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С 83

E2-89-713

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πa_1- TRANSITIONS AND THE LOW-ENERGY LIMIT IN THE LINEAR σ-MODEL WITH VECTOR AND AXIAL-VECTOR MESONS

Submitted to "Physics Letters B"

1989

At present there are serious reasons to believe that the effective Lagrangian for QCD in the medium-energy region is the generalised Nambu - Jona-Lasinio Lagrangian with four-quark interactions of the scalar, pseudoscalar, vector and axial-vector type [1-4]. The dynamic violation of the chiral symmetry allows one to understand how the light current quarks that are present in the QCD Lagrangian acquire large masses and turn into heavy constituent quarks of which mesons are formed [5]. Besides, on the basis of the Lagrangian with the four-fermion interactions one can obtain the chiral Lagrangians which were good in description of low energy meson physics. In the sector of scalar and pseudoscalar mesons there appears a linear σ-model, in the sector of vector and axial-vector mesons there appears a model of the Yang-Mills type [5,6].

Naturally, the involvement of vector and axial-vector mesons gives rise to the questions about the correct low-energy behaviour of this extended theory, since the σ-model alone is enough to reproduce the amplitudes describing pion physics in complete agreement with the requirements of the current algebra theorems. For example, it is well known that the direct consideration of the diagram with the intermediate ρ-meson in calculations of the $\pi\pi$-scattering can lead to the doubling of the result obtained within the standard σ-model. An elegant solution of this problem, based on tha use of the non-linear chiral Lagrangian, was proposed by S. Weinberg [7]. For the case of the linear realisation of the chiral symmetry we know an inconvincing method [8] with the introduction of a formfactor in the $\rho\pi\pi$ vertex, this formfactor tending to zero at low energies.

In this paper we propose another solution of this problem. It is based on consideration of the $a\to\partial\pi$ transitions in the corresponding meson vertices. The existence of these transitions was pointed out in paper [9]. To make our considerations definite, we shall use the quark model of the superconducting type [5] in the calculations.

The phenomenological meson Lagrangian of the model can be obtained by means of the effective quark-meson Lagrangian

$$L = \bar{q}[i\hat{\partial}-M+g'(\bar{\sigma}+i\gamma_5\bar{\phi}) + \frac{g_\rho}{2}(\gamma_\mu\bar{v}^\mu+\gamma_5\gamma_\mu\bar{a}^\mu)]q \ , \qquad (1)$$

if one uses the one-loop approximation with diverging quark diagrams cut at the upper limit. The cut-off parameter Λ characterises the energy region where the spontaneous breaking of chiral symmetry takes place, quark condensate is produced and light current quarks turn into heavy constituent quarks. In formula (1) $q=(u,d,s)$ are the colour quark fields; σ_α, ϕ_α, v^μ_α, a^μ_α are the fields of scalar, pseudoscalar, vector and axial-vector mesons; $\bar{a}=a_\alpha\lambda_\alpha$, where λ_α are the Gell-Mann matrices ($0\le\alpha\le8$, $\lambda_o=\sqrt{2/3}$), $M=\mathrm{diag}(m_u,m_d,m_s)$ is the mass matrix of the constituent quarks. Below only u and d quarks will occur; we shall consider that $m_u=m_d=m$.

Between the constants g_ρ and g' there is the relation $g_\rho=\sqrt{6}g'$, and they both are expressed through the logarithmically diverging integral I_2:

$$g'=1/\sqrt{4I_2} \ , \qquad I_2 = -3i\int\frac{d^4q}{(2\pi)^4}\frac{\theta(\Lambda^2-q^2)}{(q^2-m^2+i\varepsilon)^2} \ .$$

The constant g_ρ characterises the $\rho\to\pi\pi$ decay and is equal to $g_\rho^2/4\pi=\alpha_\rho\cong3$.

Using (1) one can easily obtain the phenomenological Lagrangian describing the interactions of the four above-mentioned meson nonets, and the corresponding mass formulae. As the details can be found in Ref.[5,6], we give its immediate form:

$$L = -\frac{g'^2}{4}\,\mathrm{tr}\left\{ \left[(\bar\sigma - M/g')^2 + \bar\phi^2\right]^2 - [(\bar\sigma - M/g'),\ \bar\phi\ ]_-^2 \right\}$$
$$-\frac{1}{8}\,\mathrm{tr}(G_V^{\mu\nu}G^V_{\mu\nu} + G_a^{\mu\nu}G^a_{\mu\nu}) + \frac{1}{4}\,\mathrm{tr}\left\{\left[\mathcal{D}_\mu(\bar\sigma - M/g') + \frac{g_\rho}{2}[\bar a_\mu, \bar\phi]_+\right]^2\right.$$
$$\left. + \left[\mathcal{D}_\mu\bar\phi - \frac{g_\rho}{2}\,[\bar a_\mu, (\bar\sigma - M/g')]_+\right]^2\right\} + \Delta L\ . \tag{2}$$

Here
$$G^V_{\mu\nu} = \partial_\mu\bar v_\nu - \partial_\nu\bar v_\mu - \frac{i}{2}\,g_\rho(\ [\bar v_\mu, \bar v_\nu]_- + [\bar a_\mu, \bar a_\nu]_-\ ),$$
$$G^a_{\mu\nu} = \partial_\mu\bar a_\nu - \partial_\nu\bar a_\mu - \frac{i}{2}\,g_\rho(\ [\bar a_\mu, \bar v_\nu]_- + [\bar v_\mu, \bar a_\nu]_-\ ),$$
$$\mathcal{D}_\mu\bar\sigma = \partial_\mu\bar\sigma - \frac{i}{2}\,g_\rho[\bar v_\mu, \bar\sigma\ ]_-\ .$$

In ΔL there are finite parts of the quark loops which will not be considered in this paper. Besides, below we shall only deal with a specific case of formula (2) corresponding to the SU(2) symmetry of strong interactions.

The quadratic part of Lagrangian (2) contains non-diagonal transitions $a \to \partial\pi$. We cancell this term by the standard method of the quadratic form diagonalisation with a shift $\vec a_\mu \to \vec a_\mu - \varkappa\partial_\mu\vec\pi$, where $\varkappa = \sqrt{6}m/m^2_{a_1}$. In this case the pseudoscalar field π undergoes an additional renormalisation $\pi \to Z^{1/2}\pi$, where (see [10])

$$Z = (1 - 6m^2/m^2_{a_1})^{-1}. \tag{3}$$

The constant g', describing the interaction of pseudoscalar mesons with quarks, changes over into $g = Z^{1/2}g'$. It is evident that in the case of scalar mesons this renormalisation does not take place and we shall use the coupling constant g' for them.

The Goldberger-Treiman relation $g = m/F_\pi$ ($F_\pi = 93$ MeV) and the formula

$$Z^{1/2}g_\rho = \sqrt{6}g\ , \tag{4}$$

allow a relation between the constituent u-quark and a_1-meson masses:

$$m^2 = \frac{1}{12}\left[\ 1 - \sqrt{\ 1 - (2g_\rho F_\pi\ /m_{a_1}\ )^2}\ \right] m^2_{a_1}\ . \tag{5}$$

Using (5) one can express the constant Z only through the observable quantities

$$Z^{-1} = \frac{1}{2}\left[\ 1 + \sqrt{\ 1 - (2g_\rho F_\pi\ /m_{a_1}\ )^2}\ \right]\ . \tag{6}$$

Besides, it is seen from these formulae that the a_1-meson mass has the lower limit

$$m_{a_1} \geq 2g_\rho F_\pi \cong 1140\ \text{MeV}. \tag{7}$$

This value corresponds to the mass m=330 MeV. If one assumes that the mass formula $m^2_{a_1} = m^2_\rho + 6m^2$, obtained in this model, is exactly fulfilled, one can easily see that the equality $m_{a_1} = 2g_\rho F_\pi$ in (7) leads to the Weinberg sum rule $m^2_{a_1} = 2m^2_\rho$ [11] and can be transformed into $m^2_\rho = 2g^2_\rho F^2_\pi$, known as the KSFR-relation [12].

Let us go over to the consideration of the low-energy limit for the pion-pion scattering amplitude.The corresponding diagrams are shown in Fig 1. As distinct from the standard approach, we shall take into account the $a \to \partial\pi$ transition effects in the given

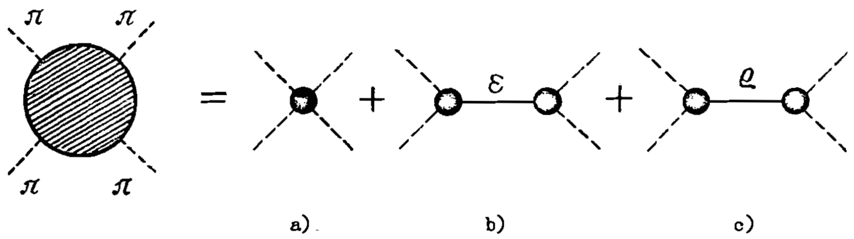

FIG 1. Born diagrams for the $\pi\pi$-scattering amplitude. The bold-faced point indicates that the $a\to\partial\pi$ transitions are taken into account in the meson vertices.

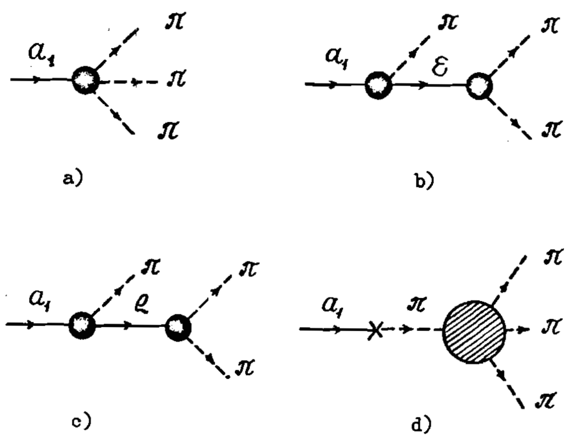

FIG 2. Born diagrams describing the $\tau\to\nu_\tau 3\pi$ decay. Like in Fig. 1, the bold-faced point indicates that the $a\to\partial\pi$ transitions are taken into account in the corresponding vertices. The four-pion vertex entering into 2d is shown in Fig.1.

vertices. Diagrams 1a and 1b are known from the linear σ-model; they correctly describe the s-wave lengths of $\pi\pi$-scattering if there are no $a\to\partial\pi$ transitions ($Z=1$). In this case diagram 1c involves the double count difficulties.

Lagrangians of the main meson vertices have the following form:

$$L(\pi^4) = -\frac{g^2}{2} Z \left\{ \vec{\pi}^4 - \left[\frac{Z-1}{Z}\right]^2 \frac{(\vec{\pi}\partial\vec{\pi})^2}{m^2} + \left[\frac{Z-1}{Z}\right]^4 \frac{(\partial_\mu\vec{\pi}\partial_\mu\vec{\pi})^2-(\partial_\mu\vec{\pi}\partial_\nu\vec{\pi})^2}{12m^4} \right\},$$

$$L(\varepsilon\pi\pi) = 2mgZ^{1/2}\varepsilon\left\{ \vec{\pi}^2 + \frac{1}{2m^2}\left[\frac{Z-1}{Z}\right]\left[\vec{\pi}\partial^2\vec{\pi} + \left[\frac{Z+1}{Z}\right](\partial\vec{\pi})^2\right]\right\},$$

$$L(\rho\pi\pi) = g_\rho(\vec{\pi}\times\partial\vec{\pi})\vec{\rho}. \tag{8}$$

Using the Lagrangians, taking into account the mass formula $m_\varepsilon^2=4m^2+m_\pi^2\cong 4m^2$, and confining ourselves to the terms with the minimum of derivatives, we obtain for diagrams 1a and 1b in the low-energy limit

$$\mathcal{L}'(\pi^4) = \frac{g^2}{2m^2 Z}\left[(\vec{\pi}\partial\vec{\pi})^2 + (Z-1)\vec{\pi}^2(\partial\vec{\pi})^2\right]. \tag{9}$$

Similarly, one establishes for diagram 1c that

$$\mathcal{L}''(\pi^4) = \frac{g_\rho^2}{2m_\rho^2}\left[(\vec{\pi}\partial\vec{\pi})^2 - \vec{\pi}^2(\partial\vec{\pi})^2\right] = -\frac{g_\rho^2}{2m_\rho^2}(\vec{\pi}\times\partial\vec{\pi})^2. \tag{10}$$

As seen from formula (7) and the considerations that follow, in the region where the low-energy sum rules are valid the equalities $m_\rho^2=6m^2$ and $Z=2$ occur. In this case, uniting (9) and (10) and using relations (4), we obtain the effective Lagrangian

$$\mathcal{L}(\pi^4)=\mathcal{L}'(\pi^4)+\mathcal{L}''(\pi^4)=\frac{g^2}{2m^2 Z}\left[2(\vec{\pi}\partial\vec{\pi})^2+(Z-2)\vec{\pi}^2(\partial\vec{\pi})^2\right]=\frac{g^2}{2m^2}(\vec{\pi}\partial\vec{\pi})^2, \qquad (11)$$

satisfying all requirements of the low-energy theorems for pion-pion scattering.

Another example we are going to dwell upon is related to determination of the chiral limit for the axial-vector hadron current $\mathcal{J}^\mu$, entering into the semileptonic $\tau\to\nu_\tau 3\pi$ decay amplitude*)

$$T_{\tau\to\nu_\tau 3\pi} = i g_\rho^2 F_\pi G_F \cos\theta \, [\bar{\nu}_\tau(p')\gamma_\mu(1-\gamma_5)\tau(p)]\mathcal{J}^\mu. \qquad (12)$$

Here G_F is the Fermi constant; θ is the Cabibbo angle; $p-p'=Q^\mu$ is the momentum transfer to the hadron block.

Since the Lagrangian of the weak interaction of the τ-lepton with mesons had the form [15]

$$L_W = \frac{G_F}{g_\rho}[\bar{\nu}_\tau\gamma^\mu(1-\gamma_5)\tau]\cdot[m_\rho^2\rho_\mu^+ + Z^{-1}m_{a_1}^2 a_{1\mu}^+] + \text{h.c.}, \qquad (13)$$

it is enough for us to obtain an expression for the current J^μ which describes the decay $a_1\to 3\pi$ and is related to the weak hadron current $\mathcal{J}^\mu$ as

$$\mathcal{J}^\mu = \frac{m_{a_1}^2 g^{\mu\nu}-Q^\mu Q^\nu}{m_{a_1}^2 - Q^2} J^\mu. \qquad (14)$$

Diagrams corresponding to this process are shown in Fig 2. To estimate their contribution to the effective chiral Lagrangian, we shall need the following Lagrangians in addition to the already known four-pion vertex (11), entering into diagram 2d :

*) Remember that a similar problem was considered in [13,14] for the case of the non-linear pion Lagrangian.

$$L(a_1\pi) = g_\rho F_\pi Z\, \vec{a}\partial\vec{\pi},$$

$$L(a_1\varepsilon\pi) = -\, g_\rho Z^{-1/2}\varepsilon[2\vec{a}\partial\vec{\pi} + Z\vec{\pi}\partial\vec{a}]. \qquad (15)$$

The former describes the $a\to\partial\pi$ transition, which was not considered in determination of the weak interaction Lagrangian (13). The latter corresponds to the $a_1\varepsilon\pi$ vertex which follows from (2) after diagonalisation.

The low-energy part of the effective Lagrangian appearing in the consideration of diagrams 2a and 2b has the form

$$\mathcal{L}'(a\pi^3) = g_\rho F_\pi^{-1}[(\vec{\pi}\partial\vec{\pi})(\vec{\pi}\vec{a})+(Z-2)/2\ \vec{\pi}^2(\vec{a}\partial\vec{\pi})], \qquad (16)$$

for diagram 2c it is

$$\mathcal{L}''(a\pi^3) = g_\rho F_\pi^{-1}[(\vec{\pi}\partial\vec{\pi})(\vec{\pi}\vec{a})-\vec{\pi}^2(\vec{a}\partial\vec{\pi})]. \qquad (17)$$

Making a sum of them, we obtain

$$\mathcal{L}(a\pi^3)=\mathcal{L}'(a\pi^3)+\mathcal{L}''(a\pi^3)= g_\rho F_\pi^{-1}[2(\vec{\pi}\partial\vec{\pi})(\vec{\pi}\vec{a})+(Z-4)/2\ \vec{\pi}^2(\vec{a}\partial\vec{\pi})]$$

$$= g_\rho F_\pi^{-1}[2(\vec{\pi}\partial\vec{\pi})(\vec{\pi}\vec{a})-\vec{\pi}^2(\vec{a}\partial\vec{\pi})]. \qquad (18)$$

Using Lagrangians (11),(15) and (18) we can find the form of the axial-vector current J^μ in the low-energy limit:

$$J^\mu_{\lim}(q_1,q_2|q_3) = \frac{2}{3m^2}\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right]q_3^\nu. \qquad (19)$$

Here q_1, q_2 are the momenta of two similar pions, and q_3 is the momentum of the pion that differs in its charge. The given current coincides with the current obtained in Ref.[13] to the accuracy of the available difference in the choise of the normalising factor, and thus satisfies all requirements of the chiral symmetry.

In conclusion we would like to point out that the full expression for the current J^{μ} and the results of its application in description of the $\tau\to\nu_{\tau}3\pi$ decay will be discussed in our separate paper.

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Received by Publishing Department
on October 13, 1989.

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πa_1-переходы и низкоэнергетический предел в линейной σ-модели с векторными и аксиально-векторными мезонами

В работе обсуждается известная проблема, связанная с получением правильного низкоэнергетического предела, которая возникает при расширении линейной σ-модели за счет включения векторных и аксиально-векторных мезонов. При первом рассмотрении древесные диаграммы с промежуточными векторными (аксиально-векторными) мезонами в области низких энергий могут воспроизводить результаты, получающиеся из линейной σ-модели, и, таким образом, ведут к нежелательному двойному счету. На примере амплитуды $\pi\pi$-рассеяния и амплитуды полулептонного распада $\tau\to\nu_{\tau}3\pi$ показано, что решить данную проблему можно, если учесть πa_1-переходы в соответствующих мезонных вершинах.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

Osipov A.A., Volkov M.K. E2-89-713

πa_1-Transitions and the Low-Energy Limit in the Linear σ-Model with Vector and Axial-Vector Mesons

We discuss a known problem associated with obtaining a correct low-energy limit, which occurs in the extension of the linear σ-model because of vector and axial-vector mesons included. In the naive consideration the tree diagrams with intermediate vector (axial-vector) mesons in the low-energy region can reproduce results obtained from the linear σ-model, thus leading to the undesirable double count. Taking the $\pi\pi$-scattering and the semileptonic $\tau\to\nu_{\tau}3\pi$ decay amplitudes as an example, the paper shows that this problem can be solved if πa_1-transitions are taken into account in the corresponding meson vertices.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1989