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THERMODYNAMICS OF A PLASMA WITH CONFINEMENT INTERACTIONS

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1. Introduction

One of the main tasks of Quantum Chromodynamics (QCD) is the description of strongly interacting many-particle systems (quarks and gluons). Of special interest are conclusions for the possible occurrence of phase transitions. But even these nonperturbative properties of QCD-matter are hardly to obtain from first principles so one is forced to use computer simulations /1/ or effective models such as the widely used bag model. However, despite of its applicability and the methodic efforts due to analytic tractability, this model fails if one is looking at the details of the phase transition. Especially, the recently observed pressure effect /2/ in lattice QCD calculations contradicts the bag model behaviour. One alternative is the use of quark confinement potentials, such as the Cornell potential

$$V(r) = -\frac{\alpha}{r} + \sigma r + c \tag{1}$$

for the interaction among the massive quark constituents of the plasma. The parameters in (1) can be adjusted to describe the low energy hadron spectroscopy. However, a straighforward use of confinement potentials, such as (1), in order to obtain thermodynamical properties of the QCD-paslma phase meets several problems in evaluating the partition function

$$Z = \underset{\substack{\text{all physical} \\ \text{states}}}{Tr} \left\{ e^{-\beta(H_0 + V - \mu N)} \right\}. \tag{2}$$

All unphysical states (Color van der Waals forces, etc.) have to be removed from the theory. In order to avoid divergencies one has to apply suitable renormalisation procedures (similar to the case of screening in Coulomb plasmas). One possible scheme which has already been successfully applied to a quark matter system /3/ is the saturation of the confinement interaction (1) within the sphere of the nearest neighbor. This is similar to strong coupling problems in solid state physics as, e.g., the electron-hole recombination /4/, where an effective potential is obtained from the naked one by weighting each distance r with the probability of finding the nearest neighbor there, i.e.

$$V^{eff}(r) = V(r)\; C(r), \qquad (3)$$

where

$$C(r) = \Omega_d\, d\, n\, r^{d-1}\, e^{-\Omega_d n r^d},$$

$$\Omega_d = \pi^{d/2}\, \Gamma(1+\tfrac{d}{2})^{-1}, \qquad (4)$$

is the nearest neighbor distribution function in d dimensions for uncorrelated particles.

After these preliminary remarks on the use of confinement type interactions (1) in a plasma, we will give in Sect. 2 a consistent set of thermodynamical relations for a plasma of quasi-particles obeying a self-energy shift in Hartree approximation

$$\Delta^H(n) = \frac{1}{2}\, n \int_{vol} d^d r\; V^{eff}(r). \qquad (5)$$

An application of the resulting thermodynamics, which is expanded up to first order in the shift (5), to quark matter is given in Sect. 3, where the problem of the quark-hadron phase transition is investigated and strangeness effects on the critical temperature are discussed. Sect. 4 contains the concluding discussion with prospects for the further exploitation of the model approach given here.

2. Equation of state

The equation of state for a system of particles obeying a density-dependent one-particle energy shift as, e.g., the Hartrees shift (5) for the confinement plasma under consideration has recently been investigated by Zimanyi et al. /5/ for the case of nuclear matter in the Walecka model approach and was afterwards applied by several authors, see e.g. /6/. Especially, the consistency of thermodynamic quantities has been regarded. We do not want to repeat these calculations here for the Hartree shift (5) but instead, we want to give an expansion for small shifts, which also leads to a consistent set of thermodynamical relations.

We give the Hamiltonian in an occupation number representation

$$H = \sum_f \sum_r n_r^f \,(\varepsilon_r^f + \Delta_f^H), \qquad (6)$$

where over the quantum numbers r and the flavor degrees of freedom f has to be summed.

We examine the thermodynamics in the region

$$\Delta_f^H / T \ll 1, \qquad (7)$$

so that the partition function may be expanded into a Taylor series with respect to this small quantity

$$p(T) = T\,\Omega^{-1} \ln Z$$

$$= \sum_f \left\{ \frac{g_f}{6\pi^2} \int dp\, \frac{p^4}{E_f(p)} \left[1 + \exp\{(E_f(p) + \Delta_f^H)/T\}\right]^{-1} \right\} \qquad (8)$$

with $\quad E_f(p) = (p^2 + m_f^2)^{1/2}$. $\qquad (9)$

Utilizing our approximation, we get

$$p(T) = \sum_f \left\{ p_f^{id}(T) + \left.\frac{\partial p}{\partial \Delta_f^H}\right|_{\Delta_f^H = 0} \cdot \Delta_f^H + \dots \right\}$$

$$\approx \sum_f \left\{ p_f^{id}(T) - n_f^{id}\, \Delta_f^H \right\}. \qquad (10)$$

From the pressure as thermodynamical potential, the other thermodynamical functions can be derived, e.g. the entropy

$$S(T) \approx \sum_f \left\{ s_f^{id}(T) - \frac{\Delta_f^H g_f}{2\pi^2 T} * \int dp\, p^2\, \frac{E_f(p)\exp\{E_f(p)/T\}}{[1+\exp\{E_f(p)/T\}]^2} \right\} \qquad (11)$$

and the energy density

$$\varepsilon(T) \approx \sum_f \left\{ \varepsilon_f^{id}(T) - \frac{\Delta_f^H g_f}{2\pi^2} \int dp\, p^2 \left[\frac{E_f(p)\exp\{E_f(p)/T\}}{T[1+\exp\{E_f(p)/T\}]^2} - \frac{1}{1+\exp\{E_f(p)/T\}} \right] \right\}. \qquad (12)$$

The desired thermodynamic consistency of eqs. (10), (11), and (12) is demonstrated by simple substitution in the Euler relation

$$\varepsilon(T) = T s(T) - p(T). \tag{13}$$

3. Application to quark matter

The thermodynamics which is defined by Eq.(10) was previously used for investigations of quark matter systems within the string-flip model, see,e.g. /7/, where especially a comparison was performed with parametrisations of the bag mode. Moreover, a stable massive quark matter phase was obtained only within a very narrow range of bag parameters at μ =0. This is in accordance with lattice calculations, where one observes that the chiral and the decomfinement phase transition quark matter nearly coincide.

This may serve as an justification for the use of massless light quark flavors in the evaluation of the Hartree shift (5) with the Cornell potential (1) in d=3 dimensions, where we have

$$n_q^{id} = \frac{3\zeta(3)}{4\pi^2} g_q T^3, \tag{14}$$

and thus

$$\Delta_q^H(T) = -\left[\frac{\zeta(3)}{3\pi} g_q\right]^{1/3} \Gamma\left(\tfrac{2}{3}\right)\alpha T + \left[\frac{\zeta(3)}{3\pi} g_q\right]^{-1/3} \Gamma\left(\tfrac{4}{3}\right)\sigma T^{-1} + c. \tag{15}$$

With

$$p_q^{id}(T) = \frac{7}{8}\frac{\pi^2}{90} g_q T^4 \tag{16}$$

and eq. (10) we have for the pressure of the interacting quark matter phase

$$p_Q = \frac{7\pi^2}{360} g_q T^4 + \frac{9}{2\pi}\left[\frac{\zeta(3)}{3\pi} g_q\right]^{4/3} \Gamma\left(\tfrac{2}{3}\right)\alpha T^4 - \frac{9}{2\pi}\left[\frac{\zeta(3)}{3\pi} g_q\right]^{2/3} \Gamma\left(\tfrac{4}{3}\right)\sigma T^2 - \frac{3\zeta(3)}{4\pi^2} g_q c T^3. \tag{17}$$

Considering phase equilibrium with non-interacting massless pions

$$p_H \equiv p_\pi = \frac{\pi^2}{90} g_\pi T^4 \tag{18}$$

by requiring $\quad p_H(T_c) = p_Q(T_c), \quad$ (19)

the critical temperature for the quark-hadron transition is obtained

$$T_c = 166 \ \mathrm{MeV}, \tag{20}$$

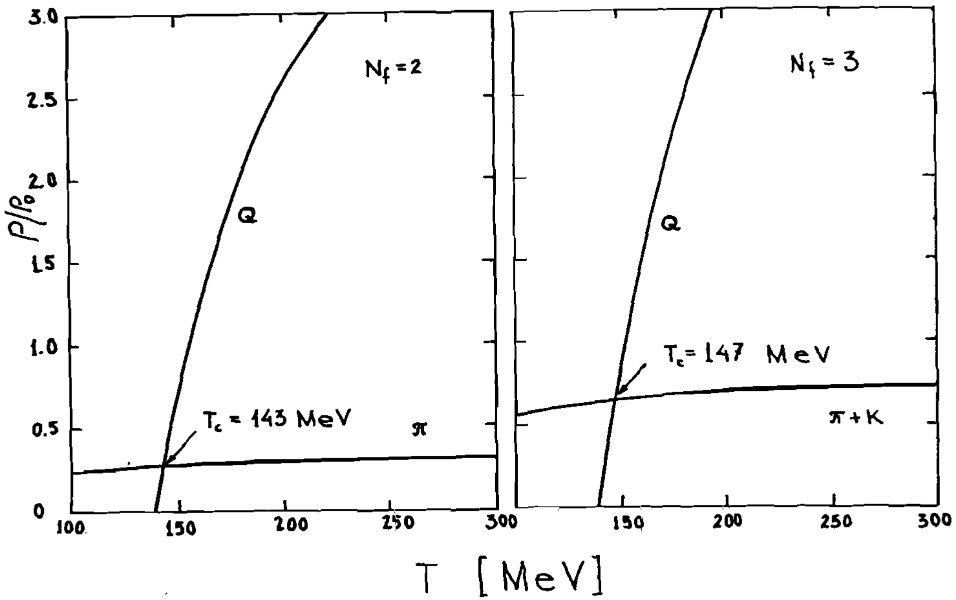
which accords very well with previous estimates within bag model approaches. For the parameter of the Cornell potential, the data are taken from Ref. 8 (α = .52, σ = .19 GeV^2, c= -.568 GeV). A calculation with massive quarks gives T_c= 177 MeV, so that the use of massless quarks can be viewed as a good approximation if one is looking at the critical temperature. Notice, that the critical parameters of the thermodynamic behaviour as, e.g., the critical temperature are fully determined by the microscopic parameters of the potential which can be adjusted in order to reproduce the hadron mass spectrum.

Furthermore, for the advocated parameter set /8/, the shift (15) becomes zero at T_0 = 200 MeV, so that the condition (8) for the applicability of the presented linear-shift-thermodynamics is fulfilled just in the region of the phase transition where the approach is applied.

For the discussion of the influence of strangeness on the critical temperature T_c at μ = 0, we consider kaons as strange hadrons in Maxwell-Boltzmann approximation ($m_K \approx$ 400 MeV)

$$p_K(T) = (2 m_K T/\pi)^{3/2}\, T\, e^{-m_K/T} \tag{21}$$

and quarks with the masses m_u=m_d=330 MeV, m_s= 600 MeV according to the parametrisation by Badalyan /9/ (α = .52, σ= .183 GeV^2, c= .94). As can be seen from the Figure, the effects of including strangeness compensate nearly and other, and the difference in T_c is negligible.



The pressure (in units of $p_0 = (\pi^2/45)\,T^4$) as a function of the temperature. According to the Gibbs condition for phase equilibrium the critical temperature T_c is determined by the crossing of the quark branch (Q) and the hadron branch (π, $\pi + K$). T_c depends only weakly on the flavour number N_f; left-hand side: only u,d quarks and pions; right-hand side: u,d,s quarks and pions and kaons. For details consult the text.

4. Future Prospects

The most interesting point for future investigations within the present model is the extension of the description to finite baryochemical potential $\mu \neq 0$. This issue is of interest for estimating the density contrast which might originate from the big bang hadronisation transition independently from the bag model calculations /10-12/ within a potential model. This is worth, because differently choosen bag models lead to some arbitrariness in the results. Within the suggested potential model approach this is possibly circumvented.

Nevertheless, the nearest neighbor distribution function (4) has to be improved by the consideration of statistical correlations at high densities (e.g., the formation of a Fermi-hole /3/) and of bound states at lower densities; see also /4/ for the availability of the nearest neighbor approach. Finally, the string picture of quark-quark interaction should break down somewhere above T_c, and free gluons have to be included in the thermodynamic description.

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