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QED CORRECTIONS AT Z^0-POLE WITH REALISTIC KINEMATICAL CUTS

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One of the most important processes which will be investigated with a high precision in a near future at LEP and SLC colliders will be e^+e^--annihilation into fermions. A high experimental accuracy for this process, which at Z^0-pole is expected to be about 0.2%, requires a very careful treatment of the electroweak radiative corrections. It is clear that the most essential QED radiative corrections are noticeably influenced by experimental conditions, which can be taken into account exactly only by means of Monte-Carlo calculations. However, for fitting the data it is necessary to use an analytical expression into which it would be desirable to include principle kinematical cuts. In the most currently available calculations [1] only a cut on the invariant mass of produced particles is taken into account. Recently, new approximate expressions with more realistic cuts have been presented in [2].

In this note, we present some results of the analytical calculation of the angular distribution for the process

$$e^+e^- \longrightarrow \mu^+\mu^- + (n\gamma) \qquad (1)$$

with taking account of cuts on an acollinearity angle between muons and the energies of both muons, which seems to be rather realistic.

The differential cross-section for process (1), with taking into account first order QED corrections to the initial state, can be written in the form

$$\frac{d\sigma}{dc} = \frac{d\sigma^0}{dc}(1+\beta_1\log(\epsilon)+\Delta_1^{S+V}) + \iint_\Omega dR dV_2\ \theta(1-R-\epsilon)\ \chi_1(s;R,V_2,c)\ , \qquad (2)$$

where $c=\cos(\theta)$, θ is an angle between μ^+ and e^+ momenta, $R = M^2_{\mu\mu}/s$, $M^2_{\mu\mu}$ is an invariant mass of two muons , $V_2 = M^2_{\mu\gamma}/s$, $M^2_{\mu\gamma}$ is an invariant mass of a photon and μ^+ . The first term in Eq.(2) is a contribution from virtual and soft photon corrections with (see for example [3])

$$\Delta_i^{S+V}(s) = \frac{3}{4}\beta_i(s) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) , \tag{3}$$

where $\beta_i(s) = \frac{2\alpha}{\pi} t_e$, $t_e = \log(s/m_e^2)-1$, and $\varepsilon = \frac{2\bar\omega}{\sqrt{s}} << 1$ ($\bar\omega$ being the soft photon energy cut-off). The Born-like angular distribution $\frac{d\sigma^o}{dc}$ can be written as:

$$\frac{d\sigma^o}{dc} = \sigma_o^+(s) f^+(c) + \sigma_o^-(s) f^-(c) , \tag{4}$$

where $f^+ = 1+c^2$, $f^- = 2c$ and expressions for $\sigma^+(s)$ and $\sigma^-(s)$ which include contributions of weak radiative corrections can be found in [4]. The second term in Eq.(2) represents hard photon bremsstrahlung from the initial state with the region of integration Ω given by the following kinematical restrictions:

$$\xi < \bar\xi , \qquad E_{\mu^+} > \bar E , \qquad E_{\mu^-} > \bar E , \tag{5}$$

where $\bar E$ is the minimum energy of muons and $\bar\xi$ is the maximum value of the acollinearity angle, defined as $\xi = \pi - \xi'$, with ξ' being the angle between μ^+ and μ^- momenta.

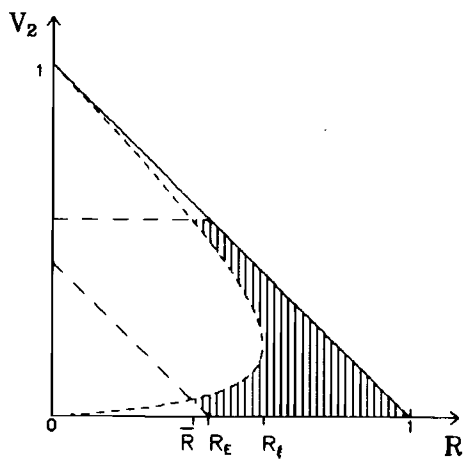

Fig.1 The Dalitz-plot for process (1). The dashed region is defined by cuts on the acollinearity angle and muon energies [see Eqs. (5-9)].

The Dalitz plot for $e^+e^- \longrightarrow \mu^+\mu^-\gamma$ is shown in Fig.1. The kinematical boundaries without cuts in the ultra-relativistic approximation is given by

$$0 < V_2 < 1-R . \tag{6}$$

Conditions (5) determine the dashed region (see Fig.1) limited by the curves defined by the following equations:

$$\begin{aligned} R &= \frac{4R_\xi V_2(1-V_2)}{(1-R_\xi)^2 + 4R_\xi V_2} , && (\xi\text{-cut}) , \\ V_2 &= 1 - R_E , && (E_{\mu^+} - \text{cut}) , \\ V_2 &= R - R_E , && (E_{\mu^-} - \text{cut}) . \end{aligned} \tag{7}$$

where $R_\xi = \frac{1-\sin(\bar\xi/2)}{1+\sin(\bar\xi/2)}$ and $R_E = \frac{2\bar E}{\sqrt{s}}$.

After integration over V_2 Eq.(2) can be written in the form

$$\begin{aligned} \frac{d\sigma}{dc} &= \int_{\bar R}^{1} dR \sum_{\rho=+,-} [S_i(s;R) f^\rho(c)\, \Theta(R-R_E) + H_i^\rho(s,\bar\xi,\bar E; R,c)]\sigma_o^\rho(sR) \\ &= \int_{\bar R}^{1} dR \sum_{\rho=+,-} G_i^\rho(s,\bar\xi,\bar E; R,c)\, \sigma_o^\rho(sR) , \end{aligned} \tag{8}$$

where $\bar R = R_E\left(1 - \frac{\sin^2(\bar\xi/2)}{1-R_E\cos^2(\bar\xi/2)}\right)$ is the minimum value of R (see Fig.1) and we define

$$S_i(s;R) = [1 + \beta_i(s)\log(\varepsilon) + \Delta_i^{S+V}(s)]\delta(1-R) + \frac{\beta_i(s)}{1-R}\Theta(1-R-\varepsilon) . \tag{9}$$

Initial-state bremsstrahlung functions $H_i^\pm$ of first-order regular at R=1, can be written in the following form

$$\begin{aligned} H_i^\pm(s,\bar\xi,\bar E; R,c) = \frac{\alpha}{\pi}\Big(&\Theta(R-R_E) h_i^\pm(c,R,1) + \Theta(R_E-R) h_i^\pm(c,R,A_E) \\ &- \Theta(R_\xi-R) h_i^\pm(c,R,A_\xi)\Big) . \end{aligned} \tag{10}$$

Here

$$A_E(R) = v - \frac{2R_E}{1-R} , \qquad A_\xi(R) = \left(\frac{1-v^2\sin^2(\bar\xi/2)}{\cos^2(\bar\xi/2)}\right)^{\frac{1}{2}} , \tag{11}$$

where $v = \frac{1+R}{1-R}$. Functions $h_i^\pm$ symmetric and antisymmetric in $\cos(\theta)$ can be written in the form

$$h_i^+(c,R,A) = \tilde h_i^+(c,R,A)\frac{R}{(1-R)^3} , \qquad h_i^-(c,R,A) = \tilde h_i^-(c,R,A)\frac{R}{(1-R)^2} , \tag{12}$$

where functions $\tilde{h}_i^{\pm}$ for three different kinematical regions are given by :

<u>I : $Y+XA > 0, \quad Y-XA < 0$,</u>

$$\tilde{h}_i^{+(I)} = \left[\log\left(\frac{Y+XA}{Y+X}\right)+\log\left(\frac{X^2}{R}\right)+t_e+1\right]\frac{1}{X^2}(R^2+1)(R^2+1-\frac{2}{X}R(R+1)+\frac{2}{X^2}R^2)$$
$$+\frac{2}{3X}(R^4+R^2+R+\frac{1}{R})+\frac{1}{2X^2}(A^2(R^4-2R^3+2R^2-2R+1)-7R^4-14R^3-\frac{58}{3}R^2-14R-7)$$
$$+\frac{1}{3X^3}R(24R^3+44R^2+44R+24)-\frac{1}{3X^4}R^2(22R^2+12R+22)-A^2(R-1)^2$$
$$+A(R-1)\frac{41}{R}+\frac{1}{3}(R^3+R^2+1+\frac{1}{R})-t_e f^+(R-2+\frac{1}{R})\ \eta(A)\ +(\ c\longrightarrow -c\ )\ , \tag{13}$$

$$\tilde{h}_i^{-(I)} = \left[\log\left(\frac{Y+XA}{Y+X}\right)+\log\left(\frac{X^2}{R}\right)+t_e+1\right]\frac{1}{X^2}(R^2+1)(R+1-\frac{2}{X}R)$$
$$+\frac{1}{2X}(1-A^2)(R^3+2R^2+2R+2+\frac{1}{R})-\frac{4}{X^2}(R^3+2R^2+2R+1)+\frac{3}{X^3}R(3R^2+2R+3)-$$
$$t_e f^-(\frac{1}{R}-1)\ \eta(A)\ -(\ c\longrightarrow -c\ )\ , \tag{14}$$

<u>II : $Y+XA < 0, \quad Y-XA < 0$,</u>

$$\tilde{h}_i^{+(II)}=\frac{1}{2X}(\frac{1}{3}A^3+A)(R^4-3R^3+4R^2-4R-\frac{1}{R}+3)-\frac{1}{X^2}A(R^4-1)+\frac{2}{X^3}AR(R^3-R^2+R-1)$$
$$-\frac{1}{6}A^3(R^3-2R^2-\frac{2}{R}+1)-\frac{1}{2}A(R^3-6R^2+12R+\frac{3}{R}-10)+(\ c\longrightarrow -c\ )\ , \tag{15}$$

$$\tilde{h}_i^{-(II)} = -\frac{1}{X^2}\,2A(R^2+1)(R-1)\ -(\ c\longrightarrow -c\ )\ , \tag{16}$$

<u>III : $Y+XA > 0, \quad Y-XA > 0$,</u>

$$\tilde{h}_i^{+(III)} = \log\left(\frac{Y+XA}{Y-XA}\right)\frac{1}{X^2}(R^2+1)(R^2+1-\frac{2}{X}R(R+1)+\frac{2}{X^2}R^2)$$
$$+A(R^3-2R^2+2-\frac{1}{R})-\tilde{h}_i^{+(II)}\ +(\ c\longrightarrow -c\ )\ , \tag{17}$$

$$\tilde{h}_i^{-(III)}=\frac{1}{X^2}\log\left(\frac{Y+XA}{Y-XA}\right)(R^2+1)(R+1-\frac{2}{X}R)-\tilde{h}_i^{-(II)}\ -(\ c\longrightarrow -c\ ), \tag{18}$$

where $X=\frac{1}{2}(1+R+c(1-R))$ and $Y=\frac{1}{2}(1-R+c(1+R))$, and $\eta(A=1)=1$, $\eta(A<1)=0$. These expressions were obtained in the approximation, which works well for $\bar{\xi}\geq 2^\circ$ and $\bar{E}\leq 0.9\ E_{beam}$.

Following [5] it is possible to take into account leading contributions of all orders of perturbation theory. After a summation of infrared and collinear singularities instead of (9) we have

$$S_i(s;R) = \beta_i(s)\ (1-R)^{\beta_i(s)-1}[1+\Delta_i^{S+V}(s)] \quad . \tag{19}$$

The angular distribution (8) can naturally be generalized to include final-state QED corrections. We obtain

$$\frac{d\sigma}{dc} = \int_{\bar{R}}^{1} dR \sum_{\rho=+,-} G_i^{\rho}(s,\bar{\xi},\bar{E};\ R,c)\ G_f^{\rho}(sR,\bar{\xi},\bar{E};\ \bar{R}/R,c)\ \sigma_o^{\rho}(sR)\ . \tag{20}$$

Final radiation functions $G_f^{\pm}$ (with soft-photon exponentiation) are given by

$$G_f^{\pm}(s,\bar{\xi},\bar{E};\ \bar{z},c) = \int_{\bar{z}}^{1} dz\ [S_f(s;z)\Theta(z-R_E)+H_f^{\pm}(s,\bar{\xi},\bar{E};\ z,c)]$$
$$= (1-R_E)^{\beta_f(s)}(1+\Delta_f^{S+V}(s))+\int_{\bar{z}}^{1} dz\ H_f^{\pm}(s,\bar{\xi},\bar{E};\ z,c)\ , \tag{21}$$

where β_f and Δ_f^{S+V} can be obtained from β_i and Δ_i^{S+V} by changing the electron mass to the muon one. Final-state functions $H_f^{\pm}$ are regular at z=1. They are given by expression (10) in which functions $h_i^{\pm}$ have to be replaced by the following functions

<u>$A = 1$</u>

$$h_f^+ = -(1+z)t_f - 2\ (1-z)\left(3-\frac{4}{f^+}\right)+\frac{1+z^2}{1-z}\log(z)\ ,$$
$$h_f^- = (1-z)-(1+z)t_f+\frac{z}{1-z}\log(z)\ , \tag{22}$$

<u>$A < 1$</u> $(A = A_\xi$ or $A = A_E)$

$$h_f^+ = \frac{1+z^2}{1-z}\log\left(\frac{1+A}{1-A}\right)-A(1-z)+\frac{4A\ z(1-z)}{A^2(1-z)^2-(1+z)^2}\left(3-\frac{4}{f^+}\right)\ ,$$
$$h_f^- = \frac{1+z^2}{1-z}\log\left(\frac{1+A}{1-A}\right)-\log\left(\frac{v+A}{v-A}\right)(z+1)\ . \tag{23}$$

By using the 4-momentum conservation law, it is easy to show [6] that the lower limit of integration in Eq.(21) $\bar{z}$ is equal to $\bar{R}/R$.

It is easy to show, that in first order approximation, equation (20) gives correct angular distribution. Note also that if a simple cut on R variable is applied (when leaving only first term in Eq.(10)), the total cross-section obtained after integration of Eq.(20) over $\cos(\theta)$ agrees with that derived in the framework of the structure function formalism [6].

As shown in [7], the $O(\alpha)$ initial-final interference term becomes essential for rather hard cuts. It is not difficult to take into account the interference term in the angular distribution with realistic cuts by the method used by us.

Now we present few pictures illustrating some results obtained with the use of the semianalytic ZCUTCOS Fortran code [8] based on Eq. (20) with the following choice for the parameters: M_Z= 91 GeV, Γ_Z= 2.476 GeV, $\sin^2\theta_W$= 0.232. To achieve an accuracy better than 0.3% we included in the program the dominant $O(\alpha^2)$ photon and real pair corrections taken from [5]. In Fig.2 we have plotted angular distributions around the Z°-pole. The dependence on the ξ-cut of the total cross-section and asymmetry at the Z°-pole for different energy cuts are shown in Figs.3 and 4 . It is seen that as far as energy cuts become more hard, the energy-momentum conservation law "forces" muons to be more collinear and the dependence on the ξ-cut

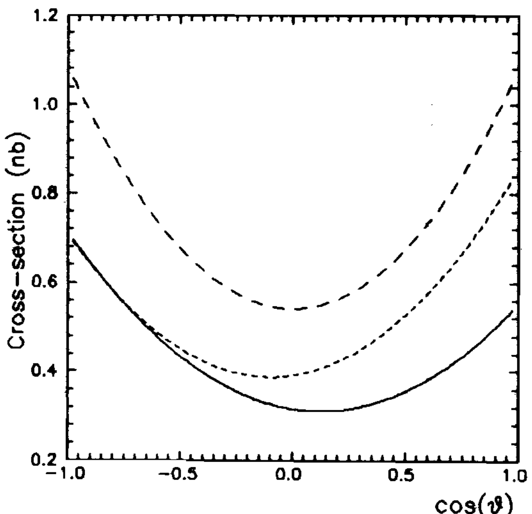

Fig.2 The angular distribution (20) for the different values of the center mass energy: $\sqrt{s}$ = 91 Gev (solid line), $\sqrt{s}$ = 90 Gev (dashed line), $\sqrt{s}$ = 92 GeV (fine dashed line). Values of cuts: $\bar{\xi}$ = 10° , $\bar{E}$ =20 GeV.

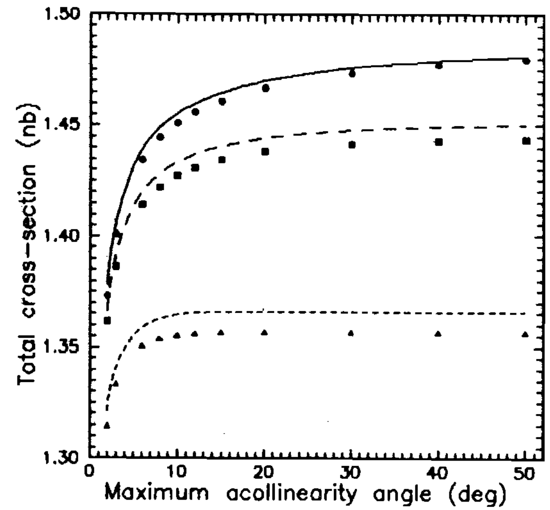

Fig.3 The total cross-section at Z°-pole as a function of the ξ-cut obtained after numerical integration of (20) and DYMU2 points for 10^6 events: without the E-cut (solid line), $\bar{E}$ = 25 GeV (dashed line), $\bar{E}$ = 40 GeV (fine dashed line). Here MC-errors are negligible.

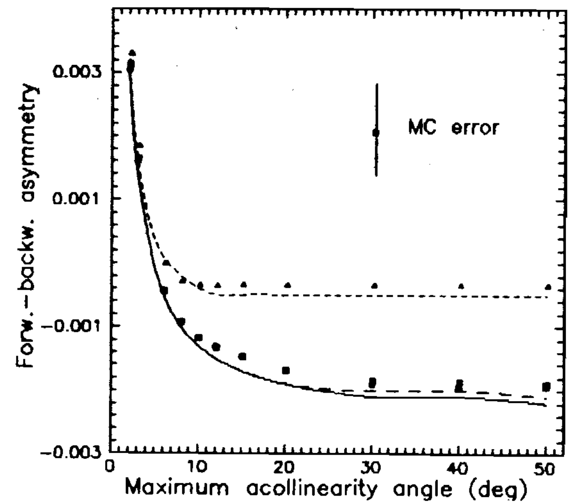

Fig.4 The same as in Fig.3 but for the integrated forward-backward asymmetry.

weakens. For $\bar{E} > \frac{1}{2} E_{beam}$ a big "plato" appears. It is seen from Fig.5 that if one includes the final-state corrections, the essential dependence of σ_T on the muon energy cuts appears. It is connected with a fact that a final photon emitted mostly at a small angle to the muon momentum decreases the energy of the muon but practically does not change its direction. So, final-state corrections make the dependence on the E-cut stronger but do not change the behavior of the ξ-cut dependence. On the other hand, a photon emitted from the initial state collinear to the beam strongly influences the acollinearity cut dependence.

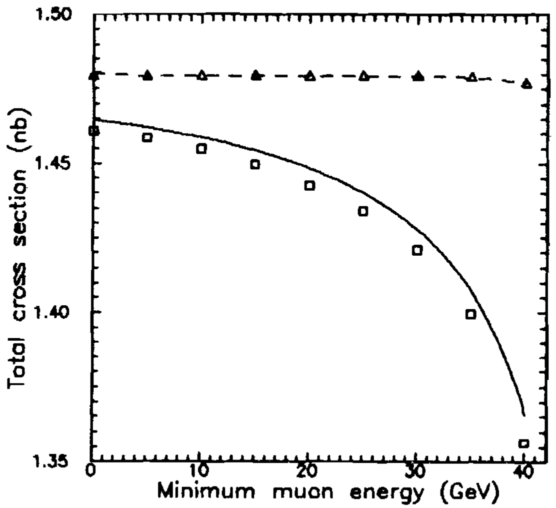

Fig.5 The total cross-section at Z°-pole as a function of the cut on muon energies for $\bar{\xi} = 15^{\circ}$: with exponentiation of the final state corrections (Eq. (20)) (solid line), no final state radiation (Eq. (8)) (dashed line). MC results are obtained for 10^6 events (errors are negligible).

In Figs.3-5 we present also some points obtained with the DYMU2 event generator [9] for the same cuts as applied in ZCUTCOS (MC results were obtained in collaboration with M.Lokajicek). It is

seen from the figures the discrepancy is not worse than 0.4% for σ_T and 0.05% for A_{FB} for reasonable values of the cuts. The disagreement for σ_T increases when the cuts become harder. The comparison of our results with MC ones reveals the same tendency in the case of the R-cut . On the other hand, in this case we have found an agreement better than 0.1% between ZCUTCOS results for σ_T and those obtained with the use of ZSHAPE analytical program [10]. We did not find a satisfactory agreement between our code and the COMPACT one [2] (the discrepancy is worse than 2% at Z°-pole for reasonable cuts). Details of the comparison will be published elsewhere.

We think, the angular distribution (20) which includes independent cuts on muon energies and on the acollinearity angle will be a suitable tool for fitting the high-luminosity data to be soon obtained at LEP and SLC colliders.

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КЭД радиационные поправки вблизи Z-полюса с учетом реалистических кинематических ограничений

Вычисляется аналитически угловое распределение e^+e^--аннигиляции в мюонную пару с учетом кинематических ограничений на угол аколинеарности импульсов мюонов и энергии мюонов. КЭД радиационные поправки первого порядка к начальному и конечному состоянию вычислены полностью. Для включения высших порядков используется экспоненцирование мягких фотонов.

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QED Corrections at Z^0-Pole with Realistic Kinematical Cuts

The angular distribution for e^+e^--annihilation into a muon pair with a cut on the acollinearity of muon momenta and cuts on muon energies is calculated analytically. First order QED corrections to initial and final states are treated completely. Soft-photon exponentiation is applied for higher orders effects.

The investigation has been performed at the Laboratory of Nuclear Problems and Laboratory of Theoretical Physics, JINR.

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