $O(N)$ in this representation.

The superconformal transformations of some superfield $\Phi(Z)$ (not necessarily the primary one) are generated by the supercurrent $J^{(N)}$ according to the general rule

$$\delta_E\Phi(Z_2) = \oint_{C_2}\frac{dz_1}{2\pi i}\int d^N\theta_1 E(Z_1)J^{(N)}(Z_1)\Phi(Z_2) . \tag{2.10}$$

In (2.10), all the information about the transformation properties of $\Phi(Z)$ is encoded in singular terms of the product $J^{(N)}(Z_1)\Phi(Z_2)$ at $Z_1 \to Z_2$, because only such terms contribute to the contour integral. In particular, for the primary superfields $\Phi^\alpha_\Delta(Z)$ the relevant SOPE is of the following form

$$J^{(N)}(Z_1)\Phi^\alpha_\Delta(Z_2) = \left[-\Delta\frac{\theta^N_{12}}{Z^2_{12}} + \frac{1}{2}\frac{\theta^{N-i}_{12}}{Z_{12}}D^i_2 - \frac{\theta^N_{12}}{Z_{12}}\partial_{z_2}\right]\Phi^\alpha_\Delta(Z_2) + \frac{1}{4}i(T^{ij})^{\alpha\beta}\frac{\theta^{N-ij}_{12}}{Z_{12}}\Phi^\beta_\Delta(Z_2) + \cdots , \tag{2.11}$$

where dots stand for the regular terms. Substituting (2.11) into (2.10) yields just eq.(2.9). The supercurrent $J^{(N)}(Z)$ is not a primary superfield, it transforms with an inhomogeneous piece

$$\delta_EJ^{(N)} = -E(\partial_zJ^{(N)}) + \frac{1}{2}(D^iE)(D^iJ^{(N)}) - (2 - \frac{1}{2}N)(\partial_zE)J^{(N)} + \mathcal{O}^{(N)}E \tag{2.12}$$

or, in the SOPE language

$$J^{(N)}(Z_1)J^{(N)}(Z_2) = \left[-(2 - \frac{N}{2})\frac{\theta^N_{12}}{Z^2_{12}} + \frac{1}{2}\frac{\theta^{N-i}_{12}}{Z_{12}}D^i_2 - \frac{\theta^N_{12}}{Z_{12}}\partial_{z_2}\right]J^{(N)}(Z_2) + \mathcal{C}^{(N)}(Z_1, Z_2) + \cdots , \tag{2.13}$$

where the central term $\mathcal{C}^{(N)}$ and the central operator $\mathcal{O}^{(N)}$ are related by

$$(\mathcal{O}^{(N)}E)(Z_2) = \oint_{C_2}\frac{dz_1}{2\pi i}\int d^N\theta_1 E(Z_1)\mathcal{C}^{(N)}(Z_1, Z_2) . \tag{2.14}$$

Jacobi identities impose severe restrictions on the operators $\mathcal{O}^{(N)}$ ($\mathcal{C}^{(N)}$) which prove to exist only for $N \leq 4$[20,21]. For $N = 3$ and $N = 4$ the operators $\mathcal{O}^{(N)}$ ($\mathcal{C}^{(N)}$) are [21]

$$\mathcal{O}^{(3)} = -\frac{c}{12}D^3, \quad \mathcal{C}^{(3)}(Z_1, Z_2) = \frac{c}{12}\frac{1}{Z_{12}} \tag{2.15}$$

$$\mathcal{O}^{(4)} = \frac{c_1}{12}\frac{D^4}{\partial_z} + \frac{c_2}{12}\partial_z, \quad \mathcal{C}^{(4)}(Z_1, Z_2) = -\frac{c_1}{12}\log(Z_{12}) + \frac{c_2}{12}\frac{\theta^4_{12}}{Z^2_{12}} , \tag{2.16}$$

where c, c_1, c_2 are central charges of $N = 3$ and $N = 4$ superconformal algebras (c and c_1 coincide with the conventionally normalized central charges of corresponding Virasoro subalgebras) (Appendix A). Appearance of two independent central charges in $N = 4$ case is related to the presence of two commuting $\hat{SU}(2)$ KM subalgebras in $N = 4$ SCA ($\hat{O}(4)_{k_1,k_2} \sim \hat{SU}(2)_{k_1} \times \hat{SU}(2)_{k_2}$, $k_1 = \frac{1}{3}(c_1 + c_2)$, $k_2 = \frac{1}{3}(c_1 - c_2)$).

To avoid nonlocalities in $\mathcal{O}^{(4)}$, it is convenient to define a new supercurrent $J^i(Z)$

$$J^i(Z) \equiv D^iJ^{(4)}(Z) , \tag{2.17}$$

which transforms according to

$$\delta_EJ^i = -E\partial_zJ^i + \frac{1}{2}(D^jE)(D^jJ^i) + \frac{1}{2}(D^iD^jE)J^j + \frac{c_1}{12}D^{4-i}E + \frac{c_2}{12}D^i\partial_zE . \tag{2.18}$$

This supercurrent has the following SOPE's with itself

$$\begin{aligned} J^i(Z_1)J^j(Z_2) = & \frac{1}{2Z^2_{12}}\left\{-\delta^{ij}\theta^{4-l}_{12} + \delta^{il}\theta^{4-j}_{12} + \delta^{jl}\theta^{4-i}_{12} - \delta^{jl}\theta^4_{12}D^i_2\right\}J^l(Z_2) + \\ & + \frac{1}{2Z_{12}}\left\{\theta^{4-ijl}_{12} + \delta^{jl}\theta^{4-ik}_{12}D^k_2 + 2\delta^{jl}\theta^{4-i}_{12}\partial_{z_2}\right\}J^l(Z_2) + \\ & + \frac{c_1}{12}\left\{\frac{\delta^{ij}}{Z_{12}} + \frac{\theta^{ij}_{12}}{Z^2_{12}}\right\} - \frac{c_2}{12}\left\{2\delta^{ij}\frac{\theta^4_{12}}{Z^3_{12}} - \frac{\theta^{4-ij}_{12}}{Z^2_{12}}\right\} + \ldots \end{aligned} \tag{2.19}$$

and with a primary superfield Φ^α_Δ

$$\begin{aligned} J^i(Z_1)\Phi^\alpha_\Delta(Z_2) = & \left[\Delta\frac{\theta^{4-i}_{12}}{Z^2_{12}} + \frac{\theta^{4-i}_{12}}{Z_{12}}\partial_{z_2} - \frac{1}{2}\frac{\theta^4_{12}}{Z^2_{12}}D^i_2 + \frac{1}{2}\frac{\theta^{4-il}_{12}}{Z_{12}}D^l_2\right]\Phi^\alpha_\Delta(Z_2) - \\ & - \frac{1}{4}\left[i(T^{lj})^{\alpha\beta}\frac{\theta^{4-lji}_{12}}{Z_{12}} - 2i(T^{ij})^{\alpha\beta}\frac{\theta^{4-j}_{12}}{Z^2_{12}}\right]\Phi^\beta_\Delta(Z_2) + \ldots . \end{aligned} \tag{2.20}$$

The general integral formula (2.10) for the superconformal variation of a primary superfield in the $N = 4$ case can also be concisely rewritten via the supercurrent $J^i(Z)$.

To this end, let us pass from the parameter superfunction $E(Z)$ to its spinor "potential" $\Lambda^i(Z)$

$$E(Z) = D^i\Lambda^i(Z) \qquad (2.21)$$

$$D^i\Lambda^j(Z) + D^j\Lambda^i(Z) = \frac{1}{2}\delta^{ij}D^k\Lambda^k(Z)\,. \qquad (2.22)$$

The constraint (2.22) leaves in $\Lambda^i(Z)$ the same number of independent functions as in $E(Z)$

$$\begin{aligned}\Lambda^i(Z) &= \frac{1}{2}\mu^i(z) + \frac{1}{4}\theta^i f(z) - b^{[il]}(z)\theta^l - \frac{1}{2}\partial_z\mu^j(z)\theta^i\theta^j - \frac{i}{2}\theta^{4-ik}\eta^k(z)\\ &+ \frac{1}{2}\partial_z b^{[mn]}(z)\epsilon^{mnil}\theta^{4-l} - d(z)\theta^{4-i} - \frac{i}{2}\partial_z\eta^i(z)\theta^4 \qquad (2.23)\\ E(Z) &= f(z) + 2\partial_z\mu^i(z)\theta^i + \partial_z b^{[ij]}(z)\epsilon^{ijkl}\theta^{4-kl} - 2i\partial_z\eta^i(z)\theta^{4-i} + 4\partial_z d(z)\theta^4 \qquad (2.24)\end{aligned}$$

Substituting (2.21) into (2.10) (for $N = 4$) and integrating by parts one gets

$$\begin{aligned}\delta_E\Phi(Z) &= \oint_{C_2}\frac{dz_1}{2\pi i}\int d^4\theta_1 D_1^i\left(\Lambda^i(Z_1)J^{(4)}(Z_1)\Phi(Z_2)\right) +\\ &+ \oint_{C_2}\frac{dz_1}{2\pi i}\int d^4\theta_1\Lambda^i(Z_1)D_1^iJ^{(4)}(Z_1)\Phi(Z_2) =\\ &= \oint_{C_2}\frac{dz_1}{2\pi i}\int d^4\theta_1\Lambda^i(Z_1)J^i(Z_1)\Phi(Z_2)\,. \qquad (2.25)\end{aligned}$$

It is worth mentioning that the representation (2.25) is actually more general than the original one and opens up a way of extending the standard $N = 4$ SCA. To show this, we first notice that $\Lambda^i(Z)$ contains more parameters than $E(Z)$. Indeed, the constant parts of functions $\mu^i(z), \eta^i(z), b^{[ij]}(z)$ and $d(z)$ in $\Lambda^i(Z)$ do not contribute to $E(Z)$ as is seen from eqs. (2.23), (2.24). For the primary superfields $\Phi(Z)$ having the standard SOPE's (2.11),(2.20) with the supercurrents these additional parameters drop out from the expression (2.25). So, for such superfields eqs.(2.10) and (2.25) are completely equivalent. However, this ceases to be the case if one assumes a more general form for SOPE of $\Phi(Z)$ and $J^i(Z)$, namely, adds one more pole term to (2.20)

$$J^i(Z_1)\Phi^\alpha_{\Delta,\omega}(Z_2) = (2.20) - \omega\cdot\frac{\theta^i_{12}}{Z_{12}}\Phi^\alpha_{\Delta,\omega}(Z_2)\,, \qquad (2.26)$$

which amounts to allowing a logarithmic singularity in the product with $J^{(4)}$

$$J^{(4)}(Z_1)\Phi^\alpha_{\Delta,\omega}(Z_2) = (2.11) + \omega\cdot\log Z_{12}\Phi^\alpha_{\Delta,\omega}(Z_2)\,. \qquad (2.27)$$

Here ω is a new constant characterizing the primary superfield. This term, being substituted into (2.25), generates an additional $U(1)$ transformation of $\Phi^\alpha_{\Delta,\omega}(Z)$ with the parameter d_0 coming out as the constant part of the function $d(z)$ in $\Lambda^i(Z)$

$$d(z) = \sum_n d_n z^{-n}\,. \qquad (2.28)$$

The remaining constant parameters $\mu^i_0, \eta^i_0, b^{[ij]}_0$ do not contribute as before.

Thus, the class of $N = 4$ primary superfields can be enlarged by allowing them to carry a new quantum number, the eigenvalue of the $U(1)$ generator U_0 associated with the parameter d_0 in $\Lambda^i(Z)$:

$$iU_0 \mid \Phi^\alpha_{\Delta,\omega} >= \omega \mid \Phi^\alpha_{\Delta,\omega} > \,. \qquad (2.29)$$

This generator acts only on the superfield as a whole (it does not affect the superspace coordinates (z, θ^i)) and is naturally accomodated by the general formula (2.25). It appears in the r.h.s. of anticommutators of spinor charges and, together with the old generators $U_n, n \neq 0$, generates the whole $\hat{U}(1)$ KM symmetry. This new $U(1)$ extended $N = 4$ SCA [12,13,14] is lacking among those listed by Ademollo *et al* [15]. The generator U_0 enters into the supercurrent $J^i(Z)$ as a coefficient of z^{-1} in the decomposition of $D^iJ^i\mid_{\theta=0}$ in Laurent series. On the other hand, in terms of the original supercurrent $J^{(4)}$ it comes out as the coefficient of the logarithmic singularity in $J^{(4)}(Z)\mid_{\theta=0}$, which was ignored in standard considerations.

Thus, on the primary superfields $\Phi^\alpha_{\Delta,\omega}$ with $\omega \neq 0$ just this new $U(1)$ extended $N = 4$ SCA is realized. It is reduced to the standard $N = 4$ SCA only for $\omega = 0$. Note that for the superfields with $\omega \neq 0$ one cannot return to the standard form of variation (2.10). The reason is that in this case the integral of a full spinor derivative in the first line of formulas (2.25) does not vanish in view of the presence of $\log Z_{12}$ under ∂/∂_z in the integrand. As we shall see in Part II, the basic superfields of $N = 4$ WZNW sigma model necessarily possess $\omega \neq 0$ and so provide a representation of the $U(1)$-extended $N = 4$ superconformal symmetry, in accordance with the component consideration of [12,13,14]. It is worthwhile to say that the supercurrents $J^{(4)}$ and J^i possess $\omega = 0$ and thus are inert under the action of U_0.

In the next Sections we shall also need an integral representation for the normally ordered product of two superfields at a given point [18]

$$:AB:(Z_1) = \frac{1}{2}\int\frac{dz_2}{2\pi i(z_2 - z_1)}\int d^N\theta_2\theta^N_{12}[A(Z_1)B(Z_2) + A(Z_2)B(Z_1)]\,. \qquad (2.30)$$

Here $Z_1 = (z_1, \theta^i_1), Z_2 = (z_2, \theta^i_2)$.

Finally, we mention that for the coefficients in the z, θ expansion of $J^{(3)}$ and J^i defined according to

$$J^{[i]}_r = \oint_{C_0}\frac{dz}{2\pi i}\int d^3\theta\cdot i^{fR}\theta^{[i]}z^{r+1-\frac{R}{2}}J^{(3)}(Z) \qquad (2.31)$$

$$I^{[i]l}_r = \oint_{C_0}\frac{dz}{2\pi i}\int d^4\theta\cdot i^{fR}\theta^{[i]}z^{r+\frac{3}{2}-\frac{R}{2}}J^l(Z) \qquad (2.32)$$

eqs.(2.13) (with $N = 3$) and (2.19) produce, respectively, $N = 3$ and $N = 4$ SCA's. These SCA's are written down in Appendix A.

3. N=3 WZNW SIGMA MODEL: CLASSICAL THEORY

The aim of this Sect. is to present a superfield formulation of $N = 3$ WZNW sigma model at the classical level. A generalization to the quantum case will be given in the next Section.

As has been shown in [9], the basic object of this model is the $O(3)$ matrix superfield $q^{i\underline{j}}(Z,\overline{Z})$

$$q^{i\underline{j}}(Z,\overline{Z}) = e^{-\sqrt{\frac{2}{k}}u(Z,\overline{Z})}\tilde{q}^{i\underline{j}}(Z,\overline{Z}) \tag{3.1}$$

$$\tilde{q}^{i\underline{j}}(Z,\overline{Z}) \in \frac{O(3)\times \overline{O(3)}}{O(3)_{diag}} \tag{3.2}$$

subjected to the equations

$$\begin{aligned} D^i q^{\underline{l}j} + D^j q^{\underline{l}i} &= \frac{2}{3}\delta^{ij} D^n q^{\underline{l}n} \quad (a) \\ \overline{D}^{\underline{i}} q^{l\underline{i}} + \overline{D}^{\underline{l}} q^{i\underline{i}} &= \frac{2}{3}\delta^{\underline{il}} \overline{D}^{\underline{n}} q^{\underline{n}i} \quad (b) \end{aligned} \tag{3.3}$$

$$\overline{D}^{\underline{i}}(q^{-1}D^i q)^{jl} = 0 \,. \tag{3.4}$$

Here the non-underlined and underlined indices refer, respectively, to the vector representations of two $O(3)$ groups, $O(3)$ and $\overline{O(3)}$, which enter the analytic and antianalytic branches of $N = 3$ superconformal group realized on the coordinates (z,θ^i), $(\bar{z},\overline{\theta}^{\underline{i}})$. The renormalization of superfield $u(Z,\overline{Z})$ in (3.1) by the factor $\sqrt{\frac{2}{k}}$, $k \in \mathbf{Z}$, has been performed for future convenience.

The meaning of eqs. (3.3),(3.4) was explained in [9] and it is as follows. Eqs. (3.3) are the off-shell irreducibility conditions extracting from $q^{i\underline{j}}(Z,\overline{Z})$ an irreducible number of $8+8$ field components. Eqs. (3.4) are dynamical equations eliminating auxiliary fields and resulting in the correct equations of motion for the physical components of $q^{i\underline{j}}$. The latter are defined as the first components of the following superfields

$$\begin{aligned} q^{i\underline{j}},\ \xi^i &= -\frac{1}{3}\sqrt{\frac{k}{2}}(q^{-1}D^j q)^{ij}, \quad \chi = \frac{1}{6}\sqrt{\frac{k}{2}}(q^{-1}D^i q)^{jl}\epsilon^{ijl} \\ \overline{\xi}^{\underline{i}} &= -\frac{1}{3}\sqrt{\frac{k}{2}}(\overline{D}^{\underline{l}} q q^{-1})^{\underline{li}}, \quad \overline{\chi} = \frac{1}{6}\sqrt{\frac{k}{2}}(\overline{D}^{\underline{i}} q q^{-1})^{\underline{nl}}\epsilon^{\underline{inl}}, \end{aligned} \tag{3.5}$$

the renormalization factors being included to get a correctly normalized action (see below). In principle, eq. (3.4) could be obtained by varying a proper superfield action which justifies the interpretation of the remaining independent components of $q^{i\underline{j}}$ as auxiliary fields. For our purpose, it is sufficient to know the component action with the auxiliary fields eliminated [14]

$$\begin{aligned} S_{phys} = \int dz d\bar{z} &\left\{\frac{1}{2}\partial_z u \partial_{\bar{z}} u + \frac{1}{2}\overline{\xi}^{\underline{i}}\partial_z\overline{\xi}^{\underline{i}} + \frac{1}{2}\xi^i\partial_{\bar{z}}\xi^i + \frac{1}{2}\overline{\chi}\partial_z\overline{\chi} + \frac{1}{2}\chi\partial_{\bar{z}}\chi\right\} + \\ &+\frac{1}{k}\left(\int dz d\bar{z} W_z^i W_{\bar{z}}^i + \frac{1}{3k}\int d^3X \epsilon^{\alpha\beta\gamma}\epsilon^{ijk} W_\alpha^i W_\beta^j W_\gamma^k\right) , \end{aligned} \tag{3.6}$$

where

$$W_A^i = \frac{k}{4}\epsilon^{ijk}(\tilde{q}^{-1}\frac{\partial}{\partial X^A}\tilde{q})^{jk} \tag{3.7}$$

and X^A may stand both for the 2D coordinates $z, \bar{z}$ and for the 3D ones $X^\alpha (\alpha = 1,2,3)$. The action (3.6) is invariant under $N = 3$ superconformal transformations and involves, as an essential ingredient, the action of the bosonic $O(3)$ WZNW sigma model. The presence of integer k (which is the same as in (3.1)) reflects quantization of the coupling constant in front of WZNW action.

As usual, $N = 3$ superconformal group can be realized in a closed form on the analytic and antianalytic components of fields involved in (3.6). These components are concisely encompassed by the general solution of the superfield equation (3.4)

$$q^{i\underline{j}}(Z,\overline{Z}) = q^{\underline{i}A}(\overline{Z})\cdot q^{Aj}(Z) \,, \tag{3.8}$$

where the index $A = 1,2,3$ is inert under both $O(3)$'s and so under $N = 3$ superconformal group as a whole. The latter acts independently on $q^{\underline{i}A}(\overline{Z})$ and $q^{Aj}(Z)$, respectively by its analytic (antianalytic) branches. Thus, we may confine ourselves to considering the analytic superfields $q^{Aj}(Z)$ (for $q^{\underline{i}A}(\overline{Z})$ all the considerations are carried out quite analogously). The only equation to be satisfied by $q^{Aj}(Z)$ follows from the irreducibility constraint (3.3a):

$$D^i q^{Aj} + D^j q^{Ai} = \frac{2}{3}\delta^{ij} D^l q^{Al} \,. \tag{3.9}$$

The superfield $q^{Ai}(Z)$ is primary with the conformal weight 1/2:

$$\delta_E q^{Ai} = -E(\partial_z q^{Ai}) + \frac{1}{2}(D^l E)(D^l q^{Ai}) - \frac{1}{2}\partial_z E\cdot q^{Ai} - \frac{1}{2}D^{3-j}E\cdot\epsilon^{ijl}q^{Al} \tag{3.10}$$

$$E(Z) = f(z) + 2\mu^i(z)\theta^i + b^i(z)\theta^{3-i} - 2i\eta(z)\theta^3 \,,$$

where the parameters $f(z), \mu^i(z), b^i(z), \eta(z)$ correspond to the transformations with generators $L_n, G_r^i, V_n^i, \Gamma_r$, respectively (see (A.1)). Eq.(3.9) is covariant under (3.10), which can be checked straightforwardly.

So far, we tightly followed the consideration in [9]. A novel point we have not discussed in [9] is the construction of $N = 3$ supercurrent $J^{(3)}(Z)$ in terms of primary superfield $q^{Ai}(Z)$. Generally speaking, it could be derived from the superfield action corresponding to eq. (3.4). However, it is uniquely determined already from the simple dimensionality and symmetry considerations. It should be a spinor, have the dimension 1/2 , be the $O(3)$ singlet and contain the KM current among its components. All these requirements are met by the expression

$$J^{(3)}(Z) = \frac{k}{24}\epsilon^{ijl}(q^{-1}D^i q)^{jl} \,. \tag{3.11}$$

Being aware of the transformation law (3.10) of $q^{Ai}(Z)$, one may easily find the corresponding law of $J^{(3)}(Z)$. It is as in eq.(2.12) where the central term is given by expression (2.15)

$$\mathcal{O}^{(3)} = -\frac{c}{12}D^3$$

with

$$c = \frac{3}{2}k\,. \tag{3.12}$$

The definition of the component $N = 3$ currents and the explicit expressions for them in terms of fields (3.5) are given in Appendix B.

It should be pointed out that in the given model the central term in the transformation of the supercurrent arises already at the classical level. This is related to the fact that the bosonic part $W^i = W^i_z$ (3.7) of the KM current contained in $J^{(3)}$ is transformed inhomogeneously under $\hat{O}(3)$ KM transformations [5].By supersymmetry, an analogous classical inhomogeneity appears in the conformal transformation of the stress tensor $T(z)$. This is achieved due to the Feigin-Fuchs term of field $u(z)$ in $T(z)$ (Appendix B).

Expression (3.11) completely defines the supercurrent $J^{(3)}(Z)$ as a function of $q^{Ai}(Z)$. However, this object is merely one irreducible piece of the expression $(q^{-1}D^iq)^{jl}$. There is another one as well. The constraint (3.9) implies the following general structure for $(q^{-1}D^iq)^{jl}$

$$\frac{k}{4}(q^{-1}D^iq)^{jl} = J^{(3)}(Z)\epsilon^{ijl} + \delta^{ij}\Psi^l(Z) - \delta^{il}\Psi^j(Z) - \delta^{jl}\Psi^i(Z)\,. \tag{3.13}$$

Thus one may associate with $q^{Ai}(Z)$ one more superfield of the supercurrent type

$$\Psi^i(Z) = \frac{k}{4}(q^{-1}D^jq)^{ji} = \frac{1}{2}\sqrt{\frac{k}{2}}D^iu(Z)\,. \tag{3.14}$$

Different representations for $\Psi^i(Z)$ follow immediately by applying the constraint (3.9). From (3.14) it also follows that $\Psi^i(Z)$ satisfies the constraint

$$D^i\Psi^j + D^j\Psi^i = \frac{2}{3}\delta^{ij}D^l\Psi^l\,. \tag{3.15}$$

Let us inspect Ψ^k in more detail. Its transformation law stems immediately from eqs.(3.14), (3.10)

$$\delta_E\Psi^i = -E\partial_z\Psi^i + \frac{1}{2}(D^lE)(D^l\Psi^i) - \frac{1}{2}\partial_zE\cdot\Psi^i - \frac{1}{2}D^{3-l}E\cdot\epsilon^{ilj}\Psi^j + \frac{\tilde{c}}{12}D^i\partial_zE \tag{3.16}$$

$$\tilde{c} = \frac{3}{2}k\,. \tag{3.17}$$

Now we observe two peculiarities.

First, as is seen from (3.16), $\Psi^i(Z)$ behaves, up to the central term, as the primary superfield of the weight 1/2 and in this respect it resembles $J^{(3)}(Z)$. However, the central term has an entirely different form. To our knowledge, no such superfields have been considered earlier within the superspace approach to $N = 3$ superconformal theories.

Second, the transformation law (3.16), in its own right, does not require $\tilde{c}$ to be as in eq.(3.17); the Jacobi identities of $N = 3$ SCA are satisfied by (3.16) with any $\tilde{c}$, so the specific value (3.17) of $\tilde{c}$ is an artefact of the given model in which $\Psi^i(Z)$ is expressed via $q^{Ai}(Z)$ by eqs.(3.14). In fact, one may forget about (3.14) and define $\Psi^i(Z)$ by its transformation law (3.16) and the constraint (3.15) which is easily checked to be covariant under (3.16) irrespective of the value of $\tilde{c}$. Then, considering $J^{(3)}(Z)$ and $\Psi^i(Z)$ together leads to the situation with the two independent central charges c and $\tilde{c}$.

Thus, the $N = 3$ WZNW sigma model admits two conserved supercurrents, $J^{(3)}(Z)$ and $\Psi^i(Z)$. As we know, $J^{(3)}(Z)$ generates $N = 3$ SCA. It remains to learn which invariance is associated with $\Psi^i(Z)$.

Actually, this question can be answered immediately by looking at the component structure of $\Psi^i(Z)$ in the present model ((B.2)) and keeping in mind that the action (3.6) possesses a wider symmetry, namely that with respect to $O(4)\times U(1)$ $N = 4$ SCA [14]. The coefficients in the expansion of $\Psi^i(Z)$

$$\Psi^{[i]l}_r = \oint_{C_0}\frac{dz}{2\pi i}\int d^N\theta\cdot i^{J_R}\theta^{[i]}z^{r+1-\frac{R}{2}}\Psi^l(Z) \tag{3.18}$$

are easily seen to complete $N = 3$ SCA (A.1) generated by $J^{(3)}(Z)$ to the $N = 4$ SCA just mentioned (eqs.(A.2)). As was given in Sect.2, the latter possesses twc independent central charges and this accounts for the appearance of two different numbers c and $\tilde{c}$ in the transformation rules of $J^{(3)}(Z)$ and $\Psi^i(Z)$. These numbers actually coincide with c_1 and c_2 figuring in eq.(2.16) and reflect the two-levels structure of $\hat{O}(4)$-KM subalgebra. Let us stress that the constraint (3.15) is entirely neccessary in order to match the component content of $\Psi^i(Z)$ with the set of component currents completing $N = 3$ SCA to $N = 4$ SCA. It is also worthwhile to emphasize that, within the superfield approach we deal with, $N = 4$ superconformal invariance of the $N = 3$ WZNW sigma model in question is directly related to the possibility to construct the second conserved supercurrent $\Psi^i(Z)$ out of $q^{Ai}(Z)$.

Being aware of the transformation properties of $N = 4$ superconformal currents [14], one may derive the transformation law of supercurrents $J^{(3)}(Z)$ and $\Psi^i(Z)$ under the action of generators belonging to the coset $(N = 4)/(N = 3)$:

$$\begin{aligned}\delta_A\Psi^i = \frac{1}{2}\epsilon^{ijl}D^{3-j}A^l\cdot J^{(3)} + \frac{1}{2}(D^jA^j)D^iJ^{(3)} + \frac{1}{2}\partial_zA^iJ^{(3)} - \\ -\frac{c}{12}D^3A^i + \frac{c}{12}\epsilon^{ijl}D^j\partial_zA^l\end{aligned} \tag{3.19}$$

$$\delta_AJ^{(3)} = -\frac{1}{6}D^jA^jD^i\Psi^i - \frac{1}{2}\partial_zA^i\Psi^i - \frac{1}{2}\epsilon^{ijl}D^{3-i}A^j\Psi^l - \frac{\tilde{c}}{12}D^i\partial_zA^i \tag{3.20}$$

$$D^iA^j(Z) + D^jA^i(Z) = \frac{2}{3}\delta^{ij}D^lA^l(Z)\,,$$

where the new superparameter $A^i(Z)$ collects all the parameters of $N = 4$ SCA lacking in $E(Z)$:

$$A^i(Z) = d^i(z) + \theta^i\rho(z) + \epsilon^{ijl}\theta^j\eta^l(z) - \theta^{3-i}a(z) + \theta^{3-l}\epsilon^{lij}\partial_zd^j(z) + \theta^3\partial_z\eta^i(z)\,. \tag{3.21}$$

Here parameters $\partial_z a^i(z), a(z), \rho(z)$, and $\partial_z \eta^i(z)$ correspond to the generators A^i_n, U_n, Q_r and S^i_r defined in (A.2), (B.5). Let us emphasize that the transformations (3.19), (3.20) do not act on the coordinates (z, θ^i) of $N = 3$ superspace and only change the form of superfields $J^{(3)}(Z), \Psi^i(Z)$. The defining constraint (3.15) is covariant under (3.19) for any c.

When deriving eqs.(3.19),(3.20), we did not use the explicit expressions of $J^{(3)}(Z)$ and $\Psi^i(Z)$ via $q^{Ai}(Z)$ and did not assume any relation between c and $\tilde{c}$. So, these transformations together with $N = 3$ transformations of supercurrents can be regarded as the general realization of above $N = 4$ SCA in terms of $N = 3$ superfields. Specializing to the particular form of supercurrents (3.11) and (3.14) amounts to expressing the central charges by eqs.(3.12), (3.17) via the integer k appearing in the WZNW action (3.6). With this choice of c and $\tilde{c}$, the transformations of supercurrents (3.19), (3.20) are produced by the following transformation of $q^{Ai}(Z)$

$$\begin{aligned}\delta_A q^{Bj} &= \frac{1}{2}(D^{3-j}A^i - D^{3-i}A^j)q^{Bi} - \frac{1}{4}D^i A^i \cdot \epsilon^{jmn} D^m q^{Bn} - \\ &- \frac{1}{2}\partial_z A^m \cdot q^{Bn}\epsilon^{jmn} + \frac{1}{2}(D^{3-i}A^i)q^{Bj}\,. \end{aligned} \tag{3.22}$$

It is noteworthy that the irreducibility constraint (3.9) for q^{Ai} puts severe restrictions on the structure of supercurrents $J^{(3)}$ and Ψ^i (3.11),(3.14).The superfield projections of the latter with dimensions 3/2 and 2 (the ordinary supersymmetry currents and the conformal stress-tensor) are expressed, in virtue of (3.9),as the composites of the projections with lower dimensions,viz 1 and 1/2 (KM currents and the additional supersymmetry currents). Thus the constraint (3.9) automatically gives rise to the generalized Sugawara form for the component currents of higher dimensions.A more detailed treatment of this phenomenon will be given in Part II in the framework of $N = 4$ superspace.

By this we finish the description of classical superconformal theory associated with the $N = 3$ WZNW sigma model (3.6) and turn to the quantum case.

4. N=3 WZNW SIGMA MODEL: QUANTUM THEORY

The component action (3.6) involves, besides the fields of bosonic $O(3)$ WZNW sigma model, only free bosonic and fermionic fields. So we may quantize this system following the standard prescriptions of ref.[22]. As has been shown in [14] and in accordance with the general reasoning of [5,7], the basic novel features brought about by quantization are the changes in the values of the central charges (actually, only in c)

$$c_q = c + 3 = \frac{3}{2}(k+2) \quad , \tilde{c}_q = \tilde{c} = \frac{3}{2}k \tag{4.1}$$

and possible appearance of anomalous conformal weight Δ and $U(1)$-charge ω for $q^{Ai}(Z)$. The shift of c by 3 reflects the contributions of quantum fermions ($c_{\chi,\xi} = 2$), of bosonic

KM current ($c_W = \frac{3k}{k+2}$) and of quantum dilaton $u(z)$ ($c_u = 1 + \frac{3k^2}{2(k+2)}$)

$$c_q = c_{\chi,\xi} + c_W + c_u.$$

The noncanonical contribution from u is due to the presence of the Feigin-Fuchs term (properly renormalized in the quantum case) in the stress tensor $T(z)$ (see Appendix B).

The transformation laws are most transparently represented by SOPE's between the supercurrents themselves and between the supercurrents and the primary superfield $q^{Ai}(Z)$

$$J^{(3)}(Z_1)J^{(3)}(Z_2) = \left[-\frac{1}{2}\frac{\theta^3_{12}}{Z^2_{12}} + \frac{1}{2}\frac{\theta^{3-i}_{12}}{Z_{12}}D^i_2 - \frac{\theta^3_{12}}{Z_{12}}\partial_{z_2}\right] J^{(3)}(Z_2) + \frac{c_q}{12}\frac{1}{Z_{12}} + \ldots$$

$$J^{(3)}(Z_1)\Psi^i(Z_2) = \left[-\frac{1}{2}\frac{\theta^3_{12}}{Z^2_{12}} + \frac{1}{2}\frac{\theta^{3-j}_{12}}{Z_{12}}D^j_2 - \frac{\theta^3_{12}}{Z_{12}}\partial_{z_2}\right]\Psi^i(Z_2) - \frac{1}{2}\frac{\theta^{3-il}_{12}\Psi^l(Z_2)}{Z_{12}} + \frac{\tilde{c}_q}{12}\frac{\theta^{3-i}_{12}}{Z^2_{12}} + \ldots$$

$$\Psi^i(Z_1)\Psi^j(Z_2) = \frac{1}{2}\left[\frac{\delta^{ij}\theta^3_{12}}{Z^2_{12}} - \frac{\theta^{3-ij}_{12}}{Z_{12}} + \frac{\theta^{3-i}_{12}}{Z_{12}}D^j_2\right] J^{(3)}(Z_2) + \frac{c_q}{12}\left[\frac{\delta^{ij}}{Z_{12}} + \frac{\epsilon^{ijk}\theta^{3-k}_{12}}{Z^2_{12}}\right] + \ldots \tag{4.2}$$

$$\begin{aligned}J^{(3)}(Z_1)q^{Ai}(Z_2) &= \left[-\Delta\frac{\theta^3_{12}}{Z^2_{12}} + \frac{1}{2}\frac{\theta^{3-j}_{12}}{Z_{12}}D^j_2 - \frac{\theta^3_{12}}{Z_{12}}\partial_{z_2}\right] q^{Ai}(Z_2) - \frac{1}{2}\frac{\theta^{3-il}_{12}}{Z_{12}}q^{Al}(Z_2) + \ldots \\ \Psi^i(Z_1)q^{Aj}(Z_2) &= \frac{1}{2}\frac{\delta^{ij}\theta^l_{12} - \delta^{il}\theta^j_{12} - 2\omega\cdot\delta^{jl}\theta^i_{12}}{Z_{12}}q^{Al}(Z_2) - \frac{1}{4}\frac{\theta^{3-i}_{12}}{Z_{12}}\epsilon^{jmn}D^m_2 q^{An}(Z_2) + \\ &+ \frac{1}{2}\frac{\theta^3_{12}}{Z^2_{12}}\epsilon^{ijk}q^{Ak}(Z_2) + \ldots\,. \end{aligned} \tag{4.3}$$

Remarkably, the underlying constraint (3.9), being applied on both sides of eqs.(4.3), unambiguously fixes Δ and ω to be equal to their classical values

$$\Delta = \frac{1}{2}\,, \quad \omega = \frac{1}{2}\,. \tag{4.4}$$

The relations (4.1)-(4.4) completely specify the quantum transformation properties of the basic superfields and supercurrents of the $N = 3$ WZNW sigma model in question. Note that the infinitesimal $(N = 4)/(N = 3)$ transformations generated by $\Psi^i(Z)$ are represented by the following general contour integral formula

$$\delta_A\Phi(Z_2) = \oint_{C_2}\frac{dz_1}{2\pi i}\int d^3\theta_1 A^i(Z_1)\Psi^i(Z_1)\Phi(Z_2)$$

(cf. (2.25)). In particular, for q^{Aj} we have the same transformation law (3.22) as in the classical case.

An important problem is how to define the supercurrents via the basic superfield $q^{Ai}(Z)$ in quantum case. The classical expressions (3.13) now make no sense (cf. an analogous situation in $N=0$ and $N=1$ WZNW sigma models [5,7]), so one needs to seek another way of relating the supercurrents to the basic superfields. By analogy with the $N=0$ and $N=1$ cases, we define the quantum supercurrents $J^{(3)}(Z)$ and $\Psi^i(Z)$ by the following equation

$$\frac{a}{2}D^l q^{Aj}(Z) =: \left[J^{(3)}(Z)\epsilon^{lkj}q^{Ak}(Z) + \Psi^j(Z)q^{Al}(Z) - \delta^{lj}\Psi^k(Z)q^{Ak}(Z) - \Psi^l(Z)q^{Aj}(Z)\right] : , \quad (4.5)$$

where the symbol : : means the normal ordering (i.e., it is assumed that all the singular pieces have been subtracted). In the classical case, eq.(4.5) with $a=\frac{k}{2}$ is just another form of eq.(3.13). In quantum case, the value of constant a is different. It can be evaluated by resorting to the standard arguments of refs.[5,7]. Namely, one defines the supermatrix state

$$\mid Q^{Ai}(Z)\rangle \equiv q^{Ai}(Z)\mid 0\rangle \quad (4.6)$$

and demands it to obey the conditions ($J^{[i]}_r, \Psi^{[i]l}_r$ are defined in (2.31), (3.18))

$$J^{[i]}_r \mid Q^{Aj}(Z)\rangle = \Psi^{[i]l}_r \mid Q^{Aj}(Z)\rangle = 0 \quad (4.7)$$

$$r \geq \frac{1}{2} ,$$

which are required for consistency with SOPE's (4.3). Eqs.(4.3),(4.7) and (4.5) further imply

$$\Theta^{ljA} \equiv \left(\Gamma_{-\frac{1}{2}}\epsilon^{lkj} + S^j_{-\frac{1}{2}}\delta^{lk} - S^k_{-\frac{1}{2}}\delta^{lj} - S^l_{-\frac{1}{2}}\delta^{jk} + iaG^l_{-\frac{1}{2}}\delta^{jk}\right) \mid Q^{Ak}(Z) >= 0 \quad (4.8)$$

$$\Omega^{jA} \equiv \left(Q_{-\frac{1}{2}}\delta^{jk} + \epsilon^{jmk}G^m_{-\frac{1}{2}}\right) \mid Q^{Ak}(Z) >= 0 \quad (4.9)$$

(for the definition of generators Γ, S, G, Q see Appendix A). So, the states Θ^{ljA} and Ω^{jA} must be interpreted as zero-norm states. For consistency of such an interpretation the following relations should be valid

$$J^{[i]}_r\Theta^{ljA} = J^{[i]}_r\Omega^{jA} = 0, \Psi^{[i]l}_r\Theta^{ljA} = \Psi^{[i]l}_r\Omega^{jA} = 0$$

$$r \geq \frac{1}{2}$$

from which, just as in the $N=0$ and $N=1$ cases, the constant a is unambiguously fixed this time to the value

$$a = \frac{k+2}{2} , \quad (4.10)$$

thereby completely specifying the supercurrents $J^{(3)}(Z)$ and $\Psi^i(Z)$ in terms of $q^{Ai}(Z)$. It is worth mentioning that in the quantum case it is impossible to define two supercurrents via q^{Ai} separately; they are introduced simultaneously by the matrix equation (4.5).

The rest of this Sect. is devoted to discussing of two important particular cases of the $N=3$ WZNW supermultiplet, corresponding to the two versions of its partial fermionization.

The first option is

$$k = 0 \Rightarrow c_q = 3, \ \tilde{c}_q = 0 . \quad (4.11)$$

This choice makes sense only quantum-mechanically. In this case the bosonic component $W^i(z)$ of KM current does not contribute to all OPE's [12,14] and the whole $N=3$ and $N=4$ SCA can be realized on a shortened multiplet consisting of scalar field $u(z)$ and spinors $\chi(z), \xi^i(z)$ [23]. The relevant currents can be obtained from the general ones by putting elsewhere $k=0$ and

$$W^i(z) = 0 . \quad (4.12)$$

Eq.(4.12) at $k=0$ can be shown to be covariant both under $N=3$ and $(N=4)/(N=3)$ transformations [14] [4], so it may be considered as the additional irreducibility condition which singles out an invariant subspace (u, χ, ξ^i) from the $N=3$ WZNW supermultiplet.

Our aim is to translate eq.(4.12) into the superfield language. Before all , we need to correctly choose the superfield representation adequate to this situation. Clearly, $q^{Ai}(Z)$ already does not suit for this purpose as the above shortened multiplet contains no trace of the bosonic WZNW fields. On the other hand, the constraint (3.15) is valid as well in the quantum case and one may still represent the supercurrent $\Psi^i(Z)$ as

$$\Psi^i(Z) = \frac{1}{2}D^i\tilde{u}(Z) , \quad (4.13)$$

where $\tilde{u}(Z)$ is unconstrained for the moment and is not obliged to coincide with the superfield $u(Z)$ figuring in our previous consideration [5]. What one actually needs is that $\tilde{u}(Z)$ starts with the field $u(z)$. Also, the field $\chi(z)$ enters as the first component into $J^{(3)}(Z)$, so we are led to include the relevant supercurrent $\tilde{J}^{(3)}(Z)$ together with $\tilde{u}(Z)$ into the sought minimal superfield set. It remains to write down the superfield constraints equivalent to eq.(4.12). Looking at the component content of $J^{(3)}(Z)$ and $\Psi^i(Z)$, such constraints are easily found to be

$$\begin{aligned} D^i\tilde{J}^{(3)} - \frac{1}{4}\epsilon^{ijk} : D^j\tilde{u}D^k\tilde{u} : &= 0 \\ \epsilon^{ijk}D^jD^k\tilde{u} + 4 : \tilde{J}^{(3)}\cdot D^i\tilde{u} : &= 0 . \end{aligned} \quad (4.14)$$

Indeed, the lowest components of the l.h.s. of these equations just coincide with the bosonic part $W^i(z)$ of the $\hat{O}(3)$ and $\hat{O}(4)/\hat{O}(3)$ KM currents, which proves equivalence of (4.14) and (4.12).

[4] In the language of $N=4$ SCA, $W^i(z)$ is the bosonic part of the current $\frac{1}{2}(V^i(z)+A^i(z))$ generating first of $\hat{SU}(2)$'s entering into the product $\hat{O}(4)_{k+1,1} \sim \hat{SU}(2)_{k+1} \times \hat{SU}(2)_1$ (for definition of V^i, A^i see Appendix B). The second $\hat{SU}(2)$ is generated by the combination $\frac{1}{2}(V^i(z)-A^i(z))$ which includes only the fermionic parts. The fermions χ, ξ^i contribute 1 to the levels of both $\hat{SU}(2)$ KM algebras, because they can be assembled into doublets with respect to each of these $\hat{SU}(2)$'s.

[5] Expression (4.13) is the general superfield solution of (3.15). Any other possible structures are reduced to (4.13) after performing some D^i algebra.

The transformation properties of $\tilde{u}(Z)$ and $\tilde{J}^{(3)}(Z)$ subjected to constraints (4.14) are easily established by substituting the representation (4.13) into the general supercurrent SOPE's (4.2), putting there $k = 0$ and taking off one spinor derivative from $\tilde{u}(Z_2)$

$$\tilde{J}^{(3)}(Z_1)\tilde{u}(Z_2) = -\frac{\theta_{12}^3}{Z_{12}}\partial_{z_2}\tilde{u}(Z_2) + \frac{1}{2}\frac{\theta_{12}^{3-k}}{Z_{12}}D_2^k\tilde{u}(Z_2) + \ldots \qquad (a)$$

$$\tilde{\Psi}^i(Z_1)\tilde{u}(Z_2) = -\frac{\theta_{12}^{3-i}}{Z_{12}}\tilde{J}^{(3)}(Z_2) + \frac{1}{2}\frac{\theta_{12}^i}{Z_{12}} + \ldots \qquad (b) \qquad (4.15)$$

The supercurrent $\tilde{J}^{(3)}(Z)$ still satisfies eqs.(4.2) with $k = 0$.

It is instructive to see how eqs.(4.14) necessitate $k = 0$. Applying D^i on both sides, e.g., of the first of these equations and summing over index i one gets

$$D^iD^i\tilde{J}^{(3)} = -3\partial_z\tilde{J}^{(3)} = \frac{1}{2}\epsilon^{ijk} : D^iD^j\tilde{u}\cdot D^k\tilde{u} := -2 : \left(: \tilde{J}^{(3)}D^k\tilde{u} :\right) D^k\tilde{u} :, \qquad (4.16)$$

where we have used the second of eqs.(4.14). In the classical case the r.h.s. of this relation identically vanishes, which leads to $\partial_z\tilde{J}^{(3)} = 0 \Rightarrow \tilde{J}^{(3)} = const.$ The set of eqs.(4.14) becomes meaningful in the quantum case where (4.16) in view of SOPE's (4.2) merely fixes c_q to the value $c_q = 3 \Rightarrow k = 0$ (the r.h.s. of (4.16) is evaluated with using the definition (2.30)).

One may directly check that eqs.(4.14) are consistent with SOPE's (4.15) and (4.2). It is also a simple exercise to show that $\tilde{J}^{(3)}(Z)$ and $\tilde{u}(Z)$ subject to (4.14) include just the necessary set of independent fields $(u(z), \chi(z), \xi^i(z))$

$$\begin{aligned}
\tilde{u}(Z)\mid_{\theta=0} &= u(z) \\
D^i\tilde{u}(Z)\mid_{\theta=0} &\equiv S^i(z) = \xi^i(z) \\
D^iD^j\tilde{u}(Z)\mid_{\theta=0} &= i[\delta^{ij}U(z) + \epsilon^{ijk}A^k(z)] \\
U(z) = i\partial_z u(z), &\quad A^i(z) = i\chi(z)\xi^i(z) \\
D^3\tilde{u}(Z)\mid_{\theta=0} &\equiv iQ(z) = \frac{i}{6}\epsilon^{ijk}\xi^i\xi^j\xi^k + i\chi(z)\partial_z u
\end{aligned} \qquad (4.17)$$

$$\begin{aligned}
\tilde{J}^{(3)}(Z)\mid_{\theta=0} &\equiv \frac{1}{2}\Gamma(z) = \frac{1}{2}\chi(z) \\
D^i\tilde{J}^{(3)}(Z)\mid_{\theta=0} &\equiv \frac{i}{2}V^i(z) = \frac{1}{4}\epsilon^{ijk}\xi^j\xi^k \\
D^{3-i}\tilde{J}^{(3)}(Z)\mid_{\theta=0} &\equiv \frac{i}{2}G^i(z) = \frac{i}{2}[-i\partial_z u\xi^i + \frac{i}{2}\chi\epsilon^{ijk}\xi^j\xi^k] \\
D^3\tilde{J}^{(3)}(Z)\mid_{\theta=0} &\equiv T(z) = -\frac{1}{2}\{: \partial_z u\partial_z u : + : \xi^i\partial_z\xi^i : + : \chi\partial_z\chi :\}\,.
\end{aligned} \qquad (4.18)$$

The components $S^i(z)$, $U(z)$, $A^k(z)$, $V^i(z)$, $\Gamma(z)$, $G^i(z)$, $T(z)$ satisfy the standard OPE's of $U(1)$-extended $N = 4$ SCA [13] with $c_q = 3, \tilde{c}_q = 0$. Note that the superfield

$\tilde{u}(Z)$ (4.17) is just the one conjectured by Schoutens [23]. It is worth mentioning that the set irreducible under $N = 4$ SCA is formed by both superfields $\tilde{J}^{(3)}(Z)$ and $\tilde{u}(Z)$; as is seen from eqs.(4.15), the $(N = 4)/(N = 3)$ transformations mix these superfields among themselves [6].

One more interesting peculiarity of the superfield $N = 3$ representation considered is related to the following interpretation of the constraints (4.14). One may construct their solution starting from the supercurrents $J^{(3)}(Z)$ and $\Psi^i(Z)$ with $k = 0$. Indeed, one may check that the expressions

$$\begin{aligned}
\tilde{J}^{(3)} &= J^{(3)} - \theta^i(D^iJ^{(3)} - \epsilon^{ijk} : \Psi^j\Psi^k :) + \ldots \\
\tilde{\Psi}^i &= \Psi^i - \frac{1}{2}\theta^k\epsilon^{kij}(\epsilon^{jmn}D^m\Psi^n + 4 : J^{(3)}\Psi^j :) + \ldots
\end{aligned} \qquad (4.19)$$

(the higher terms can also be easily restored) satisfy, in their own, the SOPE's (4.2) at $c_q = 3, \tilde{c}_q = 0$. So, we may realize on the original set of fields (involving the bosonic current $W^i(z)$) two $N = 3$ (or $N = 4$) SCA's, both with $c_q = 3, \tilde{c}_q = 0$. Their realizations are quite different, but the most important point is that the second SCA closes on the shortened multiplet (u, χ, ξ^i), while the first one mixes the latter with $W^i(z)$[7]. With respect to this second SCA the supercurrents $J^{(3)}$, Ψ^i are not superfields, but $\tilde{J}^{(3)}$ and $\tilde{\Psi}^i$ are. On the other hand, the newly defined $\tilde{J}^{(3)}$ and $\tilde{\Psi}^i$ are not superfields with respect to the previous $N = 3, 4$ SCA's generated by the supercurrents $J^{(3)}$, Ψ^i. This manifests itself as the presence of explicit θ's in the expressions (4.19). Note that it is impossible to invert eqs.(4.19) and express $J^{(3)}$, Ψ^i through $\tilde{J}^{(3)}$, $\tilde{\Psi}^i$ since the latter objects involve a lesser number of independent fields than the former ones.

The second interesting case corresponds to

$$k = 2 \Rightarrow c_q = 6, \ \tilde{c}_q = 3\,. \qquad (4.20)$$

With these values of $c_q, \tilde{c}_q$ the contributions of the bosonic KM current $W^i(z)$ to the central charges of the Kac-Moody and Virasoro subalgebras equal 1 and $\frac{3}{2}$ respectively, and thus this current can be fermionized in terms of the $O(3)$-triplet of extra fermions ζ^i [22,5]

$$W^i = -\frac{i}{2}\epsilon^{ijk}\zeta^j\zeta^k \qquad (4.21)$$

(this corresponds to fermionizing the bosonic part of KM current for one of $\hat{SU}(2)$'s entering into $\hat{O}(4)_{k+1,1} \sim \hat{SU}(2)_{k+1}\times\hat{SU}(2)_1$; another $\hat{SU}(2)$ is realized from the beginning

[6] This property becomes manifest in $N = 4$ superspace where $\tilde{J}^{(3)}(Z)$ and $\tilde{u}(Z)$ turn out to be accomodated by the single $N = 4$ superfield $\tilde{u}(Z, \theta_4) = \tilde{u}(Z) + \theta_4\tilde{J}^{(3)}(Z)$ (Part II).

[7] Generally speaking, the currents of this second SCA, being functions only of the shortened multiplet, can be constructed out of $J^{(3)}$ and Ψ^i with arbitrary k, but the expressions for them look more complicated than (4.19). The possibility of such a construction is closely related to the general observation of Goddard and Schwimmer [16].

only on fermions χ, ξ^i). The relevant superfield constraints look as follows

$$
\begin{aligned}
D^i J^{(3)} &= \frac{1}{2}\epsilon^{ijk} : \Psi^j \Psi^k : + \frac{1}{4}\epsilon^{ijk} : \zeta^j \zeta^k : \\
\epsilon^{ijk} D^j \Psi^k &= -2 : J^{(3)} \Psi^i : + \frac{1}{2}\epsilon^{ijk} : \zeta^j \zeta^k :
\end{aligned}
\tag{4.22}
$$

$$
\Psi^i = \frac{\sqrt{2}}{2} D^i u \,,
$$

where we have introduced a new superfield $\zeta^i(Z)$

$$
\zeta^i(Z)\mid_{\theta=0} = \zeta^i(z)\,. \tag{4.23}
$$

As the integrability condition for (4.22), one gets the constraint for $\zeta^i(Z)$

$$
D^i \zeta^j(Z) =: \left[J^{(3)}(Z)\epsilon^{ikj} + \delta^{ik}\Psi^j(Z) - \delta^{ij}\Psi^k(Z)\right]\zeta^k(Z) : \,. \tag{4.24}
$$

It is not difficult to figure out the relevant SOPE's (up to the regular terms)

$$
J^{(3)}(Z_1)\zeta^i(Z_2) = -\frac{\theta_{12}^3}{Z_{12}}\partial_{z_2}\zeta^i(Z_2) - \frac{\theta_{12}^3}{2Z_{12}^2}\zeta^i(Z_2) + \frac{\theta_{12}^{3-k}}{2Z_{12}}D_2^k\zeta^i(Z_2) - \frac{\theta_{12}^{3-ik}}{2Z_{12}}\zeta^k(Z_2)
$$

$$
\Psi^i(Z_1)\zeta^j(Z_2) = \frac{1}{2Z_{12}}\left[\theta_{12}^j\zeta^i(Z_2) - \delta^{ij}\theta_{12}^k\zeta^k(Z_2)\right] + \frac{\theta_{12}^{3-i}\epsilon^{jkl}}{Z_{12}}D_2^k\zeta^l(Z_2) + \frac{\theta_{12}^{3-i}}{Z_{12}} : J^{(3)}(Z_2)\zeta^j(Z_2) :
$$

$$
\begin{aligned}
J^{(3)}(Z_1)u(Z_2) &= -\frac{\theta_{12}^3}{Z_{12}}\partial_{z_2}u(Z_2) + \frac{\theta_{12}^{3-i}}{2Z_{12}}D_2^i u(Z_2) + \frac{1}{2\sqrt{2}}\frac{\theta_{12}^3}{Z_{12}^2} \\
\Psi^i(Z_1)u(Z_2) &= -\frac{1}{\sqrt{2}}\frac{\theta_{12}^{3-i}}{Z_{12}}J^{(3)}(Z_2) + \frac{1}{\sqrt{2}}\frac{\theta_{12}^i}{Z_{12}}\,.
\end{aligned}
\tag{4.25}
$$

It follows that the superfields $\zeta^i(Z)$, $u(Z)$ and $J^{(3)}(Z)$ are mixed under $(N=4)/(N=3)$ transformations. Just these superfields accomodate the irreducible set of fields in the present case. It can be shown that after imposing the constraints (4.22),(4.24) the only independent fields in these superfields are

$$
\begin{aligned}
u(Z)\mid_{\theta=0} &= u(z), \\
J^{(3)}(Z)\mid_{\theta=0} &= \frac{1}{\sqrt{2}}\chi(z) \\
D^i u(Z)\mid_{\theta=0} &= \xi^i(z), \\
\zeta^i(Z)\mid_{\theta=0} &= \zeta^i(z)\,.
\end{aligned}
\tag{4.26}
$$

The remaining components are expressed in terms of (4.26) and their derivatives. In particular, the conformal stress tensor is given by

$$
D^3 J^{(3)}(Z)\mid_{\theta=0} \equiv T(z) = -\frac{1}{2}[: \partial_z u \partial_z u : + \frac{1}{\sqrt{2}}\partial_z^2 u + : \xi^i \partial_z \xi^i : + : \chi\partial_z\chi : + : \zeta^i \partial_z \zeta^i :]\,. \tag{4.27}
$$

The presence of the Feigin-Fuchs term in (4.27) results in the noncanonical contribution 5/2 from $u(z)$ to the central charge of Virasoro algebra.

Finally, we note that one may construct new representations of $N = 3(N = 4)$ SCA by tensoring independent copies of the superfield $q^{Ai}(Z)$. Each q^{Ai} enters with its own integer k and makes an independent contribution to the supercurrents and, respectively, to the central charges. The corresponding component action is a sum of actions (3.6), with the product bosonic manifold $(U(1) \times O(3))^n$, where n is the number of q's.

So much for $N = 3$ WZNW superfields. In the second part of the paper we discuss analogous issues in the framework of $N = 4$ 2D superspace.

APPENDIX A

Here we write down $N = 3$ and $U(1)$ extended $N = 4$ SCA's in terms of generators $J_n^{[i]}$ (2.31), $\Psi_r^{[i]l}$ (3.18) and $I_n^{[i]l}$ (2.32).

$$
N = 3 \;\; SCA
$$

$$
J_n = -L_n, \;\; J_r^i = -i\frac{1}{2}G_r^i, \;\; \epsilon^{ijk}J_n^{jk} = -T_n^i, \;\; i\frac{1}{3!}\epsilon^{ijk}J_r^{ijk} = \frac{1}{2}\Gamma_r
$$

$$
\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}; \\
&[L_n, G_r^i] = (\frac{n}{2} - r)G_{n+r}^i; [L_n, V_m^i] = -mV_{n+m}^i; \\
&\{G_r^i, G_q^j\} = 2\delta^{ij}L_{r+q} + i(r-q)\epsilon^{ijk}V_{r+q}^k + \frac{c}{3}(r^2 - 1/4)\delta^{ij}\delta_{r+q,0}; \\
&[L_n, \Gamma_q] = -(\frac{n}{2} + q)\Gamma_{n+q}; \;\; [V_m^i, \Gamma_q] = 0, \{\Gamma_q, G_r^i\} = -V_{q+r}^i; \\
&[V_m^i, G_r^j] = i\epsilon^{ijk}G_{m+r}^k - m\delta^{ij}\Gamma_{m+r}; \;\; \{\Gamma_q, \Gamma_r\} = \frac{c}{3}\delta_{q+r,0}; \\
&[V_n^i, V_m^j] = i\epsilon^{ijk}V_{n+m}^k + \frac{c}{3}n\delta^{ij}\delta_{n+m,0}
\end{aligned}
\tag{A.1}
$$

$$
i,j,k = 1,2,3; n,m \in \mathbf{Z}; r,q \in \mathbf{Z} + \frac{1}{2}\,.
$$

Completion to $N=4$ SCA

$$\Psi^{il}_r = -\frac{i}{2}\delta^{il}Q_r + \frac{1}{2}(q-1/2)\epsilon^{ilj}S^j_r,$$
$$\Psi^{ijl}_n\epsilon^{ijk} = -\delta^{lk}U_n - \epsilon^{lkj}A^j_n, \quad \frac{1}{3!}\epsilon^{ijk}\Psi^{ijkl}_q = -\frac{i}{2}S^l_q$$

$$\begin{aligned}
&[L_n,Q_r] = (\frac{n}{2}-r)Q_{n+r}; \quad [L_n,S^i_r] = -(\frac{n}{2}+r)S^i_{n+r};\\
&[L_n,U_m] = -mU_{n+m} - i\frac{\tilde{c}}{6}n(n+1)\delta_{n+m,0}; \quad [L_n,A^i_m] = -mA^i_{n+m};\\
&\{G^i_r,S^j_q\} = -\delta^{ij}U_{r+q} - \epsilon^{ijk}A^k_{r+q} + i\frac{\tilde{c}}{3}(r+1/2)\delta^{ij}\delta_{r+q,0};\\
&\{G^i_r,Q_q\} = -i(r-q)A^i_{r+q}; \quad [G^i_r,U_n] = nS^i_{n+r};\\
&[G^i_r,A^j_n] = i\delta^{ij}Q_{n+r} - n\epsilon^{ijk}S^k_{n+r};\\
&[V^i_n,S^j_r] = i\epsilon^{ijk}S^k_{n+r}; \quad [V^i_n,A^j_m] = i\epsilon^{ijk}A^k_{n+m} + \frac{\tilde{c}}{3}n\delta^{ij}\delta_{n+m,0};\\
&[V^i_n,Q_r] = -nS^i_{n+r}; \ [V^i_n,U_m] = \{\Gamma_r,S^i_q\} = [\Gamma_r,U_n] = [S^i_q,U_n] = [A^i_n,U_m] = 0;\\
&[\Gamma_r,A^i_n] = iS^i_{r+n}; \ \{\Gamma_r,Q_q\} = U_{r+q} + i\frac{\tilde{c}}{3}(r-1/2)\delta_{r+q,0};\\
&\{S^i_r,S^j_q\} = \frac{c}{3}\delta^{ij}\delta_{r+q,0}; \quad [S^i_r,A^j_n] = -i\delta^{ij}\Gamma_{r+n};\\
&[A^i_n,A^j_m] = i\epsilon^{ijk}V^k_{n+m} + \frac{c}{3}n\delta^{ij}\delta_{n+m,0}; \quad [A^i_n,Q_r] = iG^i_{n+r};\\
&[U_n,U_m] = \frac{c}{3}n\delta_{n+m,0}; \quad [U_n,Q_r] = n\Gamma_{n+r};\\
&\{Q_r,Q_q\} = 2L_{r+q} + \frac{c}{3}(r^2-1/4)\delta_{r+q,0}; \{S^i_r,Q_q\} = -V^i_{r+q}\ .
\end{aligned} \tag{A.2}$$

$O(4)$ covariant form of $N=4$ SCA

$$I^{il}_m = -\delta^{il}L_m - \frac{i}{2}(m+1)T^{il}_m, \ I^{ijl}_q = \frac{1}{2}(\delta^{jl}G^i_q - \delta^{il}G^j_q) - \frac{i}{2}(q+\frac{1}{2})\epsilon^{ijlk}\Gamma^k_q,$$
$$\frac{1}{3!}\epsilon^{ijkl}I^{ijkn}_m = -\frac{1}{2}[\epsilon^{lnst}T^{st}_m + \delta^{ln}U_m], \ \frac{1}{4!}\epsilon^{ijkl}I^{ijkln}_r = -\frac{1}{2}\Gamma^n_r$$

$$\begin{aligned}
&[L_n,L_m] = (n-m)L_{n+m} + \frac{c_1}{12}(n^3-n)\delta_{n+m,0};\\
&[L_n,G^i_r] = (\frac{n}{2}-r)G^i_{n+r}; [L_n,\Gamma^i_q] = -(\frac{n}{2}+q)\Gamma^i_{n+q};
\end{aligned}$$

$$\begin{aligned}
&\{G^i_r,G^j_q\} = 2\delta^{ij}L_{r+q} - i(r-q)T^{ij}_{r+q} + \frac{c_1}{3}(r^2-1/4)\delta^{ij}\delta_{r+q,0};\\
&[L_n,U_m] = -mU_{n+m} - i\frac{c_2}{6}n(n+1)\delta_{n+m,0}; [L_n,T^{ij}_m] = -mT^{ij}_{n+m};\\
&[T^{ij}_n,T^{kl}_m] = -i(\delta^{i[l}T^{k]j}_{n+m} + \delta^{j[k}T^{l]i}_{n+m}) + \left[(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})\frac{c_1}{3} - \epsilon^{ijkl}\frac{c_2}{3}\right]n\delta_{n+m,0};\\
&[T^{ij}_n,G^k_r] = n\epsilon^{ijkl}\Gamma^l_{n+r} - i(\delta^{ik}G^j_{n+r} - \delta^{jk}G^i_{n+r});\\
&[T^{ij}_n,\Gamma^k_q] = -i(\delta^{ik}\Gamma^j_{n+q} - \delta^{jk}\Gamma^i_{n+q});\\
&\{G^i_r,\Gamma^j_q\} = \delta^{ij}U_{r+q} + \frac{1}{2}\epsilon^{ijkl}T^{kl}_{r+q} - i\frac{c_2}{3}(r+\frac{1}{2})\delta^{ij}\delta_{r+q,0};\\
&[U_n,U_m] = \frac{c_1}{3}n\delta_{n+m,0}; \quad \{\Gamma^i_q,\Gamma^j_r\} = \frac{c_1}{3}\delta^{ij}\delta_{r+q,0};\\
&[G^i_r,U_n] = -n\Gamma^i_{n+r}; \quad [\Gamma^i_q,U_n] = [T^{ij}_n,U_m] = 0
\end{aligned} \tag{A.3}$$

$$i,j,k,l = 1,2,3,4; n,m \in \mathbf{Z}; r,q \in \mathbf{Z} + \frac{1}{2}\ .$$

Comparison of (A.3) with (A.1), (A.2) yields

$$c = c_1, \quad \tilde{c} = c_2\ . \tag{A.4}$$

APPENDIX B

Here we give a general definition of the component currents appearing in the θ-decomposition of $J^{(3)}(Z), \Psi^i(Z)$ and their explicit form in terms of the fields $u(z)$, $\xi^i(z)$, $\chi(z)$ and $W^i(z)$ entering into the action (3.6).

$$\begin{aligned}
\Gamma(z) &\equiv 2J^{(3)}(Z)\mid_{\theta=0} = \sqrt{\frac{c}{3}}\chi(z);\\
V^i &\equiv -2iD^iJ^{(3)}(Z)\mid_{\theta=0} = \left[W^i(z) - \frac{i}{2}\epsilon^{ijk}\xi^j(z)\xi^k(z)\right]\\
G^i(z) &\equiv -2iD^{3-i}J^{(3)}(Z)\mid_{\theta=0} = -\Big[i\xi^i(z)\partial_z u(z) + \sqrt{\frac{3}{c}}\chi(z)V^i(z) +\\
&\quad + \sqrt{\frac{3}{c}}\epsilon^{ijk}\xi^j(z)W^k(z) + i\frac{\tilde{c}}{c}\sqrt{\frac{c}{3}}\partial_z\xi^i(z)\Big]
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
T(z) \equiv D^3J^{(3)}(Z)\mid_{\theta=0} = -\frac{1}{2}\Big[&:(\partial_z u(z))^2: + \frac{\tilde{c}}{c}\sqrt{\frac{c}{3}}\partial_z^2 u(z) + :\xi^i(z)\partial_z\xi^i(z): +\\
&+ :\chi(z)\partial_z\chi(z): - \frac{3}{c}:W^i(z)W^i(z):\Big]
\end{aligned}$$

$$
\begin{aligned}
S^i(z) &\equiv 2\Psi^i(Z)\,|_{\theta=0} = \sqrt{\frac{c}{3}}\xi^i(z); \quad \left[\delta^{ij}U(z) + \epsilon^{ijk}A^k(z)\right] \equiv -2iD^i\Psi^j(Z)\,|_{\theta=0} = \\
&= \left[i\delta^{ij}\sqrt{\frac{c}{3}}\partial_z u(z) + \epsilon^{ijk}\left(W^k(z) + i\chi(z)\xi^k(z)\right)\right]; \qquad \text{(B.2)} \\
Q(z) &= i\frac{2}{3}D^{3-i}\Psi^i(Z)\,|_{\theta=0} = \\
&= \left\{-\sqrt{\frac{3}{c}}\xi^i(z)\left[W^i(z) - \frac{i}{6}\epsilon^{ijk}\xi^j(z)\xi^k(z)\right] + i\chi(z)\partial_z u(z) - i\frac{\tilde{c}}{c}\sqrt{\frac{c}{3}}\partial_z\chi(z)\right\}.
\end{aligned}
$$

The remaining components of $\Psi^i(Z)$ vanish or are expressed as derivatives of (B.2) in virtue of the constraint (3.15). The values of the central charges are

$$c = \tilde{c} = \frac{3}{2}k \qquad \text{(B.3)}$$

in the classical case and

$$c = \frac{3}{2}(k+2), \tilde{c} = \frac{3}{2}k \qquad \text{(B.4)}$$

in the quantum one.

For the reader's convenience, we also recall the general z-expansion of the currents

$$
\begin{aligned}
&S^i(z) = \sum_r z^{-r-\frac{1}{2}}S^i_r; \quad \Gamma(z) = \sum_r z^{-r-\frac{1}{2}}\Gamma_r; \\
&V^i(z) = \sum_n z^{-n-1}V^i_n; \quad A^i(z) = \sum_n z^{-n-1}A^i_n; \quad U(z) = \sum_n z^{-n-1}U_n; \\
&Q(z) = \sum_r z^{-r-\frac{3}{2}}Q_r; \quad G^i(z) = \sum_r z^{-r-\frac{3}{2}}G^i_r; \qquad \text{(B.5)} \\
&T(z) = \sum_n z^{-n-2}L_n \\
&\qquad n \in \mathbf{Z}; \quad r \in \mathbf{Z} + \frac{1}{2}.
\end{aligned}
$$

The coefficients in (B.5) obey the relations (A.1), (A.2) as a consequence of SOPE's (4.2).

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Received by Publishing Department
on December 22, 1989.