объединенный
институт
ядерных
исследований
дубна

E2-89-699

A.T.Filippov, A.P.Isaev

GAUGE MODELS OF FERMIONIC DISCRETE "STRINGS"

Submitted to "Modern Physics Letters A"

1989

1. Introduction

Recently, we have proposed [1] new discrete gauge models having some resemblance to bosonic strings in the hamiltonian formulation. There were introduced discrete analogs of the chiral variables $z_{\pm} = p \pm q'$ as well as of the reparametrization symmetry. The hamiltonians of the models are linear combinations of first-class constraints which are quadratic in z_+ and z_-. The constraints generate a subalgebra of $\mathrm{sp}(N,\mathbb{R})_+ \oplus \mathrm{sp}(N,\mathbb{R})_-$ (with respect to the Poisson brackets) while the corresponding Lagrange multipliers are transformed as the standard gauge potentials defined over the one dimensional base $0<t<T$, where t is the evolution parameter. These gauge models are nontrivial due to the important fact that the requirement of the gauge invariance of the action gives certain boundary conditions on the gauge transformations at the boundaries $t=0$ and $t=T$. Therefore, we can not fix the gauge in which all gauge potentials vanish and there exist some gauge-invariant parameters constructed from the gauge potentials and defining nontrivial dynamics of the system (like the Teichmuller parameters in string theory).

Having in mind all these analogies we used for our models a generic name "discrete strings". In fact, the standard theory of the closed bosonic string may be presented in the same gauge form, the gauge algebra being a very special subalgebra $\mathrm{Vect}(S^1) \oplus \mathrm{Vect}(S^1)$ of the infinite-dimensional linear canonical algebra sp(2∞,ℝ), see Ref.[1].[1] This also was demonstrated in Ref.[2], where some examples of discrete gauge models were proposed and discussed. The new idea of Ref.[1] is to introduce a chiral ("complex") structure in these models as well as to consider arbitrary subalgebras of sp(N,ℝ) ⊕ sp(N,ℝ) as gauge algebras. This has been achieved by introducing a discrete analog of the derivative $\partial_s = \partial/\partial s$ (s is the string parameter, $0 \leqslant s \leqslant 2\pi$), which is some skew-symmetric matrix ∂^{ab}. The phase space (p^a, q_a), $a = 1,\ldots,N$, is naturally split into left and right sectors by introducing the variables $z^a_\pm = p^a \pm \partial^{ab} q_b$. The analogy with the bosonic string is completed by considering $p^a = p^{a\mu}$ and $q_a = q_{a\mu}$ as Lorentz D-vectors ($\mu = 0,1,\ldots,D-1$). As the Lorentz symmetry is, in our approach, completely disconnected from the canonical symmetry we usually suppress the Lorentz indices μ (the Lorentz invariance is trivial to satisfy at least at the classical level).

One may try to use our "discrete strings" as finite dimensional approximations to continuous strings. Then rather severe restrictions on the gauge groups must be satisfied. To obviate reducing the relativistic phase space to the physical phase space, we require the gauge group to have not less than N mutually commuting generators. This means that the rank of the group is equal to the rank of the original canonical group in which it is imbedded. Then one may use N mutually commuting constraints and N corresponding gauge-fixing conditions to express the time

---

[1] The chiral variables $z_\pm$ are transformed by the generators of the chiral subalgebras sp(∞,ℝ) ⊕ sp(∞,ℝ).

components of $p^{a\mu}$ and $q_{a\mu}$ ($a = 1,\ldots,N$) in terms of physical coordinates and momenta. Roughly speaking, the generators of this abelian (Cartan) subgroup in the continual limit have to become the generators $p^2 + q'^2$, giving t-reparametrizations of the world lines $q^\mu(t,s)$ for different s. The rest of the generators, pq', correspond to s-reparametrizations. One can make a somewhat more precise statement using a naive discretization of the closed string in which it is replaced by a system of N particles (N-even) with coordinates q_a and momenta p^a. Then it is easy to construct N mutually commuting generators:

$$M^\pm = [(p^a + p^{a+1}) \pm (q_{a+1} - q_a/\varepsilon)]^2, \quad a=1,3,\ldots,N-1.$$

So we expect that any "discrete string" having a chance to approximate the continual string for large N should have a gauge group with $2N$ generators (N of them mutually commuting). Such a group will certainly be not semisimple. As we also expect it to contain Sl(2,ℝ)∼Sp(2,ℝ) subgroups, the group will be noncompact.

The models with compact gauge groups and Lorentz-vector coordinates are interesting for describing relativistic bound states (see Ref.[2]). The most general chiral models introduced in Ref.[1] might possibly be considered also as discrete analogs of two-dimensional conformal field theories [3]. In view of such applications the term "discrete strings" seems to be somewhat misleading. However, our immediate aim is to construct discrete gauge models in closest possible parallel to bosonic and fermionic strings and, in this context, using the term is justifiable.[2]

In this letter we construct fermionic analogs of the gauge models proposed in [1] by adding anticommuting degrees of freedom (Grassmann variables). From the preceding remarks it must be clear that the gauge algebras of these models are some subalgebras of the chiral algebra, $osp(N|K,\mathbb{R}) \oplus osp(N|K,\mathbb{R})$, imbedded into the canonical symmetry algebra $osp(2N|2K,\mathbb{R})$ (such an extension for compact nonchiral gauge algebras has been discussed in Ref.[2]). With all above reservations, we will call these models "fermionic discrete strings".

2. Classical Hamiltonian Formulation of Fermionic Discrete "Strings".

Consider a system described by coordinates $z_A = (z_a, z_\alpha)$ and its conjugate momenta $\bar{z}^A = (\bar{z}^a, \bar{z}^\alpha)$ where $(z_a, \bar{z}^a)$ and $(z_\alpha, \bar{z}^\alpha)$ are even and odd variables, respectively ($a = 1,2,..,N$, $\alpha = 1,2,..,K$). Introducing a compact notation for sign factors,

$$(-)^A = +1 \text{ if } A = a, \quad (-)^A = -1 \text{ if } A = \alpha,$$

we can write the commutation relations for these variables as

$$z_A z_B = (-)^{AB} z_B z_A; \quad z_A \bar{z}^B = (-)^{AB} \bar{z}^B z_A; \quad \bar{z}^A \bar{z}^B = (-)^{AB} \bar{z}^B \bar{z}^A. \qquad (1)$$

Remark that $z_a = q_a$, $\bar{z}^a = p^a$ are the standard real coordinates and momenta while z_α and $\bar{z}^\alpha$ are nonhermitian Grassmann variables, see [4],[5]. In terms of these variables the action has the form [5]

---

[2] We postpone a discussion of attempts to construct a sequence of models with gauge groups G_N giving in the limit $N \to \infty$ the standard closed bosonic string theory. It requires rather involved considerations even at the classical level.

$$S = \frac{1}{2}\int_0^T dt\, (\bar{z}^A(t)\dot{z}_A(t) - \dot{\bar{z}}^A(t) z_A(t)) - H(z,\bar{z}), \qquad (2)$$

where the dot denotes the t-derivative, and the corresponding Poisson superbrackets are [4]

$$\{X,Y\} = X\overleftarrow{\partial}/\partial z_A\, \overrightarrow{\partial}/\partial \bar{z}^A Y - (-)^A X\overleftarrow{\partial}/\partial \bar{z}^A\, \overrightarrow{\partial}/\partial z_A Y, \qquad (3)$$
$$\{z_A, \bar{z}^B\} = \delta^B_A, \quad \{z_A, z_B\} = \{\bar{z}^A, \bar{z}^B\} = 0.$$

It is well known that the kinematical part of this action is invariant with respect to the rigid (super)canonical transformations belonging to the supergroup $Osp(2N|2K,\mathbb{R})$. In Ref.[2] it has been suggested to construct a gauge theory by first choosing some hamiltonian $H(\bar{z},z)$ and then gauging the (super)canonical transformations leaving $H(\bar{z},z)$ invariant. This approach has led to some interesting discrete gauge models but failed to reproduce, in a natural way, the chiral structure of the string theory and gave no discrete string model which could be considered as a good approximation to continual string theory (an example is a "discrete string" model with nonsemisimple but compact gauge group $(U_2)^{N/2}$, [6]).

For this reason we chose in Ref.[1] a somewhat different route which will be followed here. Consider the "action" (2) with zero hamiltonian ("kinematical action") and introduce the chiral variables $z^A_\pm$ ("left and right movers"),

$$z^A_\pm = (\bar{z}^A \pm D^{AB} z_B)(\pm 1)^{(A)/2}, \qquad (4)$$

where $(A) = 0$ if $A = a$, $(A) = 1$ if $A = \alpha$, and $(-1)^{1/2} = -i$ (this somewhat strange factor is introduced to make $z^\alpha_\pm$ hermitian). The (super)matrix D^{AB} can be chosen so as to split the kinematical action (2) into two pieces depending on z_+ and z_-, respectively.

To simplify our discussion we assume here that the Grassmann elements of the supermatrix D^{AB} vanish and that it is invertible. Then it can be shown to have the form

$$D^{AB} = \begin{pmatrix} \partial^{ab} & 0 \\ 0 & \partial^{\alpha\beta} \end{pmatrix}, \quad \partial^{ab} = -\partial^{ba}, \quad \partial^{\alpha\beta} = \partial^{\beta\alpha}.$$

The Poisson superbrackets for $z_\pm$ can be calculated by using (3),

$$\{z_+^A, z_+^B\} = 2D^{AB} = \{z_-^B, z_-^A\}, \quad \{z_+^A, z_-^B\} = 0, \tag{5}$$

and the kinematical action (2) in terms of $z_\pm$ is:

$$S_0 = \frac{1}{4}\int_0^T dt \; (z_+^A D_{AB} \dot{z}_+^B + z_-^A D^T_{AB} \dot{z}_-^B), \tag{6}$$

where $D_{AC}D^{CB} = \delta^B_A$, $D^T_{AB} = D_{BA}$. We regard the action (6) as the starting point for constructing gauge theories by gauging some of its rigid symmetries. We only consider the linear canonical transformations preserving the chiral structure of S_0. They have to commute with the chiral reflection $z_\pm^A \to \pm z_\pm^A$, and so z_+^A and z_-^A are transformed by independent $\mathrm{osp}(N|K,\mathbb{R})$ transformations which form a chiral subalgebra, $\mathrm{osp}(N|K,\mathbb{R})_+ \oplus \mathrm{osp}(N|K,\mathbb{R})_-$, of the full linear supercanonical algebra $\mathrm{osp}(2N|2K,\mathbb{R})$.

Introducing the supermatrices, $F_\pm = (F_\pm)^A_B$, of the corresponding infinitesimal transformations we have

$$\delta z_\pm = F_\pm z_\pm, \quad F^T_\pm D_\pm + D_\pm F_\pm = 0,$$

where $D_+ = D_{AB}$, $D_- = D^T_{AB}$ (we reserve for the matrix D^{AB} the notation D^{-1}), and the standard supermatrix transposition rule is used, $(F^T)^A_B = (-)^{AB+A}F^B_A$. Up to this point our consideration has been completely independent of any particular choice of the matrix D^{AB}. From here we will use that, for our choice of D^{AB}, the matrix D_{AB} is a direct sum of the matrices ∂_{ab} and $\partial_{\alpha\beta}$ which are inverse for ∂^{ab} and $\partial^{\alpha\beta}$, respectively. Accordingly, $D^T_{AB} = D_{BA}$ and $D^{TT} = D$. However, as $F^{TT}_\pm \neq F_\pm$, the conditions defining F_+ and F_- are not identical. One can easily show that they are related by the following involution: $F_- = F_+^{TT'}$, $F_+ = F_-^{TT'}$, where T' means the usual simple transposition, $(F^{T'})^A_B = F^B_A$, for which, of course, $F^{T'T'} = F$ (the involution relations follow from the identities $F^{TT'T}_\pm = F^{T'}_\pm$, $D^{T'}_\pm = D^T_\pm$, $(D_\pm F_\pm)^{T'} = F^{T'}_\pm D^{T'}_\pm$).

It is convenient to write $F_\pm$ in terms of independent c-number matrices $(T^\pm_M)^A_B$,

$$F_\pm = f^M_\pm \, (T^\pm_M)^A_B, \quad (T^\pm_M)^A_B \neq 0 \text{ only if } (M) = (A) + (B),$$

where (M) is the Grassmann parity of the transformation parameter. From the involution relation we find that $(T^-_M)^A_B = (-)^{AB+B}(T^+_M)^A_B$. In this notation the symmetry transformations are

$$\delta z^A_\pm = f^M_\pm \, (T^\pm_M)^A_B \, z^B_\pm \tag{7}$$

and $(T^\pm_M)^A_B$ satisfy the conditions defining $\mathrm{osp}(N|K,\mathbb{R})_\pm$ algebras,

$$(-)^{MA}(T^\pm_M)^C_A \, D^\pm_{CB} + D^\pm_{AC} \, (T^\pm_M)^C_B = 0. \tag{8}$$

As the transformations (7) are closed with respect to the standard commutation, $\delta_1\delta_2 - \delta_2\delta_1 = \delta_3$, the generators $T^\pm_M$ satisfy a graded commutation relation,

$$[T^\pm_M, T^\pm_N\} \equiv T^\pm_M T^\pm_N - (-)^{MN} T^\pm_N T^\pm_M = (\pm)^{MN} t^K_{MN} T^\pm_K . \tag{9}$$

To stress that the left and right superalgebras defined by Eq.(9) are in fact isomorphic, we express the correspondent structure constants in terms of one set, t^K_{MN} (remind that t^K_{MN} depend on the chosen basis and satisfy the well known symmetry and (super)Jacobi identities). To demonstrate the isomorphism, one can introduce a second-kind (conjugate) commutation relation for T^-_M,

$$[T^-_M, T^-_N\}^c \equiv (-)^{MN} T^-_M T^-_N - T^-_N T^-_M = t^K_{MN} T^-_K ,$$

as proposed by Berezin [7]. This simple observation might possibly provide a more deep foundation for considering chiral variables but we will not pursue this idea here.

To construct gauge models from the action S_0, we choose some maximal subalgebra[3] of osp($N|K,\mathbb{R}$) with generators $T^{\pm}_M$ satisfying the commutation relations (9). Then, considering t-dependent parameters, $f^M_{\pm} \to f^M_{\pm}(t)$, introducing the "gauge potentials", $A_{\pm}(t)^A_B \equiv l^M_{\pm}(t)(T^{\pm}_M)^A_B$, and replacing the t-derivative, ∂_t, by the covariant derivative, $\nabla_{\pm} \equiv \partial_t - A_{\pm}$, we obtain the new action,

$$S_1 = \frac{1}{4}\int_0^T dt \; [z^A_+ D_{AB} (\nabla_+)^B_C z^C_+ + z^A_- D^T_{AB} (\nabla_-)^B_C z^C_-], \qquad (10)$$

in which rigid symmetries of the action (6) are localized. The lagrangian in Eq.(10) is invariant under the gauge transformations

$$\delta z_{\pm} = F_{\pm}(t) z_{\pm}, \quad \delta A_{\pm} = \dot{F}_{\pm} + [F_{\pm}, A_{\pm}], \qquad (11)$$

or, in the component notation,

$$\delta z^A_{\pm} = f^M_{\pm}(t)(T^{\pm}_M)^A_B z^B_{\pm}, \quad \delta l^M_{\pm}(t) = \dot{f}^M_{\pm}(t) + (\pm)^{NK} f^N_{\pm}(t) t^M_{NK} l^K_{\pm}(t). \qquad (12)$$

A general variation of the action (10) may be written as

$$\delta S_1 = \frac{1}{4}\int_0^T dt \; (\delta z_+ D \nabla_+ z_+ + \delta z_- D^T \nabla_- z_- - z_+ D \,\delta A_+ z_+ - z_- D^T \delta A_- z_-)$$
$$+ \frac{1}{4}[z_+ D\, \delta z_+ + z_- D^T \delta z_-]^{t=T}_{t=0} \qquad (13)$$

(recall that $D = D_{AB}$, $D^T = D_{BA}$). The first four terms give the equations of motion,

$$\nabla_{\pm} z_{\pm} \equiv (\partial_t - A_{\pm}) z_{\pm} = 0, \qquad (14)$$

and the constraints which we discuss later. The last two terms in Eq.(13) determine the boundary conditions $z_{\pm}(0), z_{\pm}(T)$ are fixed. These conditions are, of course, unphysical and we are better to change the action (13) by adding some boundary terms giving reasonable boundary conditions.

In our problem, the most natural boundary conditions fix bosonic canonical coordinates z_a while for fermionic variables, one has to fix initial (i) "coordinates" z_α and final (f) "momenta" $\bar{z}^\alpha$:

$$z_a(0) = z^i_a, \quad z_a(T) = z^f_a; \qquad (15)$$
$$z_\alpha(0) = z^i_\alpha, \quad \bar{z}^\alpha(T) = \bar{z}^{f\alpha}. \qquad (16)$$

The conditions (16) for the fermionic variables are necessary for a correct definition of the path-integral quantization [8]; in the context of string theory they have recently been discussed in Ref.[9]. To include the boundary conditions (15), (16) into the variational principle, we add, to the action (10), the boundary terms thus defining the following new action:

$$S_2 = S_1 + \frac{1}{2}[z^f_\alpha \bar{z}^\alpha(T) - z^i_\alpha \bar{z}^\alpha(0)] - \frac{1}{2}[\bar{z}^{f\alpha} z_\alpha(T) + \bar{z}^\alpha(0) z^i_\alpha]. \qquad (17)$$

The variational principle $\delta S_2 = 0$ now gives the equations of motion (4), the constraints, and the boundary conditions (15),(16). The new action can be rewritten in the form,

$$S_2 = \int_0^T dt \; [\bar{z}^\alpha(t)\dot{z}_\alpha(t) + \frac{1}{2}(z^\alpha_+ \partial_{\alpha\beta} \dot{z}^\beta_+ + z^\alpha_- \partial_{\alpha\beta} \dot{z}^\beta_-) - l^M_+ T^+_M - l^M_- T^-_M] -$$
$$- \frac{1}{2}[\bar{z}^{f\alpha} z_\alpha(T) + \bar{z}^\alpha(0) z^i_\alpha], \qquad (18a)$$

where the constraints,

$$T^{\pm}_M = \frac{1}{4} z^A_{\pm} (\Gamma^{\pm}_M)_{AB} z^B_{\pm}, \qquad (18b)$$

are expressed in terms of the new matrices $\Gamma^{\pm}_M$ ($D^+ = D_{AB}$, $D^- = D^T_{AB}$):

---

[3] Note that $z^A_{\pm}$ form a reducible representation of this subalgebra, it is called a reduced representation of the algebra osp($N|K,\mathbb{R}$) on the subalgebra. This fact is very important for understanding our gauge models.

$$(\Gamma^{\pm}_{M})_{AB} \equiv -(T^{\pm}_{M})^{C}_{A}\, D^{\pm}_{CB}\ , \quad (T^{\pm}_{M})^{A}_{B} = -(\Gamma^{\pm}_{M})_{BC}\, D^{CA}_{\pm}. \tag{18c}$$

These matrices generalize the corresponding Γ-matrices introduced in [1]. Remark that $(\Gamma^{-}_{M})_{AB} = (-)^{AB+A+B}(\Gamma^{+}_{M})_{AB}$ and, according to Eq.(8), $(\Gamma^{\pm}_{M})_{AB} = (-)^{AB}(\Gamma^{\pm}_{M})_{BA}$. Note also that

$$(\Gamma^{\pm}_{M})_{AB} \neq 0 \text{ only if } (M) = (A) + (B). \tag{19}$$

The Poisson brackets for $T^{\pm}_{M}$ are easily obtained from Eq.(5):

$$\{T^{\pm}_{M}, T^{\pm}_{N}\} = (\mp)^{MN} t^{K}_{MN}\, T^{\pm}_{K}\ , \quad \{T^{+}_{M}, T^{-}_{N}\} = 0. \tag{20a}$$

These relations are equivalent to the "commutation" relations for the matrices $(\Gamma^{\pm}_{M})_{AB}$, which follow from Eqs.(18c) and (9):

$$\Gamma^{\pm}_{M}\, D^{-1}_{\pm}\Gamma^{\pm}_{N} - (-)^{MN}\, \Gamma^{\pm}_{N}\, D^{-1}_{\pm}\Gamma^{\pm}_{M} = (\mp)^{MN} t^{K}_{MN}\, \Gamma^{\pm}_{K}\ , \tag{20b}$$

where $D^{-1}_{+} = D^{AB}$, $D^{-1}_{-} = D^{BA}$. Following ideas of Ref.[1] we regard the matrices $\Gamma^{\pm}_{M}$ and $D^{-1}_{\pm}$ as fundamental objects defining the gauge group. The generators, $T^{\pm}_{M}$, of the corresponding superalgebra are also expressed in terms of them (see Eq.(18c)), and the action S_2 depends on $D_{\pm}$ only trough $\partial_{\alpha\beta}$. It follows that ∂^{ab} may be not an invertible matrix having zero eigenvalues. As explained in [1], this allows us to introduce a conserved total momentum of the system. Recall that, to describe a relativistic "string", we simply define the relativistic phase superspace by extending $(z_A, \bar{z}^A)$ to $(z^{\mu}_{A}, \bar{z}^{A\mu})$ where μ is the D-dimensional space-time index, $\mu = 0,1,\ldots,D-1$. By contracting these indices one trivially obtains Lorentz-invariant discrete strings.

Returning to the action S_2 (Eq.(18a)), we stress that it differs from S_1 (Eq.(10)) by boundary terms. Accordingly, S_2 is invariant under the gauge transformations (7),(11) only if the following boundary conditions are fulfilled:

$$f^{\alpha}_{+}(\mathrm{T}) - f^{\alpha}_{-}(\mathrm{T}) = 0, \quad f^{\alpha}_{+}(0) - f^{\alpha}_{-}(0) = 0; \tag{21a}$$

$$f^{\alpha}_{+}(\mathrm{T}) + i f^{\alpha}_{-}(\mathrm{T}) = 0, \quad f^{\alpha}_{+}(0) + i f^{\alpha}_{-}(0) = 0. \tag{21b}$$

The conditions (21a) are identical to the boundary conditions in the bosonic discrete string models [1] and are analogous to the corresponding conditions in the bosonic string theory [10].

The complete system of equations of motion is given by the evolution equations (14) and by constraints $T^{\pm}_{M} = 0$. As in the bosonic case [1], the Cauchy problem can formally be solved,

$$z_{\pm}(t) = V_{\pm}(t,t_0)\ z_{\pm}(t_0), \tag{22}$$

$$V_{\pm}(t,t_0) = \mathrm{Pexp}\left\{ \int_{t_0}^{t} dt'\, l^{M}_{\pm}(t')T^{\pm}_{M} \right\}. \tag{23}$$

The finite transformations corresponding to Eqs.(7),(11) can be represented as

$$z_{\pm}(t) \to U_{\pm}(t)\ z_{\pm}(t), \quad \nabla_{\pm} \to U_{\pm}(t)\ \nabla_{\pm}\ U^{-1}_{\pm}(t), \tag{24}$$
$$V_{\pm}(t,t_0) \to U_{\pm}(t)\ V_{\pm}(t,t_0)\ U^{-1}_{\pm}(t_0), \quad U_{\pm}(t) = \exp(f^{M}_{\pm}(t)T^{\pm}_{M}).$$

3. Ghosts and BRST

To complete a foundation for quantizing our supergauge models, we extend the phase space by adding ghost variables $B^{\pm}_{M}$, $C^{M}_{\pm}$ having the Grassmann parities opposite to those of the gauge potentials $l^{M}_{\pm}$, i.e.

$$B^{\pm}_{M}\, B^{\pm}_{N} = (-)^{(M+1)(N+1)} B^{\pm}_{N}\, B^{\pm}_{M}\ , \quad C^{M}_{\pm}\, C^{N}_{\pm} = (-)^{(M+1)(N+1)} C^{N}_{\pm}\, C^{M}_{\pm}.$$

Following the general rules for treating hamiltonian systems with first-class constraints [11] we consider the extended action,[4]

---

[4] In Ref.[1] some unnecessary multipliers "i" and "$-$" signs appeared in formulas containing ghost variables. These should be corrected by using corresponding relations of the present paper.

$$S_3 = \int_0^T dt\ [\bar{z}^{\alpha}(t)\dot{z}_{\alpha}(t) + \tfrac{1}{2}(z^{\alpha}_{+}\partial_{\alpha\beta}\dot{z}^{\beta}_{+} + z^{\alpha}_{-}\partial_{\alpha\beta}\dot{z}^{\beta}_{-}) + (B^{+}_{M}\dot{C}^{M}_{+} + B^{-}_{M}\dot{C}^{M}_{-}) -$$
$$- \{l^{M}_{+}B^{+}_{M},\Omega^{+}\} - \{l^{M}_{-}B^{-}_{M},\Omega^{-}\}] - \tfrac{1}{2}[\bar{z}^{f\alpha}\ z_{\alpha}(T) + \bar{z}^{\alpha}(0)z^{i}_{\alpha}], \qquad (25)$$

where $\Omega^{\pm} = (-)^{N}[\mathcal{T}^{\pm}_{N} - (\pm)^{MN}\tfrac{1}{2}B^{\pm}_{L}\ t^{L}_{MN}\ C^{M}_{\pm}]C^{N}_{\pm}$ are the standard BRST charges corresponding to our constraints $\mathcal{T}^{\pm}_{N}$, and the Poisson superbrackets for the ghosts are

$$\{C^{N}_{\pm},B^{\pm}_{M}\} = \delta^{N}_{M}\ ,\ \{B^{\pm}_{M}C^{N}_{\mp}\} = 0. \qquad (26)$$

The ghost equations of motion,

$$\dot{C}^{M}_{\pm} = (\pm)^{NL}l^{N}_{\pm}\ t^{M}_{NL}\ C^{L}_{\pm},\quad \dot{B}^{\pm}_{M} = -(\pm)^{MN}B^{\pm}_{L}\ l^{N}_{\pm}\ t^{L}_{NM}\ , \qquad (27)$$

can be solved explicitly:

$$C_{\pm}(t) = \tilde{V}_{\pm}(t,t_0)\ C_{\pm}(t_0),\quad B^{\pm}(t) = B^{\pm}(t)\ (\tilde{V}_{\pm}(t,t_0))^{-1}, \qquad (28)$$
$$\tilde{V}_{\pm}(t,t_0) = \mathrm{Pexp}\left\{\int_{t_0}^{t}dt'\,l^{M}_{\pm}(t')\tilde{T}^{\pm}_{M}\right\},$$

where $(\tilde{T}^{\pm}_{M})^{L}_{N} = (\pm)^{MN}t^{L}_{MN}$ are the generators of the gauge group in the adjoint representation.

In some applications (see [1]) it is convenient to change the chiral ghost variables $B^{\pm}_{M}$ and $C^{M}_{\pm}$ to the standard canonical coordinates $(\rho^{M},\ \bar{\rho}_{M})$ and momenta $(\pi_{M}\ ,\ \bar{\pi}^{M})$:

$$B^{\pm}_{M} = a_{\pm}\bar{\rho}_{M} + b_{\pm}\pi_{M}\ ,\quad C^{M}_{\pm} = a_{\mp}\rho^{M} + (-)^{M}b_{\mp}\bar{\pi}^{M},$$

where $a_{+}b_{-} + b_{+}a_{-} = 1$. The new ghosts have the canonical Poisson superbrackets $\{\rho^{M},\pi_{N}\} = \{\bar{\rho}_{N},\bar{\pi}^{M}\} = \delta^{M}_{N}$, others being zero. To quantize fermionic discrete "strings", one can use, from this point, the route outlined in Ref.[1]. Corresponding calculations being rather lengthy will be published elsewhere.

3. Gauge Formulation of Standard Closed Fermionic String Theory

Finally, we will demonstrate that the theory of the standard fermionic string (FS) can be presented in the gauge form by applying our approach to the infinite-dimensional case. Recall that the action in the hamiltonian formulation of FS is [12],[13]

$$S_{FS} = \int_0^T dt\int_0^{2\pi} ds\ [p^{\mu}\dot{q}_{\mu} + \tfrac{i}{2}(\psi^{\mu}_{+}\dot{\psi}_{+\mu} + \psi^{\mu}_{-}\dot{\psi}_{-\mu}) - l^{M}_{+}\ T^{+}_{M} - l^{M}_{-}\ T^{-}_{M}], \qquad (29)$$
$$T^{\pm}_{0} = \tfrac{1}{4}(\pm\ z^{\mu}_{\pm}\ z_{\pm\mu} + 2i\ \psi^{\mu}_{\pm}\ \partial_{s}\ \psi_{\pm\mu}),\quad T^{\pm}_{1} = z^{\mu}_{\pm}\ \psi_{\pm\mu}$$

where $M = 0,1$; all the variables are functions of t and s (which are periodic or antiperiodic in s, $0<s<2\pi$), $\partial_{s} = \partial/\partial s$, μ is the Minkowski space-time index, and $z^{\mu}_{\pm} = p^{\mu} \pm \partial_{s}q^{\mu}$. The action (29) is invariant under the gauge transformations which are most clearly expressed in terms of the bosonic, $z^{\mu}_{\pm}$, and fermionic, $\psi^{\mu}_{\pm}$, chiral variables:

$$\delta\begin{bmatrix} z^{\mu}_{\pm} \\ \psi^{\mu}_{\pm}\end{bmatrix} = \begin{bmatrix}\partial_{s}f^{0}_{\pm} & \mp 2i\partial_{s}f^{1}_{\pm} \\ f^{1}_{\pm} & \tfrac{1}{2}f^{0}_{\pm}\partial_{s} + \tfrac{1}{2}\partial_{s}f^{0}_{\pm}\end{bmatrix}\begin{bmatrix} z^{\mu}_{\pm} \\ \psi^{\mu}_{\pm}\end{bmatrix} \qquad (30)$$
$$\delta\begin{bmatrix} l^{0}_{\pm} \\ l^{1}_{\pm}\end{bmatrix} = \begin{bmatrix}\dot{f}^{0}_{\pm} \\ \dot{f}^{1}_{\pm}\end{bmatrix} + \begin{bmatrix}\partial_{s}l^{0}_{\pm} - 2l^{0}_{\pm}\partial_{s} & \pm 2il^{1}_{\pm} \\ \partial_{s}l^{1}_{\pm} - \tfrac{3}{2}l^{1}_{\pm}\partial_{s} & \tfrac{1}{2}\partial_{s}l^{0}_{\pm} - \tfrac{3}{2}l^{0}_{\pm}\partial_{s}\end{bmatrix}\begin{bmatrix} f^{0}_{\pm} \\ f^{1}_{\pm}\end{bmatrix}. \qquad (31)$$

Here $f^{M}_{\pm}$ are the gauge transformation functions depending on t and s. These transformations may be written in a more compact form by using the superfield hamiltonian approach [13]. Another transparent representation of the FS gauge symmetry has been given in [2].

To complete the hamiltonian structure, we write Poisson brackets for the dynamical variables:

$$\{z^{\mu}_{\pm}(s),z^{\nu}_{\pm}(s')\} = \pm 2\partial_{s}\ \delta(s-s')g^{\mu\nu},\quad \{z_{+},z_{-}\} = 0;$$
$$\{\psi^{\mu}_{\pm}(s),\psi^{\nu}_{\pm}(s')\} = -i\delta(s-s')g^{\mu\nu},\quad \{\psi_{+},\psi_{-}\} = 0. \qquad (32)$$

With the new variables, $z^{A\mu}_{\pm}(s) = [z^{\mu}_{\pm}(s),\psi^{\mu}_{\pm}(s)]$, these equations can be presented in the form (5) where

$$D^{AA'} = \begin{bmatrix} \partial_s \delta(s - s') & 0 \\ 0 & -\frac{1}{2}\delta(s - s') \end{bmatrix}. \qquad (33)$$

The action (29) can now be rewritten similarly to Eq.(18a) if we suppress the space-time indices and treat s,s' as the matrix indices A,A'. The formulation is completed by introducing the "continual" analogs of the matrices $(\Gamma_M)_{AA'}$:

$$(\Gamma^{\pm}{}_{1s})_{s's''} = -2i\delta(s-s')\delta(s-s'')\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad (34)$$

$$(\Gamma^{\pm}{}_{0s})_{s's''} = \begin{bmatrix} \pm\delta(s-s')\delta(s-s'') & 0 \\ 0 & i[\delta(s-s')\partial_s\delta(s-s'') - \delta(s-s'')\partial_s\delta(s-s')] \end{bmatrix}.$$

One may check that by substituting Eqs.(33),(34) into Eq.(18a) the FS action is obtained(up to a boundary term).

4. Conclusion

We have demonstrated that the ideas of Ref.[1] can be applied to constructing discrete analogs of the fermionic string, in the sense explained in the Introduction. We also have tried to clarify the meaning of the chiral decomposition of the dynamical variables and gauge groups. Note that our gauge approach can be applied to construct chiral asymmetric models by choosing different left and right fermionic variables. Then our starting point must be Eq.(6) rather than (2), with completely unrelated matrices $\partial^{+}_{\alpha\beta}$ and $\partial^{-}_{\alpha\beta}$ ($\partial^{-}_{\alpha\beta} \neq \partial^{+}_{\beta\alpha}$). Such discrete analogs of heterotic strings will be considered in subsequent publications. The results related to quantizing our fermionic models are being prepared for publication.

The authors are obliged to D.Gangopadyay for valuable discussions.

References

1. A.T.Filippov, A.P.Isaev, *"Gauge Models of Discrete "Strings""*, JINR preprint E2-89-280, Dubna, 1989 (*Modern Phys. Lett.* **A**).
2. A.T.Filippov, *JETP Lett.* **46** (1987) 425; *Modern Phys. Lett.* **A4** (1989) 463.
3. A.A.Belavin, A.M.Polyakov and A.B.Zamolodchikov, *Nucl.Phys.* B241 (1984) 333; D.Friedan, Z.Qiu and S.Shenker, *Phys.Rev.Lett.* 52 (1984) 1575; E.Witten, *Comm. Math. Phys.* 92 (1984) 455.
4. R.Casalbuoni, *Nuovo Cim.* 33A (1976) 115; F.A.Berezin and M.S.Marinov, *Ann. Phys.* 104 (1977) 336.
5. J.Schwinger, *Quantum Kinematics and Dynamics* (W.A.Benjamin, 1970).
6. A.T.Filippov, preprint NBI-HE-88-07, Copenhagen, 1988.
7. F.A.Berezin, *Introduction to Algebra and Analysis with Anticommuting Variables* (Moscow State University Press, 1983; in Russian).
8. L.D.Faddeev and A.A.Slavnov, *Gauge Fields, Introduction to Quantum Theory* (W.A.Benjamin, 1980).
9. L.Mesincescu, R.I.Nepomechie and P.K.Townsend, *"Path Integral Calculation of the Closed Spinning String Propagator"*, preprint UMTG-147 (1989).
10. A.P.Isaev, *"On Quantization of Relativistic String"*, preprint IHEP 82-193, Serpukhov, 1982; A.Neveu and P.West, *Nucl.Phys.* B293 (1987) 266.
11. L.D.Faddeev, *Teor. Mat. Fiz.* **1** (1969) 3; E.S.Fradkin and G.A.Vilkovisky, *Phys.Lett.* 55B (1975) 224; I.A.Batalin and G.A.Vilkovisky, *Phys.Lett.* 69B (1977) 309.
12. C.Marshall and P.Ramond, *Nucl.Phys.* B85 (1975) 375; P.A.Collins and R.W.Tucker, *Nucl.Phys.* B121 (1977) 307.
13. V.I.Borodulin and A.P.Isaev, *Zs.Phys.* C38 (1988) 659.

Received by Publishing Department
on October 6, 1989.