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ELECTROMAGNETIC MOMENTS OF HADRONS AND QUARKS IN A HYBRID MODEL

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1. INTRODUCTION

The quark model calculations of magnetic moments and radiative decays of hadrons are known to provide a simple and clear understanding of the SU(3)- and SU(6)-symmetry predictions [1-3], to locate a number of the unitary symmetry breaking effects, to enable a check of the universality of the quark characteristics in mesons and baryons, and to estimate the effective (i.e., dynamical) masses of constituent quarks [4-7]. The new precise measurements of the hyperon magnetic moments [8] might be a source of new, more subtle and detailed information on the internal structure of baryons. The problem is now to choose or elaborate reliable enough (that is, adequate to the achieved experimental accuracy) tools to extract this information.

This work is devoted to the consideration of relations between the magnetic moments of baryons and quarks following from the sum rules which result,in turn,from a limited number of rather general assumptions. These sum rules do not result from any non-relativistic dynamics or the translationally non-invariant description of the relativistic quark motion in multiparticle systems. Furthermore, they take into account additional contribution from a nonvalence partons in hadrons. We rely here on the picture of the hadronized clusters of nonvalence quarks which display themselves as meson - mainly pion - exchange currents acting on the periphery of hadrons. On the basis of sum rules and data we extract first the structure parameters of hadrons and quarks. Then to calculate some of these parameters, the model is used of the local interaction of the pion field with quarks. The qualitative agreement obtained between two sets of the numerical values testifies to "hybrid" models, in which quarks and mesons are introduced as independent dynamical variables, and which can serve as an intermediate substitute for still not available full hadronization theory.

In connection with the general question of flavour and spin structure of nucleons, we discuss briefly and emphasize the sensitivity of selected structure parameters (analogues of the magnetic moments) of the weak neutral current for isoscalar hadrons and nuclei (e.g. the deuteron) to a possible violation of the quark line rule (the OZI rule), namely, to the presence and polarization characteristics of strange quarks inside nucleons.

2. MAGNETIC MOMENTS OF BARYONS AND QUARKS IN BROKEN SU(3)

We have earlier proposed [9] the parametrization of the baryon magnetic moments based both on the theory of broken unitary symmetries and on the composite quark model. The electromagnetic current operator of a quark system is assumed to be a sum of the covariant operators bilinear in the u, d, s quark field. The magnetic moment operator is defined, in a standard way, by the well-known moment of the total current. The following parametrization is introduced for baryon magnetic moments through an additive sum of single quark operators:

$$\mu(B) = \sum_{q=u,d,s} \mu(q) \langle B|\bar{q}q|B\rangle = \frac{1}{\sqrt{2}}(\mu(u) - \mu(d)) \langle B|\hat{\rho}_3|B\rangle + \frac{1}{\sqrt{6}}(\mu(u) + \mu(d) - 2\mu(s)) \langle B|\hat{\omega}_8|B\rangle + \frac{1}{\sqrt{3}}(\mu(u) + \mu(d) + \mu(s)) \langle B|\hat{\omega}_1|B\rangle, \qquad (1)$$

$$\hat{\rho}_3 = \frac{1}{\sqrt{2}}\Sigma(\bar{u}u - \bar{d}d), \qquad (2)$$

$$\hat{\omega}_8 = \frac{1}{\sqrt{6}}\Sigma(\bar{u}u + \bar{d}d - 2\bar{s}s), \quad \hat{\omega}_1 = \frac{1}{\sqrt{3}}\Sigma(\bar{u}u + \bar{d}d + \bar{s}s).$$

Eq.(1) defines only the structure of the corresponding operators in the SU(3) internal variable space. No assumption about the nonrelativistic quark dynamics is made.

Also, no constraints are imposed on magnitudes of $\mu(q)$ in (1) absorbing the hadronic matrix element values of the vector currents, defined in terms of the quark configuration variables (momenta, spins, etc.). The $\hat{\rho}_3$, $\hat{\omega}_8$ and $\hat{\omega}_1$ operators in (2) have evident SU(3)-transformation properties and in the limit of unbroken SU(3) all $\langle B|\hat{\rho}_3(\hat{\omega}_8)|B\rangle$ are parametrized by two coupling constants of the F- and D-type. The singlet contribution is represented as a linear combination of the octet operators and the matrix element $\delta(B)$ of the corresponding OZI-suppressed $\bar{q}q$-configuration for a given baryon: $\delta(B) = \langle B|\bar{q}q|B\rangle$, $q \notin q_\ell, q_u$, where $B = (q_\ell^2, q_u)$ (e.g. $\delta(N) = \langle N|\bar{s}s|N\rangle$, etc.)

$$\frac{1}{\sqrt{3}}\langle B|\hat{\omega}_1|B\rangle = \sqrt{\frac{2}{3}}\langle N|\hat{\omega}_8|N\rangle + \delta(N). \qquad (3)$$

Therefore, if $\delta(N)$ and other contributions violating the quark line rule (i.e., the Okubo-Zweig-Iizuka rule) are negligible, then the singlet constant in (1) is expressed via the same F- and D-octet constants.

At last, we take a simple parametrization scheme of the pion current contributions to baryon magnetic moments which is suggested by the simplest Feynman diagrams with the two-pion intermediate states in the current channel. Those diagrams, where the meson propagator line (here it can be any meson,not only pion) begins and ends on the same quark, are assumed to be absorbed in the quantities $\mu(q)$ in (1). The pion exchange current contributions are defined by the diagrams with the pion propagator connecting different quark lines. The K- (and more heavy) meson contributions are neglected here because there should be a much more strong suppressing influence of hadron structure form factors due to a larger mass of propagating mesons. The charged pion exchange currents give contribution $C_{exch}(P) = -C_{exch}(N)$ and $C_{exch}(\Lambda\Sigma)$ to magnetic moments $\mu(P)$, $\mu(N)$ and $\Lambda\Sigma^o$ - transition moment $\mu(\Lambda\Sigma)$, and give no contribution to other $\mu(Y)$ where $Y = \Lambda, \Sigma, \Xi$. So,we are led to the following parametrization of the baryon magnetic moments ($B = (q_\ell^2, q_u)$, $q = u, d, s$, $q_\ell = q_{like}$, $q_u = q_{unlike}$):

$$\mu(B) = \mu(q_\ell)\cdot g_2(B) + \mu(q_n)\cdot g_1(B) + C_{exch}(B) + \sum_q \mu(q)\cdot \delta(N),$$

$$\mu(\Lambda) = \mu(s)\cdot\left(\frac{2}{3}g_2(\Lambda) - \frac{1}{3}g_1(\Lambda)\right) + (\mu(u) + \mu(d))\cdot\left(\frac{1}{6}g_2(\Lambda) + \frac{2}{3}g_1(\Lambda)\right) + \sum_q \mu(q)\cdot\delta(N),$$

$$\mu(\Lambda\Sigma) = \frac{1}{\sqrt{3}}(\mu(u) - \mu(d))\cdot\left(\frac{1}{2}g_2(\Lambda\Sigma) - g_1(\Lambda\Sigma)\right) + C_{exch}(\Lambda\Sigma). \qquad (4)$$

In the exact SU(3) limit for all octet baryon wave functions we would have $g_i(B) = g_i(\Lambda) = g_i(\Lambda\Sigma)$, $i = 1, 2$, $B = P, N, \Sigma^{\pm}, \Xi^{o,-}$. The following sum rules were derived and discussed [9] for this case:

$$P + N + \Xi^\circ + \Xi^- - 3\Lambda - \frac{1}{2}(\Sigma^+ + \Sigma^-) = 0, \quad (5)$$

$$(\Sigma^+ - \Sigma^-)(\Sigma^+ + \Sigma^- - P - N) - (\Xi^\circ - \Xi^-)(\Xi^\circ + \Xi^- - P - N) = 0. \quad (6)$$

All particle symbols in Eqs.(5) and (6) and further on denote the corresponding magnetic moments in nuclear magnetons. Furthermore, the isotopic sum rule $\Sigma^\circ = (\Sigma^+ + \Sigma^-)/2$ is assumed to be valid. One can obtain now Σ^+ and Ξ^- from (5) and (6) in terms of other magnetic moments. For P, N and Ξ°, the PDG-tabulated values /10/ were taken, and $\Sigma^- = -1.164 \pm 0.014$ which is the weighted average of several experiments /11/ and $\Lambda = -0.58 \pm 0.01$ is the experimental value $\Lambda_{exp} = -0.613 \pm 0.004$ with a subtracted part corresponding to the calculated $\Lambda\Sigma^\circ$-mixing effect /12/. As a result, we obtain from (5) and (6)

$$\Sigma^+ = 2.46 \pm 0.04, \quad (7)$$

$$\Xi^- = -0.72 \pm 0.05. \quad (8)$$

The numerical value of (7) is closer to the last /13/ of all the obtained /14/ values of Σ^+: $\Sigma^+ = 2.479 \pm 0.025$. The deviation of (8) from the PDG-tabulated value $\Xi^- = -0.69 \pm 0.04$ /10/ is insignificant, although in view of new (still preliminary) result /8/ $\Xi^- = -0.651 \pm 0.017$ the validity of (8), hence, of the sum rules (5) and (6), appears doubtful. That is why we consider now a less restrictive assumption: $g_i(Y) = g_i(\Lambda) = g_i(\Lambda\Sigma)$ for $Y = \Sigma, \Xi$, but at the same time $g_i(N) \neq g_i(Y)$. This assumption has been used also in Ref. /15/, but contrary to the cited work /15/ we keep in (4) $C_{exch}(N) \neq 0$, $C_{exch}(\Lambda\Sigma) \neq 0$. Now we obtain the most general sum rule of our approach

$$(\Sigma^+ - \Sigma^-)\cdot(\Sigma^+ + \Sigma^- - 6\Lambda + 2\Xi^\circ + 2\Xi^-) + (\Xi^\circ - \Xi^-)\cdot(\Sigma^+ + \Sigma^- + 6\Lambda - 4\Xi^\circ - 4\Xi^-) = 0. \quad (9)$$

Taking now /8/ $\Sigma^- -1.156 \pm 0.014$ and $\Xi^- = -0.651 \pm 0.017$ and the values of Λ and Ξ° fixed earlier we get for Σ^+ from (9)

$$\Sigma^+ = 2.37 \pm 0.08. \quad (10)$$

The mean value of (10) is almost the same as in the earlier measurement /14/. However, due to a rather large uncertainty range the deviation of (10) from the latest measurement /13/ amounts only to 1σ, the dominant part of this deviation being

due to uncertainty of the theoretical calculation of $\Lambda = -0.58 \pm 0.01$. To exploit this high sensitivity of sum rule (9) to Λ we take now $\Sigma^+ = 2.479 \pm 0.025$ /13/ to obtain from (9)

$$\Lambda = -0.567 \pm 0.009. \quad (11)$$

The deviation (11) from $\Lambda_{exp} = -0.613 \pm 0.004$ may be ascribed, besides the electromagnetic $\Lambda\Sigma^\circ$-mixing to a number of other factors. To illustrate, these are: the presence, in wave functions of hyperons, of a small configuration admixtures of a more complex nature than it is reflected in (4) or the relativistic effects resulting in the difference of the effective magnetic moment for quarks of a given flavour but residing in different baryons. Nevertheless, we estimate the parameter $\theta_{\Lambda\Sigma}$ of the isotopic $\Lambda\Sigma^\circ$-mixing assuming this effect to be main cause of the difference of Λ defined by (11) and Λ_{exp}:

$$\sin\theta_{\Lambda\Sigma} \simeq \theta_{\Lambda\Sigma} = \frac{\Lambda - \Lambda_{exp}}{2\mu(\Lambda\Sigma)} = (1.43 \pm 0.31)\cdot 10^{-2}, \quad (12)$$

where Λ is the Λ-hyperon magnetic moment with "switched-off" $\Lambda\Sigma^\circ$-mixing and $\mu(\Lambda\Sigma) = 1.61 \pm 0.08$ taken from the experiment /16/. The independent estimate of $\theta_{\Lambda\Sigma}$ follows from the SU(3)-symmetry sum rule for the electromagnetic splitting of baryon masses /17/:

$$\theta_{\Lambda\Sigma} = \frac{m(P) - m(N) - m(\Sigma^+) + m(\Sigma^\circ)}{\sqrt{3}\,(m(\Sigma^\circ) - m(\Lambda))} = (1.41 \pm 0.08)\cdot 10^{-2}. \quad (13)$$

Calculation of $\theta_{\Lambda\Sigma}$ on the basis of sum rule (9) and Eq.(12) seems to be, to our knowledge, the first calculation of this important parameter from sufficiently accurate data. If no additional assumption about the terms $\sum_q \mu(q)\cdot\delta(N)$ in (4) is made, we can also obtain within our approach (thereafter we make use of the average value $\mu(\Sigma^+) = 2.419 \pm 0.022$ /8/)

$$\alpha_D(Y) = \left(\frac{D}{D+F}\right)_Y = \frac{g_2(Y) - 2g_1(Y)}{2(g_2(Y) - g_1(Y))} = \frac{1}{2}\cdot(1 - (\Xi^\circ - \Xi^-)/(\Sigma^+ - \Sigma^- - \Xi^\circ + \Xi^-)) = 0.572 \pm 0.022, \quad (14)$$

$$\frac{u-d}{u-s} = (\Sigma^+ - \Sigma^- - \Xi^\circ + \Xi^-)/(\Sigma^+ - \Sigma^\circ) = 1.137 \pm 0.012. \quad (15)$$

Eq.(14) gives a measure of the unitary symmetry breaking ($a_D^{SU(6)}$ = 0.6) in hyperons.

If the OZI rule-violating terms are negligible, then

$$\frac{u}{d} = (\Sigma^+(\Sigma^+ - \Sigma^-) - \Xi^\circ(\Xi^\circ - \Xi^-))/(\Sigma^-(\Sigma^+ - \Sigma^-) - \Xi^-(\Xi^\circ - \Xi^-)) = \qquad (16)$$
$$= -1.74 \pm 0.03\,,$$

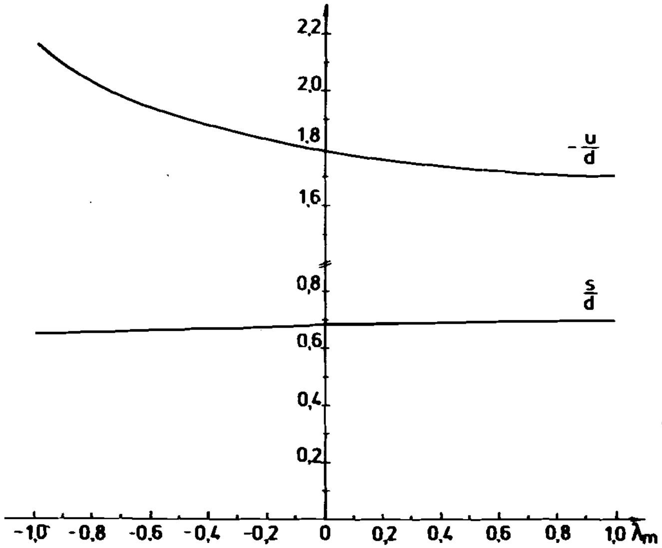
$$\frac{s}{d} = (\Sigma^+\Xi^- - \Sigma^-\Xi^\circ)/(\Sigma^-(\Sigma^+ - \Sigma^-) - \Xi^-(\Xi^\circ - \Xi^-)) = 0.67 \pm 0.02\,. \qquad (17)$$

Dependence of u/d and s/d on a parameter λ_m

$$\lambda_m = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}, \qquad (18)$$

$$\langle B|\hat{\omega}_1|B\rangle = \sqrt{2}\cdot\frac{1+\lambda_m/2}{1-\lambda_m}\cdot\langle N|\hat{\omega}_8|N\rangle,$$

characterizing the OZI rule violation is depicted in the Figure. It illustrates a very weak dependence of the quark structure parameters u/d and s/d on the magnitude of λ_m. It is just the smallness of $|\sum_q \mu(q)| \simeq 0.13\,|\mu(d)| \ll |\mu(d)|$ that gives a significant suppression of the above-mentioned dependence namely in the case of electromagnetic structure parameters



Figure

considered, besides the expected smallness of the very OZI-violation measures like $\delta(N)$ and λ_m in (4) and (18). The determination of $C_{exch}(\Lambda\Sigma)$ and $C_{exch}(N)$ requires further inputs. From Eq.(4) and experimental values of hyperon magnetic moments/8/ we get

$$C_{exch}(\Lambda\Sigma) = \mu_{exp}(\Lambda\Sigma) + \frac{1}{\sqrt{3}}(\Xi^\circ - \Xi^- - \frac{1}{2}(\Sigma^+ - \Sigma^-)) = 0.23 \pm 0.08. \qquad (19)$$

Next we outline the relation of (19) to $C_{exch}(N)$. Let the isovector operator $\hat{\mu}^\pi_{exch}$ of the exchange magnetic moment be, approximately, presented as a sum

$$\hat{\mu}^\pi_{exch} = \hat{\mu}_{exch}(8) + \hat{\mu}_{exch}(10-10^*)$$

of two operators transforming like an octet and a decuplet of SU(3). Remembering conditions $C_{exch}(Y) = 0$, $Y = \Sigma, \Xi$, fixed earlier in formulas (4), we can relate 3 unknown constants parametrizing (within assumed here SU(3)-symmetry) all matrix elements $\langle B|\hat{\mu}^\pi_{exch}|B\rangle$ over the octet baryon states. In this way, using also (19), we find

$$C_{exch}(P) = -C_{exch}(N) = \sqrt{3}\,C_{exch}(\Lambda\Sigma) = 0.40 \pm 0.14\,. \qquad (20)$$

It follows from (20) that the pion exchange currents contribute considerably (about 15-20%) as compared with the "quark" parts $P - C_{exch}(P) = 2.39$, $N - C_{exch}(N) = -1.51$ of proton and neutron magnetic moments. After that we obtain the configuration structure parameter $a_D(N)$ for nucleons

$$a_D(N) = \frac{g_2(N) - 2g_1(N)}{2(g_2(N) - g_1(N))} = 0.542 \pm 0.014. \qquad (21)$$

Comparison of (14) and (21) suggests a more strong SU(6) breaking due to spin-dependent quark interactions in nucleons relative to hyperons. This conclusion differs in the sense of direction of the deviation of $a_D(N)$ and $a_D(Y)$ from each other from the corresponding result of Ref./15/.

Further use of the hypothesis of universality/4/ for ratios (16) and (17) results in a number of testable predictions for magnetic moments of the baryon decuplet and for the transition moments $\mu(B^*_{10} \to B_8\gamma)$:

$$\mu(\Omega^-) = \frac{s}{u}\mu(\Delta^{++}) = -0.38\,\mu(\Delta^{++}), \qquad (22)$$

$$\mu(\Delta^-) = \frac{d}{u}\mu(\Delta^{++}) = -0.56\,\mu(\Delta^{++}), \tag{23}$$

$$\mu(\Sigma^{*+}\to\Sigma^+\gamma):\mu(\Sigma^{*-}\to\Sigma^-\gamma):\mu(\Sigma^{*o}\to\Sigma^o\gamma):\mu(\Xi^{*o}\to\Xi^o\gamma):\mu(\Xi^{*-}\to\Xi^-\gamma) = \tag{24}$$
$$= 1 : -0.135 : 0.432 : 1 : -0.135 .$$

The quantities $\mu(\Delta^+ \to P\gamma) = \mu(\Delta^o \to N\gamma)$ and $\mu(\Sigma^{*o}\to\Lambda\gamma)$ are affected by the pion exchange contributions intractable without further model assumptions. Invoking (still preliminary) result /8/ $\mu_{exp}(\Omega^-) = -2.0 \pm 0.2$ one gets from (22) and (23) $\mu(\Delta^{++}) = 5.20 \pm 0.52$ and $\mu(\Delta^-) = -3.0 \pm 0.3$. The magnitude of $\mu(\Delta^{++})$ thus obtained agrees with the earlier estimate /18/ $\mu_{exp}(\Delta^{++}) = 5.7 \pm 1.0$.

In conclusion of this section we briefly discuss the sensitivity to the magnitude of $\delta(N)$ or λ_m of an analogue of the magnetic moment with respect to weak neutral current. By substitution

$$Q(q) \to V^{NC}(q) = t_L(q) - 2Q(q)\sin^2\theta_W ,$$

where $Q(q)$ is the electric charge of a quark, $t_L(q)$ = the weak isospin 3d projection, we pass from the electromagnetic current to the vector part of the weak neutral current and to the corresponding structure characteristics of hadrons. First, let us assume that the contribution of strange quarks to magnetic moments of nucleons is negligible. Then, the charge symmetry of the u- and d-quark distributions in a proton and a neutron enables one to write for the general structure of the quantities at hand

$$\mu^{em(NC)}(P) = Q_u(V_u^{NC})\,g'_2 + Q_d(V_d^{NC})\,g'_1, \tag{25}$$
$$\mu^{em(NC)}(N) = Q_d(V_d^{NC})\,g'_2 + Q_u(V_u^{NC})\,g'_1 ,$$

and, after fixing the values of g'_i (i = 1, 2) through $\mu^{em}(P) = 2.793$ and $\mu^{em}(N) = -1.913$, to obtain

$$\mu^{NC}(P) = 1.067, \tag{26}$$

$$\mu^{NC}(N) = -1.474. \tag{27}$$

The isoscalar part caused by a possible presence of the strange quarks in nucleons is expresses in the form

$$\Delta\mu^{NC}(N) = \frac{1}{2}\lambda_m(V_u^{NC} + V_d^{NC} + V_s^{NC})(g'_1 + g'_2) =$$

$$= \frac{3}{2}\lambda_m \cdot \sum_q V_q^{NC}(\mu^{em}(P) + \mu^{em}(N)) \simeq -0.66\,\lambda_m , \tag{28}$$

where λ_m is defined in (18) and the numerical estimates in (26)-(28) were performed with $\sin^2\theta_W = 0.23$.

Expected smallness of λ_m suggests us to use, as the most sensitive constraints on or definitions of $\lambda_m \neq 0$, the investigation of elastic neutrino scattering or measuring μ^{NC} from the P-odd effects in elastic electron (or muon) scattering off the deuteron or other isoscalar nuclei with non-zero spin (^{6}Li, etc). For deuterons, one can write down the analogue of the well-known non-relativistic formula for $\mu^{em}(D)$:

$$\mu^{NC}(D) = \mu^{NC}(P) + \mu^{NC}(N) - \frac{3}{2}P_D\,(\mu^{NC}(P) + \mu^{NC}(N) - \tag{29}$$
$$- \frac{1}{2}(V^{NC}(P) + V^{NC}(N))) = -0.38 - 1.3\,\lambda_m ,$$

where $P_D \simeq 0.05$ is the D-wave (ℓ = 2) probability in the deuteron, $V_{P(N)}^{NC} = 2V_{u(d)}^{NC} + V_{d(u)}^{NC}$ and Eqs.(26)-(28) are used. Therefore, even a comparatively rough determination of $\mu^{NC}(D)$ would be an important source of valuable information about the magnitude of λ_m, hence of the range of applicability of the OZI rule.

3. RADIATIVE MESON DECAYS

In treating the $V \to P\gamma$ and $P \to V\gamma$ decays, the problem is to take into account the nonstatic retardation effects (i.e. the photon wave function variation over a distance of an order of the meson radii) and recoil effects (dependence of the radial overlap integrals on meson momenta). To minimize the dependence of final results on these effects, we confine ourselves to the comparison of the amplitude ratios for those processes which have close energies of final photons (e.g. $\omega \to \pi^o\gamma$ and $\rho \to \pi\gamma$, , or $\rho \to \eta\gamma$ and $\eta' \to \rho\gamma$, etc.). In these situations we have more confidence in approximating the ratios of the physical amplitudes by those of the static limits of the same amplitudes which are parameterized according to /9/

$$\langle P|\hat{\mu}^{\Delta I=0}|V\rangle = (\mu(u) + \mu(d))\,I_{VP}^q + 2\mu(s)\,I_{VP}^s, \tag{30}$$

$$\langle K|\hat{\mu}|K^*\rangle = (\mu(q) + \mu(s))\,I_{K^*K} , \tag{31}$$

$$\langle P|\hat{\mu}^{\Delta I=1}|V\rangle = (\mu(u) - \mu(d) + C_{exch})\, I^{q}_{VP}, \tag{32}$$

where $q = u, d$, C_{exch} is a constant approximately representing the exchange pion current contributions to all isovector transitions, $I^{q(s)}_{VP}$ is a static radial overlap integral of the $\bar{q}q$ ($\bar{s}s$) configurations. Two terms in Eq.(30) reflect a possibility of mixing in the isoscalar mesons of different non-strange ($N = (\bar{u}u + \bar{d}d)/\sqrt{2}$) and strange ($S = \bar{s}s$) quark configurations. The quark-gluon composition of isoscalar mesons (η and η' or ϕ and ω) is represented in the form

$$\eta = X_\eta N + Y_\eta S + \sum_i Z^{\eta}_i R_i, \qquad X^2 + Y^2 + \sum_i Z_i^2 = 1, \tag{33}$$
$$\eta' = X_{\eta'} N + Y_{\eta'} S + \sum_i Z^{\eta'}_i R_i,$$

where the state vectors N and S are assumed to correspond to a hypothetical situation with "switched-off" annihilation interaction which is responsible for mutual transitions of the $\bar{q}q$-pairs and $\bar{s}s$-pairs to each other and to gluons, while the states R_i in (33) represent either the radial excited states of the same quark pairs that compose the N and S-states, or the gluon states.

We list now a number of results concerning radiative meson decays. The most reliable confirmation of universal ratios of the quark magnetic moments in mesons and baryons [4] follows from the calculated ratio of the K-meson radiative widths:

$$\frac{\Gamma(K^{*+}\to K^{+}\gamma)}{\Gamma(K^{*o}\to K^{o}\gamma)} = \left(\frac{u/d + s/d}{1 + s/d}\right)^2 = \begin{matrix} 0.42 \pm 0.03 \text{ (th)} \\ 0.44 \pm 0.06 \text{ (exp)}^{/19/}, \end{matrix} \tag{34}$$

where values of (16) and (17) have been used in calculation.

With Eqs.(30), (32) and pertinent kinematical factors we get

$$\left(\frac{u - d + C_{exch}}{u + d}\right)^2 = 0.95\, \frac{\Gamma(\omega \to \pi^o\gamma)}{\Gamma(\rho \to \pi\gamma)}, \tag{35}$$

where upon $C_{exch}/(u-d) = -0.09 \pm 0.04$ in the meson sector, if $\Gamma(\omega \to \pi^o\gamma) = 746 \pm 51$ keV [20], $\Gamma(\rho \to \pi\gamma) = 69 \pm 3$ keV (the weighted average of three experiments [21]). Relations between the $\rho(\omega) \to \eta\gamma$ and $\eta' \to \rho(\omega)\gamma$ decays are derived analogously.

Using the experimental data $\Gamma(\rho \to \eta\gamma) = 52 \pm 13$ keV, $\Gamma(\eta' \to \rho\gamma) = 72 \pm 12$ keV [10] and $u/d = -1.74 \pm 0.03$, we have, as a consequence (all values in units of keV)

$$\Gamma(\omega \to \eta\gamma) = 4.2 \pm 1.0 \; (6.1 \pm 2.5)^{/20a/}, \tag{36}$$
$$\Gamma(\eta' \to \omega\gamma) = 6.4 \pm 1.1 \; (5.6 \pm 0.7)^{/22/}$$

(the experimental values are given in parentheses). As last, we mentioned only

$$|I^{q}_{\eta'\rho}/I^{q}_{\rho\eta}| = 0.78 \pm 0.12, \tag{37}$$
$$|I^{s}_{\phi\eta}/I^{q}_{\rho\eta}| = 0.80 \pm 11.$$

which follow from the ratios of width $\Gamma(\phi \to \eta\gamma) = 55 \pm 3$ keV [20b] and $\Gamma(\rho \to \eta\gamma)$ as well as $\Gamma(\eta' \to \rho\gamma)$ and $\Gamma(\rho \to \eta\gamma)$ and testify to the η and η' quark composition. If we adopt all $|Z_i| \ll 1$ in (33), then $X^2 + Y^2 = 1$ and from (37) we get $|X_\eta| = 0.78 \pm 0.04$ and $|Y_\eta| = 0.62 \pm 0.05$ which correspond to the effective SU(3)-singlet-octet mixing angle $\theta_P = -16 \pm 4°$ in the pseudoscalar nonet.

4. QUARK-PION INTERACTION AND THE STRUCTURE OF CONSTITUENT QUARKS

Due to interaction of u- and d-quarks with the charged pions there appears "magnetic anomaly" i.e., the relation $\mu(u):\mu(d) = Q_u:Q_d = -2$ ceases to be valid. We estimate the anomalous magnetic moments and charge radii of quarks in a simple model of the local preudoscalar coupling between pions and the isodoublet constituent (massive) u- and d-quarks. As is well-known, the perturbative calculations of the nucleon anomalous moments have been performed up to terms of the order $g^2_{\pi NN}$ and $g^4_{\pi NN}$ in the coupling constant of standard pseudoscalar interaction in the early 50th (for references to the original papers one can turn to the monograph [23]). In so far as the quark-pion effective Lagrangian is fully equivalent to that for the pion-nucleon interaction, many results for structural characteristics of quarks, treated as usual spinor particles with fixed mass m_q, can be obtained from the corresponding nucleon calculations by a simple redefinition of pertinent parameters and coupling constants.

As a result, the sum of the Dirac and anomalous magnetic moment of a massive fermion with the quantum numbers of a u-or d-quark is written as

$$\mu(u) = (\frac{e}{2m_q})(Q_u + \kappa_u) = (\frac{e}{2m_q})(Q_u + (Q_u - Q_d) B_\pi - \frac{1}{2}(Q_u + 2Q_d) B_f), \qquad (38)$$

where $m_q = m_u = m_d$, $\mu(d) = \mu(u \to d)$, $Q_{u(d)}$ = electric charges of quarks, $B_{\pi(f)}$ are functions of $\eta = (m_\pi/m_q)^2$ $Q_\pi = \pm(Q_u - Q_d)$ electric charges of pions. The explicit form of B's in the one-loop approximation is [23]

$$B_f = \frac{g^2_{\pi qq}}{8\pi^2}(1 + 2\eta + \eta(1-\eta)\ln\eta - 2\eta(3-\eta)\frac{\tan^{-1}\xi}{\xi}),$$

$$B_\pi = \frac{g^2_{\pi qq}}{8\pi^2}(1 - 2\eta - \eta(2-\eta)\ln\eta - 2\eta(2 - 4\eta + \eta^2)\frac{\tan^{-1}\xi}{\xi}) \qquad (39)$$

with $\xi = \sqrt{(4-\eta)/\eta}$ and $B_{f(\pi)}$ corresponding to the Feynman graph with a photon line ending on the intermediate quark (pion) line. Mass m_q of constituent quarks is an effective parameter of our model. We shall further assume it to be the same in baryons and mesons and for the numerical calculations the following value is taken

$$m_q^2 = \frac{2}{3}\pi^2 F_\pi^2 = (240\ \text{MeV})^2 \qquad (40)$$

found from meson data [24].

The quark-pion coupling constant is defined from the Goldberger - Treiman relation and Eq.(40)

$$g^2_{\pi qq}/(4\pi) = m_q^2/(4\pi F_\pi^2) = \pi/6 \simeq 0.5, \qquad (41)$$

F_π = 93 MeV being the leptonic decay constant.

It follows from (39) finally

$$\kappa_u = 0.029, \quad \kappa_d = -0.059, \qquad (42a)$$

$$\frac{\mu(u)}{\mu(d)} = \frac{Q_u + \kappa_u}{Q_d + \kappa_d} = -1.77 \qquad (42b)$$

in agreement with Eq.(16).

Undoubtedly, the binding effects originating from the relativistic motion of quarks in some confining potential will influence the absolute values of $\kappa_{u(d)}$ or $\mu(u)$ and $\mu(d)$ but the ratio (42b) is expected to be influenced much weaker. The one-loop estimate of the quark-gluon correction (analogues to the lowest-order radiative correction to magnetic moments of leptons) gives

$$\Delta\kappa_q^{QCD} = \frac{4}{3}(\frac{\alpha_s}{2\pi})Q_q = \frac{2Q_q\alpha_s}{3\pi}, \qquad (43)$$

where α_s is the quark-gluon interaction coupling constant.

Inclusion of $\Delta\kappa_q$ given by (43) into $\mu(q)$ increases the ratio $\mu(u)/\mu(d)$ by $0.02 \div 0.04$ for $0.5 \le \alpha_s \le 1.0$, thus leaving intact qualitative agreement of (16) and (42).

The pion cloud contribution to the charge radius $\langle r^2\rangle^q_{ch}$ of a given quark is defined by the standard linear combinations of $\langle r_1^2\rangle_q$ and κ_q which are the corresponding limits of the Dirac and Pauli form factors of spinor particles. The analytic expressions of $\langle r_1^2\rangle$ for massive fermions interacting with a pseudoscalar meson can be found, e.g., in Ref. [25]. Omitting unwieldy explicit formulas we give only the final numerical values of $\langle r^2\rangle^q_{ch}$ computed by Eqs.(40)-(42):

$$Q_u\langle r^2\rangle^u_{ch} = \frac{2}{3}\langle r^2\rangle^u_{ch} \simeq Q_d\langle r^2\rangle^d_{ch} = -\frac{1}{3}\langle r^2\rangle^d_{ch} \cong 0.05\ \text{fm}^2, \qquad (44)$$

The numerical values written down in this section give a qualitative impression on the influence of the pion corrections to electromagnetic characteristics of quark and hadrons. The problem left unsolved is the calculation of the meson exchange current contributions to hadronic observables. Its solution seems possible in essentially model-dependent approaches and it is beyond the scope of this paper.

5. CONCLUSION

Now we recapitulate the essential points of this paper. The sum rule (9) is the most general relation of our approach. It appears somewhat more general than a number of sum rules discussed in earlier papers [9,15,26-28] and meets no difficulties when compared with the precise data on the hyperon magnetic moments [8]. We emphasize the necessity of taking account of the electromagnetic $\Lambda\Sigma^\circ$-mixing for the sum rule (9) to agree with data and propose the determination of the $\Lambda\Sigma^\circ$-mixing parameter (i.e. the non-diagonal element of the corresponding mass operator) as a new result following from Eq.(9). Note that the substitution of $\mu(\Sigma^+)$ from the earlier measurement [14] gives the $\Lambda\Sigma^\circ$-transition mass close to the theory including the SU(3)-breaking effects [12] while the latest $\mu(\Sigma^+)$ value [13] leads to the $\Lambda\Sigma^\circ$-mixing parameter in accord with the unbroken SU(3) sum rule [17] (17) for this quantity. Therefore, it is highly desirable to discriminate between two available values of $\mu(\Sigma^+)$ by an independent measurement of this quantity.

It is seen from Eqs.(14) and (21) that a_D^Y and a_D^N ($a_D = D/(F + D)$) might be somewhat different from each other and both are different from $a_D^{axial} = 0.613 \pm 0.013$ following from the semi-leptonic baryon decays [29]. We suggest that this is due to a more prominent role of the pion currents in the formation of the nucleon structure characteristics as compared to the hyperon ones.

The non-additive, in the quark configuration variables, contribution of the pion exchange current to $\mu(P)$, $\mu(N)$ and $\mu(\Lambda\Sigma^\circ)$ is significant, which is in a qualitative agreement with the model calculations [30-32]. The pion exchange contribution to the isovector matrix elements (32) for the radiative meson decays is found to decrease the corresponding amplitudes thus compensating partially the otherwise essential enlargement of the isovector-to-isoscalar amplitude ratio (like, e.g., the ratio of the $\omega \to \pi^\circ\gamma$ and $\rho \to \pi\gamma$ decay amplitudes) due to the deviation of $\mu(u)|\mu(d)$ accrding to Eq.(16) from the charge ratio $Q(u)/Q(d) = -2$.

The numerical value of $\mu(u)/\mu(d)$ is in a qualitative accord with the earlier results [33-37] while the quantitative proximity of Eq.(16) to the pion-loop contribution (42a,b) to the anomalous magnetic moments of the quasi-free quarks is in line with many other examples of duality between the perturbative calculations and the (not fully worked out yet) non-perturbative theory including all the quark binding and confinement effects.

Both the $\mu(u)/\mu(d)$ and $\mu(s)/\mu(d)$ ratios are weakly dependent on the quark-line (or the OZI) rule violation. Much more sensitive to the breaking of the certainly non-exact OZI-rule turn out to be the neutral weak magnetism constants which, like usual magnetic moments, appear in the corresponding nucleon matrix elements of the vector part of the weak neutral current. Possible effects of the OZI-rule violation in various processes are intensively discussed in the current literature (see, e.g., Refs. [38,39] and quotations therein). In this connection we stress and illustrate, via Eq.(29), the special suitability for checking the OZI-rule of measurements of the nucleon weak magnetism constants in the experiments dealing with the elastic neutrino scattering or the observable P-odd effects in elastic scattering of the longitudinally polarized electrons or muons on the simplest isoscalar targets, the deuterium nuclei.

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Электромагнитные моменты адронов и кварков в гибридной модели

Соотношения между магнитными моментами барионов и кварков рассматриваются на основе общих правил сумм, которые следуют из теории нарушенных симметрий и кварковой модели и учитывают релятивистские эффекты и адронные поправки из-за мезонных обменных токов. Предложено новое нелинейное правило сумм для магнитных моментов гиперонов, которое согласуется с прецизионными экспериментальными данными и теорией электромагнитного $\Lambda\Sigma^{\circ}$ смешивания. Значения электромагнитных моментов кварков вычислены в рамках гибридной модели учета пионных поправок через локальное взаимодействие пионного поля с массивными конституентными кварками. Отмечено, что структурные константы слабого нейтрального магнетизма нуклонов и ядер чувствительны к возможному нарушению правил Окубо-Цвейга - Иизуки, т.в. к примеси странных кварков в нуклонах.

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Gerasimov S.B. E2-89-837

Electromagnetic Moments of Hadrons and Quarks in a Hybrid Model

Magnetic moments of baryons are analysed on the basis of general sum rules following from the theory of broken symmetries and quark models including the relativistic effects and hadronic corrections due to the meson exchange currents. A new sum rule is proposed for the hyperon magnetic moments, which is in accord with the most precise new data and also with a theory of the electromagnetic $\Lambda\Sigma^{\circ}$-mixing.The numerical values of the quark electromagnetic moments are obtained within a hybrid model treating the pion cloud effects through the local coupling of the pion field with the constituent massive quarks. Possible sensitivity of the weak neutral current "magnetic" moments to violation of the OZI rule is emphasized and discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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