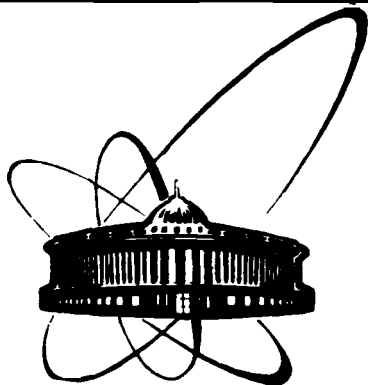


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ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
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ON THE NATURE OF PHASE TRANSITION  
IN A TWO DIMENSIONAL  $\phi^4$  THEORY

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## 1. INTRODUCTION

Many problems of modern particle physics rely on the spontaneous symmetry breaking as, for instance, the electroweak model with Higgs bosons (see, for example [1]), or the color confinement in QCD which can be explained by a vacuum instability<sup>2</sup>. There are many papers<sup>3-12</sup> devoted to investigation of the problem of the vacuum phase structure for a scalar field model with the Lagrangian

$$\mathcal{L}(\varphi) = -\frac{1}{2} \cdot [(\partial_\mu \varphi)^2 + m^2 \cdot \varphi^2] - \frac{g}{4} \cdot \varphi^4 \quad (1.1)$$

in  $\mathbb{R}^2$ . The theory is simple enough, and it is used widely for testing new ideas and methods in quantum field theory.

On the classical level for  $m^2 > 0$ , the theory (1.1) is stable and has a unique symmetric trivial ground state. On the other hand, it has been found<sup>3</sup> that high-order quantum corrections can give rise to the vacuum instability. A useful instrument for the investigation of the vacuum instability due to quantum effects is the method of the effective potential<sup>4</sup> which can be defined as

$$V(\varphi_0) = -\frac{1}{\Omega} \cdot \lim_{\Omega \rightarrow \infty} \ln( I_\Omega(\varphi_0) ),$$
$$I_\Omega(\varphi_0) = \int \delta\varphi \cdot \delta(\varphi_0 - \frac{1}{\Omega} \int_\Omega d^2x \cdot \varphi(x)) \cdot \exp( \int_\Omega d^2x \cdot \mathcal{L}(\varphi(x)) ), \quad (1.2)$$

where  $\Omega$  is a finite volume in  $\mathbb{R}^2$  and  $\varphi_0$  is a vacuum expectation value of the scalar field. A symmetry broken phase of a system is associated with the absolute minimum of the effective potential, for which  $\varphi_0 \neq 0$ . As the effective potential  $V(\varphi_0)$  is described by non-Gaussian functional integrals, one should use some

approximation schemes. These may be perturbative loop-expansion methods, variational approaches, or numerical calculations on a lattice.

The effective potential has been calculated<sup>3,5</sup> in the one-loop approximation which predicts a phase transition in the theory. Chang<sup>6</sup> has got the effective potential as a partial sum of "n-loop" diagrams only of the "cactus-type". This approximation method gives a first-order phase transition. Nonperturbative Gaussian approaches<sup>7</sup> also lead to similar results. On the other hand there exist mathematical theorems<sup>8</sup> proving that the second-order phase transition takes place in this model. There are papers<sup>9,10,11</sup> where variational methods were used for investigation of the vacuum stability problem and a correct behaviour of the vacuum energy in the critical region was obtained. The variational methods were applied to the Hamiltonian of the system under consideration but not to the functional integral (1.2) defining the effective potential.

In this paper, we obtain a variational estimation of the effective potential in (1.2) using the variational method introduced in [12]. We show that there exists a second-order phase transition in the  $\phi_2^4$  model and give estimation for the critical coupling constant. For this aim we consider the coefficient  $\alpha(G)$  in the expansion of the effective potential for small  $\phi_0$ :

$$V(\phi_0) = E(G) + \alpha(G) \cdot \phi_0^2 + O(\phi_0^4), \quad (1.3)$$

where  $G = g/2\pi m^2$  is a dimensionless coupling constant. We also obtain that the "all-loop" approximation of the effective potential gives only a first-order phase transition and cannot explain the second order transition in principle.

## 2. THE EFFECTIVE POTENTIAL AND ITS LOOP APPROXIMATION

We will consider the scalar field theory (1.1). The theory is supernormalizable in two-dimension. All ultraviolet divergences

in this model can be removed readily by using the quantum Lagrangian density

$$\mathcal{L}(\phi) = \frac{1}{2} \phi(x) \cdot [-\square - m^2] \cdot \phi(x) - \frac{g}{4} N_m \phi^4(x), \quad (2.1)$$

where  $N_m$  denotes the normal product of the fields  $\phi(x)$  with mass  $m$ ;  $g$  is the self-coupling constant.

We will investigate the effective potential  $V(\phi_0)$  defined by (1.2) and has the meaning of the vacuum energy density<sup>13,14</sup> in the vacuum state, the expectation value of the field over which is  $\phi_0$ . The functional integral in (1.2) is normalized in the following way

$$I_\Omega(\phi_0) = C_m \int \delta\phi \cdot \delta(\phi_0 - \frac{1}{\Omega} \int_\Omega d^2x \cdot \phi(x)) \cdot \exp\left\{ \int_\Omega d^2x \cdot \mathcal{L}(\phi(x)) \right\}, \quad (2.2)$$

where  $C_m = \det^{1/2}(-\square + m^2)$ . All integrations are performed here in the Euclidean space.

Let us perform some transformations of the functional integral  $I_\Omega(\phi_0)$  in (2.2). First, we can write

$$\phi(x) = \phi_0 + \phi(x), \quad (2.3)$$

where  $\phi_0$  is a constant field and  $\phi(x)$  satisfies the condition

$$\int_\Omega d^2x \cdot \phi(x) = 0. \quad (2.4)$$

We can substitute (2.3) into (2.2) and perform integration over  $d\phi_0$  taking into account the functional differential  $\delta\phi(x) = d\phi_0 \cdot \delta\phi(x)$ . Then we obtain

$$I_\Omega(\phi_0) = C_m \int \delta\phi(x) \cdot \exp\left\{ \int_\Omega d^2x \cdot \mathcal{L}(\phi_0 + \phi(x)) \right\}. \quad (2.5)$$

Second, we go to the normal product in the Lagrangian in (2.5) with respect to the field with a new mass  $\mu$  and require the coefficient of  $\phi^2(x)$  in the Lagrangian density to be equal to  $\mu^2$ . Then after some calculations we get

$$I_\Omega(\phi_0) = \exp[-\Omega \cdot V_{1\text{loop}}(\phi_0)] \cdot J_\Omega(\phi_0), \quad (2.6)$$

where

$$V_{1\text{loop}}(\phi_0) = \frac{m^2}{2} \phi_0^2 + \frac{g}{4} \phi_0^4 - \frac{m^2 + 3g\phi_0^2}{8\pi} \ln \frac{\mu^2}{m^2} + \frac{\mu^2 - m^2}{8\pi} + \frac{3g}{64\pi^2} \left[ \ln \frac{\mu^2}{m^2} \right]^2 \quad (2.7)$$

and

$$J_{\Omega}(\phi_0) = \exp[-\Omega \cdot V_{\text{sc}}(\phi_0)] = C_{\mu} \int \delta\phi \exp \left( \int_{\Omega} dx \cdot \left[ \frac{1}{2} \phi \cdot (\square - \mu^2) \cdot \phi - N_{\mu} \cdot (g\phi_0 \phi^3(x) + \frac{g}{4} \phi^4(x)) \right] \right) \quad (2.8)$$

Here the new mass  $\mu$  in (2.7) and (2.8) is defined by the equation

$$\mu^2 = m^2 + 3 \cdot g \cdot \phi_0^2 - \frac{3g}{4\pi} \ln \frac{\mu^2}{m^2} \quad (2.9)$$

Thus, the effective potential consists of two parts

$$V(\phi_0) = V_{1\text{loop}}(\phi_0) + V_{\text{sc}}(\phi_0) \quad (2.10)$$

The function  $V_{1\text{loop}}(\phi_0)$  in (2.7) will be called the "all-loop" potential because it corresponds to an infinite sum of all cactus-type loop diagrams shown in Figure 1a. We call the second function  $V_{\text{sc}}(\phi_0)$  the "strongly-connected" potential, its graphical representation is plotted in Figure 1b.

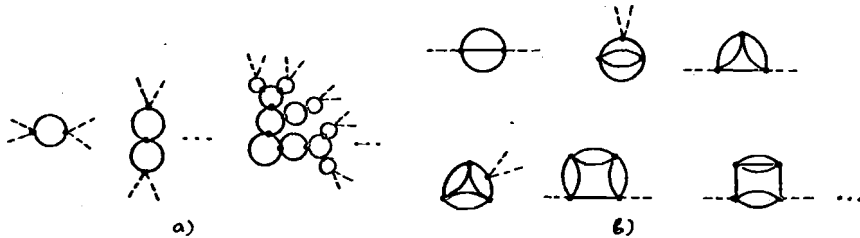


Figure 1. Graphical representations of the

- a) - "cactus-type" and
- b) - "strongly-connected" potentials.

At first let us consider the "all-loop" potential. It is convenient to rewrite (2.7) using the following notation:

$$\phi_0^2 = 4\pi \cdot \phi_0^2, \quad g = g/2\pi m^2 \quad \text{and} \quad \frac{\mu^2}{m^2} = 1 + \frac{3}{2} G \cdot \xi \quad (2.11)$$

The variables  $\phi_0$ ,  $G$  and  $\xi$  are dimensionless. Then substituting (2.11) into (2.7) we obtain

$$V_{1\text{loop}}(\phi_0) = \frac{m^2}{8\pi} \left( \phi_0^2 + \frac{1}{4} G \cdot \phi_0^4 - \frac{3}{2} G \cdot \phi_0^2 \cdot \ln(1 + \frac{3}{2} G \cdot \xi) + \frac{3}{2} G \cdot \xi - \ln(1 + \frac{3}{2} G \cdot \xi) + \frac{3}{4} G \cdot [\ln(1 + \frac{3}{2} G \cdot \xi)]^2 \right), \quad (2.12)$$

$$\xi + \ln(1 + \frac{3}{2} G \cdot \xi) = \phi_0^2.$$

The behaviour of  $V_{1\text{loop}}(\phi_0)$  is given in Figure 2, for different values of coupling the constant  $G$ .

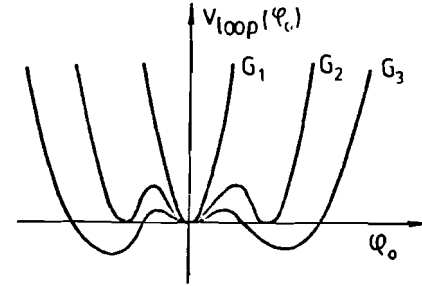


Figure 2. The behaviour of  $V_{1\text{loop}}(\phi_0)$  for different coupling strengths:  $G_1 < G_2^{\text{loop}} = G_c < G_3$ .

When  $\phi_0^2 \rightarrow 0$ , we have, for any  $G$ , the following asymptotic behaviour of the "all-loop" potential

$$V_{1\text{loop}}(\phi_0) = \frac{m^2}{8\pi} \left( \phi_0^2 + O(\phi_0^4) \right) \quad (2.13)$$

This means that the loop potential (2.12) or (2.7) is not able to describe the second order phase transition. The critical coupling constant  $G_c^{\text{loop}}$ , at which the first order phase transition takes place, is

$$G_c^{\text{loop}} = 0.83 \dots \quad (2.14)$$

### 3. VARIATIONAL ESTIMATIONS OF THE "STRONGLY-CONNECTED" POTENTIAL

In the previous section we have seen that the loop approach for the effective potential is responsible for the first-order phase transition in this model although the mathematical theorems<sup>8</sup> give the second-order one. In the method of the effective potential this means that we have to take into account the second part, the "strongly-connected" potential  $V_{sc}(\varphi_0)$  defined by (2.8). The question is which kind of phase transition takes place then. As stressed in Introduction, it is enough to investigate the effective potential at small values of  $\varphi_0$  (i.e.  $\varphi_0^2 \ll 1$ ) in order to answer this question. We will investigate the coefficient  $\alpha(G)$  in the representation of the effective potential (1.1). Our aim is to show that  $\alpha(G)$  is positive at small  $G$  and negative as  $G \rightarrow \infty$ . As the functional integral (2.8) is non-Gaussian, its explicit computation is impossible at present. We will use the variational method suggested in [12].

Let us rewrite (2.8) in the form, which is correct for small  $\varphi_0^2$

$$J_{\Omega}(\varphi_0) = C_{\mu} \int \delta\phi \cdot \exp\left(\int_{\Omega} dx \cdot \left[\frac{1}{2} \phi(x) \cdot (\sigma - \mu^2) \cdot \phi(x) - \frac{g}{4} N_{\mu} \cdot \phi^4(x)\right] + \frac{1}{2} g^2 \varphi_0^2 \left[\int_{\Omega} dx \cdot N_{\mu} \cdot \phi^3(x)\right]^2\right) = \exp[-\Omega \cdot V_{sc}(\varphi_0)]. \quad (3.1)$$

This representation can be obtained easily due to validity of the following transformation in (2.8)

$$\exp(-\varphi_0 W) = \text{ch}(\varphi_0 W) \approx \exp\left(\frac{1}{2} \varphi_0^2 W^2 + O(\varphi_0^4)\right)$$

for infinitesimal  $\varphi_0$  and finite functional  $W$ . Then applying to the integral (3.1) the variational techniques<sup>12</sup>, one can get

$$V_{sc}(\varphi_0) = -\frac{1}{\Omega} \lim_{\Omega \rightarrow \infty} \ln(J_{\Omega}(\varphi_0)) \approx V_{sc}^*(\varphi_0),$$

$$V_{sc}^*(\varphi_0) = \min_{q, \Lambda} \left\{ \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \left[ \ln(1+q(k^2)) - \frac{q(k^2)}{1+q(k^2)} \right] + \right.$$

$$+ \frac{\mu^2 \Lambda^2}{2} + \frac{g}{4} \left[ \Lambda^4 - 6 \Lambda^2 \Delta_q + 3 \Delta_q^2 \right] - 3g^2 \varphi_0^2 \int_{\Omega} \int_{\Omega} dx dy \left[ D_q^3(x-y) + 3\Lambda^2 D_q^2(x-y) + \frac{3}{2} \Lambda^4 D_q(x-y) \right], \quad (3.2)$$

where

$$\Delta_q = \int \frac{d^2 k}{(2\pi)^2} \frac{q(k^2)}{1+q(k^2)} \tilde{D}(k^2)$$

$$D_q(x-y) = \int \frac{d^2 k}{(2\pi)^2} \frac{e^{ik(x-y)}}{1+q(k^2)} \tilde{D}(k^2)$$

$$\tilde{D}(k^2) = \frac{1}{\mu^2 + k^2}. \quad (3.3)$$

Here  $\mu^2$  is defined by equations (2.11) and (2.12). The constant  $\Lambda$  and function  $q(k^2)$  are variational parameters (see Appendix A). The optimal form of the function  $q(k^2)$  is

$$q(k^2) = f \cdot \mu^2 \cdot \tilde{D}(k^2) \quad (3.4)$$

as it follows from the variational equation. Here  $f$  is a variational parameter.

For  $\varphi_0^2 \ll 1$  the equation (2.9) gives

$$\mu^2 = m^2 \left[ 1 + \frac{3G}{2+3G} 4\pi\varphi_0^2 + O(\varphi_0^4) \right]. \quad (3.5)$$

All integrals in (3.2) and (3.3) for the function  $q(k^2)$  are calculated explicitly and the upper bound of the "strongly-connected" potential can be written for  $\varphi_0^2 \ll 1$  in notation (2.11)

$$V_{sc}^*(\varphi_0^2) = \frac{m^2}{8\pi} \left( E_{sc}^*(G) + \alpha_{sc}^*(G) \cdot \varphi_0^2 + O(\varphi_0^4) \right), \quad (3.6)$$

where

$$E_{sc}^*(G) = \min_{f, B} \left\{ f - \ln(1+f) + B^2 + \frac{G}{4} \cdot \left\{ (B^2 - 3 \ln(1+f))^2 - 6 \ln^2(1+f) \right\} \right\}, \quad (3.7)$$

$$\alpha_{sc}^*(G) = \frac{3G}{2+3G} \left( f(G) - \ln(1+f(G)) + B^2(G) \right) - \frac{3G^2}{2(1+f(G))} \left[ Q + 3B^2(G) + \frac{3B^4(G)}{2} \right], \quad (3.8)$$

$$Q = \int_0^1 \int_0^1 \int_0^1 d\alpha d\beta d\gamma \delta(1-\alpha-\beta-\gamma) \cdot [\alpha\beta + \alpha\gamma + \beta\gamma]^{-1} = 2.3435\dots$$

The functions  $f(G)$  and  $B(G)$  define the minimum  $E_{sc}^+(G)$  in (3.7). They satisfy the following equations

$$\begin{aligned} f - \frac{3}{2} G \cdot [B^2 - \ln(1+f)] &= 0, \\ 1 + \frac{1}{2} G \cdot [B^2 - 3 \cdot \ln(1+f)] &= 0. \end{aligned} \quad (3.9)$$

Equations (3.9) have nontrivial real solutions for

$$G > G_0 = \min \frac{f+3}{f} \frac{1}{3 \ln(1+f)} = 1.43 \dots$$

When  $G < G_0$ , the solutions are trivial:  $f=B=0$ .

Finally, for the effective potential we have the upper estimation

$$V(\varphi_0) \leq V^+(\varphi_0) = V_{loop}(\varphi_0) + V_{sc}^+(\varphi_0). \quad (3.10)$$

The behaviour of  $V^+(\varphi_0)$  is shown in Figure 3. Substituting (2.13) and (3.6) into (3.10) one gets finally for  $\varphi_0^2 \ll 1$  (i.e.  $\varphi_0^2 \ll 1$ ):

$$\begin{aligned} V^+(\varphi_0^2) &= \frac{m^2}{8\pi} \{ E_{sc}^+(G) + \alpha^+(G) \cdot \varphi_0^2 + O(\varphi_0^4) \}, \\ \alpha^+(G) &= 1 + \alpha_{sc}^+(G). \end{aligned} \quad (3.11)$$

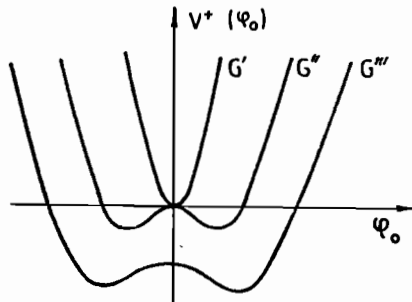


Figure 3. The upper bound of the effective potential for different couplings. Here  $G' < G_c^* < G'' < 1.63 < G'''$ .

Our estimation are undoubtedly true at large couplings  $G \rightarrow \infty$ . In this limit we have

$$V^+(\varphi_0^2) \rightarrow \frac{m^2}{8\pi} \left( -\frac{3}{2} G \cdot \ln^2 G - \frac{15}{4} G \cdot \ln G \varphi_0^2 + O(\varphi_0^4) \right), \quad (3.12)$$

i.e.  $\alpha^+(G) = -\frac{15}{4} G \cdot \ln G < 0$ . It means that in the region of strong coupling the second-order phase transition takes place in the  $\varphi_2^4$  field model due to the contributions of the "strongly-connected" part of the effective potential.

The numerical value of the critical coupling constant can be found from our formulae. In critical region

$$f(G) = 0, \quad B(G) = 0$$

and

$$\alpha^+(G) = 1 - \frac{3}{2} G^2 Q$$

becomes negative for

$$G > G_c^* = \left[ \frac{2}{3Q} \right]^{1/2} = 0.53 \dots \quad (3.13)$$

or

$$\left( \frac{Q}{m} \right)_c^* = 2\pi G_c^* = 3.35 \dots$$

We believe that the true critical coupling  $G_c$  lies not far from  $G_c^*$ , although

$$G_c \leq G_c^* \quad (3.14)$$

in any case. Besides we want to pay attention to that

$$G_c^* < G_c^{loop}.$$

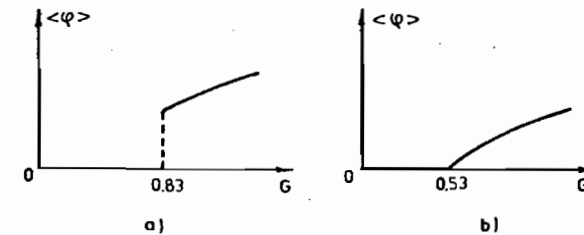


Figure 4. Field expectation values corresponding to the a)  $-V_{loop}$  and b)  $-V^+$  potentials.

This means that the second-order phase transition occurs earlier than the first order one. Figure 4 exhibits the field expectation values corresponding to the  $V_{loop}(\varphi_0)$  and  $V_{sc}^*(\varphi_0)$  potentials as functions of the coupling strength  $G$ .

#### CONCLUSION

In this paper, we have investigated the problem of phase transition in a two-dimensional  $\varphi^4$  quantum field theory. The functional integral describing the effective potential is estimated by a variational approximation. We have obtained expressions for the "cactus-type" expansion and upper bound of the effective potential. We show that the loop potential  $V_{loop}$  describes only the first-order phase transition and is not able to explain the second order one on principle. By contrast the "strongly-connected" part of the effective potential gives contributions leading to the second-order phase transition at a large coupling constant. Thus, in the theory under consideration the symmetry  $\varphi \rightarrow -\varphi$  turns out to be spontaneously broken through the second-order phase transition.

#### ACKNOWLEDGEMENTS

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#### APPENDIX A

Here we formulate our variational techniques (for details, see [15]), i.e. show how to obtain (3.2) and (3.3) from (3.1). We work in a Euclidean space volume  $\Omega \rightarrow \mathbb{R}^2$ . Let integral (3.1) be given

$$J_{\Omega}(g) = \int d\sigma_{\Phi} \cdot \exp\left(-g \int_{\Omega} dx \cdot U(\Phi)\right) \quad (\text{A.1})$$

$$d\sigma_{\Phi} = C_{\mu} \cdot \delta\Phi \cdot \exp\left(-\frac{1}{2} \int_{\Omega} dx \cdot \Phi \cdot (-\square + \mu^2) \cdot \Phi\right), \quad (\text{A.2})$$

where  $U(\Phi)$  is a real functional,  $C_{\mu} = \det^{1/2}(-\square + \mu^2)$  and  $\Phi(x)$  satisfies (2.4).

Let us diagonalize the quadratic form in (A.2) by introducing the functional variables  $\phi(x)$ :

$$\Phi(x) = (-\square + \mu^2)^{-1/2} \phi(x) = \int_{\Omega} dy \Delta(x, y) \cdot \phi(y) = (\Delta, \phi)(x), \quad (\text{A.3})$$

where

$$\Delta(x, y) = \int \frac{d^2k}{(2\pi)^2} (k^2 + \mu^2)^{-1/2} \cdot \exp[-ik(x-y)] \quad ,$$

$$\int_{\Omega} dy \cdot \phi(y) = 0 \quad . \quad (\text{A.4})$$

Then (A.1) can be rewritten

$$J_{\Omega}(g) = \int d\sigma_{\phi} \cdot \exp\left(-g \int_{\Omega} dx \cdot U[(\Delta, \phi)(x)]\right) \quad ,$$

$$d\sigma_{\phi} = C \cdot \delta\phi \cdot \exp\left\{-\frac{1}{2} \int_{\Omega} dx \cdot \phi^2(x)\right\} \quad (\text{A.5})$$

$C$  obeys the condition  $\int d\sigma_{\phi} = 1$ .

Now we proceed to the variational estimation of integral (A.5). Let us introduce new variables  $v(x)$  and  $A(x)$

$$\phi(x) = (1+q(\square))^{-1/2} \cdot v(x) + (-\square + \mu^2)^{1/2} \cdot A(x), \quad (\text{A.6})$$

where the variational function  $q(k)$  satisfies the condition

$$\int d^2k \cdot q^2(k^2) < \infty \quad . \quad (\text{A.7})$$

Substituting (A.6) into (A.5) we have the equivalent form of Eq. (A.1):

$$J_{\Omega}(g) = \prod_q (1+q(\Delta))^{-1/2} \cdot \int d\sigma_v \exp\left\{\frac{1}{2} \int_{\Omega} dx v(x) q(\square) [1+q(\square)]^{-1} v(x)\right. \\ \left. - \frac{1}{2} \int_{\Omega} dx [2A(x)(-\square + \mu^2)(\Delta_q, v)(x) + A(x)(-\square + \mu^2)A(x)]\right. \\ \left. - g \int_{\Omega} dx \cdot U[(\Delta_q, \phi)(x) + A(x)]\right\}, \quad (\text{A.8})$$

where

$$\Delta_q(x) = \int \frac{d^2k}{(2\pi)^2} [(k^2 + \mu^2)(1+q^2)]^{-1/2} \cdot \exp(-ikx) \quad . \quad (\text{A.9})$$

Now we choose the function  $\lambda(x)$  in the form:

$$\int_{\Omega} dx \cdot \lambda(x) = 0, \quad \lambda^2(x) = \lambda^2, \quad (\text{A.10})$$

where  $\lambda$  is an arbitrary number. Let us use the inequality:

$$\int d\sigma \cdot \exp(-W) \geq \exp(-\int d\sigma W),$$

which is valid for any positive definite measures  $d\sigma$  and any real functionals  $W$ . Then taking into account (3.1) one can obtain as

$$V_{\text{eff}}(\varphi_0) \approx \min_{q, \lambda} \frac{1}{\Omega} (L(q) + \frac{\mu^2 \lambda^2}{2} \Omega + g \int d\sigma_v \int_{\Omega} dx \cdot U[(\Delta_q, v)(x) + \lambda(x)])$$

$$L(q) = \frac{\Omega}{2} \int \frac{d^2 k}{(2\pi)^2} [\ln(1+q(k^2)) - \frac{q(k^2)}{1+q(k^2)}]. \quad (\text{A.11})$$

After integration over  $\int d\sigma_v$  we obtain (3.2).

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О фазовом переходе в двумерной  $\phi^4$  теории

Изучается устойчивость вакуума в двумерной скалярной  $\phi^4$  теории. Получена вариационная оценка функционального интеграла, определяющего эффективный потенциал в данной модели. Имеет место фазовый переход второго рода при сильной связи, что находится в полном согласии с теоремой Саймона - Гриффитса. Показано, что петлевое приближение ведет к неправильному поведению эффективного потенциала в критической области.

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On the Nature of Phase Transition  
in a Two Dimensional  $\phi^4$  Theory

The vacuum stability of a scalar  $\phi^4$  theory in two dimensions is studied. A variational approach is applied to estimate a functional integral defining the effective potential in this model. We find that the second-order phase transition takes place in the theory under consideration. This is in complete agreement with the Simon - Griffiths theorem. We show that the loop approximation leads to a wrong critical behaviour of the effective potential.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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