сообщения
объединенного
института
ядерных
исследований
дубна

E2-89-828

V.N.Strel´tsov

"THE PROBLEM 4/3" AND THE RINDLER-DENUR "PARADOX"

1989

1. INTRODUCTION

About eight decades separate the appearance of the problems indicated in the title. The two problems, however, have much in common which is related to the definition on electromagnetic field energy and momentum. Thus, both problems have a common solution. Perhaps, an insufficient understanding of the advent and solution of the first problem gave rise to an interesting "paradox" involving the electrostatic energy of a capacitor.

2. ELECTROMAGNETIC ENERGY AND MOMENTUM OF A CHARGE

As is known, the 4-momentum of an electromagnetic field is given by the integral

$$G^{i} = \int T^{ik} dV_{k} , \tag{1}$$

where T^{ik} is the energy-momentum tensor of an electromagnetic field

$$T^{ik} = - F^{i\ell} F_{\ell}^{\ k} + \frac{1}{4} \gamma^{ik} F_{mn} F^{mn} \tag{2}$$

with F^{ik} the electromagnetic field tensor and dV_k the 4-vector of an infinitesimal volume which can in particular take the form

$$dV_{k} (dx^{1} dx^{2} dx^{3}, - dx^{0} dx^{2} dx^{3}, - dx^{1} dx^{0} dx^{3}, - dx^{1} dx^{2} dx^{0}) . \tag{3}$$

Here $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$ and dV_0 is evidently an element of usual volume ($dV_0 = dV$).

At one time, according to the Abraham´s hypothesis of electromagnetic origin of the electron mass for 4-momentum (1), the following formulae have been obtained (see, e.g., [1])

$$G^{1} = \frac{4}{3} m \beta c \gamma, \quad \mathcal{E} = mc^{2}(1 + \frac{\beta^{3}}{3}) \gamma , \tag{4}$$

where $m = \mathcal{E}^*/c^2$, $\mathcal{E}^*$ is the electrostatic energy in the electron rest system, $\gamma = (1-\beta^2)^{-1/2}$ and $\beta c = v_x$ is the velocity of electron motion. It should be noted that the Lorentz formula of volume contraction was in particular used to derive these expressions.

It is evident that eqs.(4) obtained in this way differ markedly from the known relativistic expressions for momentum and energy of a moving mechanical particle with rest mass m

$$p^1 = m\beta c\gamma, \quad \mathcal{E} = mc^2\gamma. \tag{5}$$

Below we discuss in detail the formulated problem. As usual, let us first consider a given charge in the proper reference frame (S^*) where it is at rest ($\vec{G}^* = 0$). As the magnetic field $\vec{H}^* = 0$ and $F_*^{ik} = (-\vec{E}^*, 0)$ for the charge at rest, the components T_*^{ik} are defined by the following expressions:

$$T_*^{ik} = \begin{pmatrix} -(E_x^*)^2 + \frac{1}{2}(\vec{E}^*)^2 & -E_x^*E_y^* & -E_x^*E_z^* & 0 \\ -E_x^*E_y^* & -(E_y^*)^2 + \frac{1}{2}(\vec{E}^*)^2 & -E_y^*E_z^* & 0 \\ -E_x^*E_z^* & -E_y^*E_z^* & -(E_z^*)^2 + \frac{1}{2}(\vec{E}^*)^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\vec{E}^*)^2 \end{pmatrix} \tag{6}$$

Based on the condition of spherical field symmetry, we have

$$\int E_*^\mu E_*^\nu dV^* = \frac{\delta^{\mu\nu}}{3}\int (E^*)^2 dV^* \tag{6'}$$

and also

$$\int E_*^\mu E_*^\nu dV_\nu^* = 0 \quad (\mu, \nu = 1, 2, 3). \tag{6''}$$

The charge is at rest in the S^* system. Therefore we have to require that the components of momentum $\vec{G}^*$ may be reducible to zero

$$G_*^1 = \frac{1}{6}\int (E^*)^2 dV_1^* = 0,\ G_*^2 = \frac{1}{6}\int (E^*)^2 dV_2^* = 0,\ G_*^3 = \frac{1}{6}\int (E^*)^2 dV_3^* = 0. \tag{7}$$

As the underintegral expressions are essentially positive here, the integrals (7) can be reduced to zero only provided that [2]

$$dV_1^* = dV_2^* = dV_3^* = 0. \tag{8}$$

The last condition is automatically fulfilled if the 4-vector of volume element in the S^* system is defined by three 4-vectors of the following form:

$$dx_*^i(0, dx^*, 0, 0),\ \delta x_*^i(0, 0, dy^*, 0),\ \Delta x_*^i(0, 0, 0, dz^*). \tag{8'}$$

The physical meaning of this choice of the vectors dx_*^i, δx_*^i and Δx_*^i becomes clear if we apply the procedure of measuring spatial cuts by the radar method. It is apparent that each of the indicated vectors can be presented as a half-difference of the two "light" 4-vectors describing the processes of propagation of a light signal along the corresponding infinitesimal cut in the positive and negative directions. In other words, this choice of the vectors dx^i, δx^i and Δx^i just corresponds to the previously considered definition (concept) of relativistic length [3].

The use of the special Lorentz transformations for the transition to some frame S moving along the x^*-axis of the S^*-system with velocity $v_x = -\beta c$ allows one to obtain

$$dV = dV^*\gamma, \tag{9a}$$

$$dV_1 = -\beta dV^*\gamma,\quad dV_2 = dV_2^* = 0,\quad dV_3 = dV_3^* = 0 \tag{9b}$$

for the transformation formulae of the components dV_i.

In the considered specific case the energy and momentum of a moving charge are defined by the following expressions:

$$\mathcal{E} = \int T^{00} dV_0 + \int T^{01} dV_1, \tag{10a}$$

$$G^1 = \int T^{10} dV_0 + \int T^{11} dV_1. \tag{10b}$$

Using the transformation formulae for the components of the energy-momentum tensor T_{ik}

$$T^{00} = (T_*^{00} + \beta^2 T_*^{11})\gamma^2, \tag{11a}$$

$$T^{01} = T^{10} = \beta(T_*^{00} + T_*^{11})\gamma^2, \tag{11b}$$

$$T^{11} = (T_*^{11} + \beta^2 T_*^{00})\gamma^2, \tag{11c}$$

one can easily find

$$\mathcal{E} = \gamma\int T_*^{00} dV^* = \mathcal{E}^*\gamma, \tag{12a}$$

$$G^1 = \frac{1}{c}\beta\gamma\int T^{00}_{*}\,dV^* = \frac{1}{c}\beta\ \mathcal{E}^*\gamma \qquad (12b)$$

taking (9) into account.

It is evident that the obtained formulae (12) correspond to the conventional relativistic transformation formulae for momentum and energy (5) and differ from the known expressions (4). Thus, within the frame of the considered approach there is no need to ascribe an extra mechanical inertial mass, which is due, say, to the existence of nonelectrical forces ("Poincare stresses"), to the electron /4/.

It should be noted that the question on the covariant definition of electromagnetic momentum and energy and the derivation of formulae (12) related to it are considered by a number of authors (see, e.g.,/5/). However, it should be stressed that one requirement of covariance alone is insufficient to obtain formulae (12) as, e.g., the known expressions (4) also satisfy the indicated requirement if $G^1_* \neq 0$ is taken into account in this case. Indeed, the choice of a 4-vector dV_i in the form of (0, dV, 0, 0) providing Lorentz contraction, in the S-frame implies that we have $(\beta dV\gamma, dV\gamma, 0, 0)$ or $(\beta dV^*, dV^*, 0, 0)$ in the S*-frame. As a result, $G^1_* = \beta\,\mathcal{E}^*/3c$ and, as it is easy to make sure, the corresponding quantity is really related to the Lorentz transformations.

In connection with the above-said, we would like to touch on the paper of Gamba /6/ in which, in particular, the conventional procedure of calculation of the electromagnetic field energy and momentum of a charge in different reference frames (S and S*) related to integration over volumes for t=const and t*= const, respectively, is subjected to criticism. As integration is performed over different hypersurfaces, the results of calculations should concern, as noted by the author, a variety of physical events whereas the Lorentz transformations deal with the same set of events.

In general, as for the choice of a (space-like) integration surface in the calculation of integral (1), it would seem that a priori it is really difficult to prefer some surface /7/. However, it should be kept in mind that the momentum of a charge at rest turns out to be different from zero in all cases except for integration over the surfaces normal to the world lines. This fact leads us to the physical condition, and the requirement of fulfilment of this condition (the momentum of a charge at rest equals zero) makes it possible to choose an integration surface unambiguously.

Thus, relativistic electrodynamics (imposing stringent requirements on the choice of the indicated surface) gives factually unambiguous evidence for the extention (and not contraction) of a moving volume.

It should be noted that the conclusion of the increase of the longitudinal dimensions of fast-moving bodies ("elongation formula") is a direct consequence of the above-mentioned concept of relativistic length /3/.

3. CONCEPT OF RELATIVISTIC LENGTH (CRL)

It will be recalled that CRL /3/ is based on the definition of the dimensions of fast-moving objects, in particular their longitudinal dimensions, which differs from the traditional definition. In this case the measuring procedure starts from the known radar method of measuring distances. The consequence of CRL is the relativistic "elongation formula". In the frame of this formula, for example, the length of a rod moving fastly along its maximum dimension is defined by the mean distance which a light signal covers in the positive and negative directions, i.e. from one of its ends to another and back. Let the rod move in the S-system with velocity βc from left to right along the x-axis, and an observer begins measuring after the rod has flown past, i.e. the signal sent from the left end will first run after the right end. Therefore the travelled distance is equal to

$$\ell_f = \ell^*(1+\beta)\gamma \qquad (13a)$$

with ℓ^* the length of the given rod at rest (S*-system).

In the negative direction, light goes to meet the left end, and so

$$\ell_b = \ell^*(1-\beta)\gamma. \qquad (13b)$$

Hence for relativistic length we get

$$\ell_r = \frac{1}{2}(\ell_f + \ell_b) = \ell^*\gamma \quad \text{("elongation formula")} \qquad (14)$$

Although the considered measuring procedure is "mental", it practically reflects a real physical situation in contradistinction to the traditional procedure related to notches of the simultaneous position of the rod ends to a subsequent measurement of the distance between them by a standard scale. The point is that the basic field of applicability of the theory of relativity embraces microworld phenomena. In so doing, according to present-day representations, the interactions of microobjects, in particular elementary particles, are due to the exchange of field quanta. Roughly speaking, these interactions are of the type of quantum sending-reception* or, in other words, one can say that they (especially electromagnetic ones) are of the radar type. If this is the case, it is evident that the effective spatial dimensions, which can be called "dynamic"**, characterizing the interactions should be defined just by relativistic length (14).

It should be also noted that in a four-dimensional representation the relativistic length is given by a spatial part of the difference between two 4-vectors describing the processes of light propagation in the positive and negative directions. In the above case for the corresponding 4-vector ℓ^i_r in the S- and S*-systems we have

$$\ell^i_r(\beta\ell^*\gamma, \ell^*\gamma, 0, 0), \quad \ell^{i*}_r(0, \ell^*, 0, 0). \tag{15a,b}$$

It should be also emphasized here that the problem associated with the existence of fundamental (or elementary) length can be practically solved only within the frame of CRL[18] which, as one believes, should play a large role in elementary particle physics. In fact, the introduction of fundamental length contradicts the conventional opinion of the contraction of relativistic moving scales[9]. Apparently, Pauli[10] has remembered precisely this fact speaking that the least universal length cannot probably exist on the basis of relativistic invariance.

In connection with the foregoing, we would like to dwell on the problem of visible dimensions of fast-moving objects. The point is that the deep-rooted notion that rapidly moving objects must be always contracted has been first called in question in the study of this problem[11]. In particular, it has been found[12,3] that a point observer standing in proximity to the way of rod motion sees the approaching rod elongated by a factor of $(1+\beta)\gamma$ and the flown past rod contructed by a factor of $(1-\beta)\gamma$ and so on (according to formulae (13a) and (13b).In this case the "mean" visible dimension(taking intermediate positions into acoount) is given just by the "elongation formula" (14). Research in the behaviour of the apparent form of a relativistic sphere can be of particular attention[13].

The most important of all this is that the common process of "vision" is a kind of a modification of the above radar method related to the interaction of radiated light signals (finally, photons) with an observer or a detector. Therefore, visible dimensions must reflect the character of interaction (in this case - electromagnetic one). In general, one can say that according to the present-day representations, radar (or "vision") with the aid of photons and gluons, respectively, factually forms the basis for the mechanism of electromagnetic and strong interactions. Thus, such an ordinary question on visible dimensions of fast-moving objects turn out to be closely related to the deepest processes of the microworld.

4. RINDLER-DENUR "PARADOX"

The "paradox" about the electrostatic energy of a capacitor[14] is related to the "problem 4/3". Let us remind its basis features. A plano-parallel charged capacitor is normal to the x^*-axis, the area of its plates A, and the gap between them is ℓ^*. In this case the energy density of an electric field is given by $\rho^* = (E^*_x)^2/2$. The total electrostatic energy $\mathcal{E}^*$ is measured by multiplying ρ^* by the volume $V^* = Al^*$. Let us calculate now its electrostatic energy in the S-system where the capacitor moves with velocity v_x. As E_x is not transformed when passing to the S-system, the energy density remains unchangeable $(\rho = \rho^*)$. The areas of the plates do not change as well. According to the contraction formula, the gap between them must decrease down to ℓ^*/γ. Consequently, the energy $\mathcal{E}$ also decreases by a factor γ whereas, according to all canons, it must increase by a γ factor in motion.The essence of the Rindler-Denur "paradox" lies in this fact[14]. It is easy to see that no paradox arises in the frame of CRL as the gap of a moving capacitor increases by a factor γ according to the "elongation formula" (14). As a result, electrostatic energy also increases which is in full agreement with the common "energy $= \gamma \times$ rest energy" law.

Nevertheless, strictly speaking, to calculate energy, one should use the covariant expression for G^i which can be written in a simple form

$$G^i = T^{ik} V_k \tag{1'}$$

*I.e., the same physical signals.

**In particular, taking into account that the influence of the process of measuring (interaction) cannot be already neglected in the microworld.

in accordance with (1). In this case, according to (15), we have

$$V_k\,(V^*\gamma,\ \beta V^*\gamma,\ 0,\ 0),\quad V_k^*(V^*,\ 0,\ 0,\ 0) \qquad (16a,b)$$

with $V^* = \ell^* A$. As a consequence, for the electrostatic energy of a moving capacitor we obtain

$$\mathcal{E} = T^{00}V_0 + T^{01}V_1\ . \qquad (10a')$$

Taking into account the transformation formulae (11) for the components T^{ik} and the existing relation between energy density and pressure $T_*^{11} = T_*^{00}$, it is easily seen that

$$\rho = T^{00} = T_*^{00} = \rho^*,\quad T^{01} = 0,\quad T^{11} = T_*^{11}. \qquad (17)$$

Thus, the second term in (10a') vanishes, and we come really (already strictly) to the wanted expression $\mathcal{E} = \mathcal{E}^*\gamma$.

In addition, it should be noted that no paradox arises pro-forma even in the traditional approach. The fact is that in this case we have

$$V_E\,(V^*\gamma^{-1},\ 0,\ 0,\ 0),\quad V_E^*(V^*,\ \beta V^*,\ 0,\ 0) \qquad (18a,b)$$

instead of (16). Therefore, according to (1'), the momentum of a capacitor at rest G_*^1 does not equal 0(!) whereas, on the contrary, $G_*^1 = 0$(!) for a moving capacitor.

However, it is evident that the last result is physically meaningless. We should put $G_*^1 = 0$. As above, this condition (taking $T_*^{11} \neq 0$ into account) leads unambigously to the requirement $V_{E1}^* = 0$, i.e. this gives evidence for CRL again.

5. RESUME

We have shown that the requirement of covariance itself does not solve the known "problem 4/3". The formulation of this solution is the following physical condition: the momentum of a charge at rest is equal to zero. The last condition leads factually to the definition of the volume of a moving body which differs from the conventional one. Volume extension and not contraction is its consequence. As a result, the "problem 4/3" and the Rindler-Denur "paradox" are indeed solved completely.

REFERENCES

1. Pais A. - In: Aspects of Quantum Theory, ed. A.Salam and E.P.Winger, Cambr. Univ. Press, 1972, p.79.
2. Strel'tsov V.N. - JINR Commun. P2-6710, Dubna, 1972.
3. Strel'tsov V.N. - Found Phys., 1976, v.6, p.293.
4. Poincare H. - Rendiconti del Circolo Matematico do Palermo, 1906, v.21, p.129.
5. Rohrlich F. - Classical Charged Particles, Reading Mass., Addison-Wesley, 1965, §6-3.
6. Gamba A. - Am.J.Phy., 1967, v.35, p.83.
7. Polubarinov I.V. - JINR Commun., P2-7532, Dubna, 1973.
8. Strel'tsov V.N. - JINR Commun. P2-88-626, Dubna, 1988.
9. Liboff R.L. - Am. J. Phys., 1987, v.55, p.1041.
10. Pauli W.-Handbuch der Physik, B d.24, Teil 1, 1933, Bd.24, II-§ 5.
11. Terrell J. - Phys. Rev., 1959, v.116, p.1014.
12. Mathews P.M., Lakshmanan M. - Nuovo Cim., 1972, v.B12, p.168.
13. Suffern K.G. - Am.J.Phys., 1988, v.56, p.729.
14. Rindler W., Denur J. - Am.J.Phys., 1988, v.56, p.795.

Received by Publishing Department
on December 14, 1989.