сообщения
объединенного
института
ядерных
исследований
дубна

E2-89-655

S.P.Kurlovich*, A.V.Sidorov, N.B.Skachkov

PHOTON STRUCTURE FUNCTION AND ITS QCD-ANALYSIS BY EXPANSION IN GENERALIZED ORTHOGONAL POLYNOMIALS

* GSU, Gomel, USSR

1989

1. INTRODUCTION

Quantum chromodynamics being a recognized candidate for the theory of strong interactions nevertheless is all the time checked by confronting its predictions with still new experimental data. The most convincing check may be achieved in such a kinematical region where the methods of the QCD perturbation theory can be applied and where the influence of the phenomenological parameters, inevitably entering into its theoretical formulae for observables, allows a clear and physical control. From this viewpoint, the deep inelastic processes with a large transfer momentum squared $Q^2 = -q^2 = -(p-p')^2$ have doubtless advantages as compared with exclusive processes*, for which the data are concentrated in the region of relatively small Q^2. The theoretical apparatus for describing the behaviour of the structure functions (SF) of deep inelastic processes in the framework of QCD is well studied and has been applied many times for interpreting the experimental data.

Future start of the LEP accelerator opens the possibility for continuing measurements started at DESY and SLAC 2 (see the reviews [3,4]), of the photon structure functions $F^{\gamma}(x,Q^2)$ at higher of transfer momenta squared Q^2. The photon SF at large Q^2 contain the information about the transition of quarks and gluons into a photon or, as it is usually said, the distribution of quarks and gluons inside a photon. This information is of great interest because in a deep-inelastic lepton-hadron scattering the photon is considered as a point-like probe that allows the study of the quark structure of composite hadrons. The knowledge of the transition of a photon into strongly interacting quarks and gluons would allow us to construct a more consistent physical picture of high energy processes involving photons.

The nowaday status of QCD allows one to calculate only the Q^2-evolution of the SF. In QCD it has become customary to represent $F^{\gamma}(x,Q^2)$ as consisting of two parts

---

* For discussion see [1].

$$F^{\gamma}(x,Q^2) = F^{\gamma,PL}(x,Q^2) + F^{\gamma,HAD}(x,Q^2) . \quad (1)$$

The hadronic part $F^{\gamma,HAD}(x,Q^2)$ till the last few years was customary to represent on the basis of the vector meson dominance model (VDM) that describes the transition of a photon into vector mesons. The other part $F^{\gamma,PL}(x,Q^2)$ is caused by the possibility of the direct $\gamma \to q\bar{q}$ transition and is called the "point-like part".

It is just this component of $F_2^{\gamma}(x,Q^2)$ that there has caused a stable interest to the photon SF during the last ten years.

The reason for this interest was the discovery by Witten[5] of an unusual behaviour of this SF in the framework of QCD.

First, as it was shown in [5], the asymptotical behaviour of $F_2^{\gamma,PL}(x,Q\ )$ at large $Q^2 \to \infty$ can be calculated completely in the framework of the QCD perturbation theory. So, the only parameter that enters into the QCD formula for $F_{2,ASYM}^{\gamma,PL}(x,Q^2)$ is the QCD scale parameter Λ.

Second, and it is also very important, the point-like part $F_2^{\gamma,PL}(x,Q^2)$ appears to be proportional to the inverse power of the QCD running coupling constant, i.e.,

$$F_2^{\gamma,PL}(x,Q^2) \sim \alpha_s^{-1}(Q^2) , \quad (2)$$

that predicts its increase with $Q^2 \to \infty$ and guarantees its dominance over $F_2^{\gamma,HAD}(x,Q^2)$ at large Q^2.

Such a behaviour of $F_2^{\gamma,PL}(x,Q^2)$ is distingushed in QCD and differs from the behaviour of the lepton-hadron SF. These features of $F_2^{\gamma}(x,Q^2)$ for many years served as a ground for the statement that the experimental measurement of $F_2^{\gamma}(x,Q^2)$ would provide a unique possibility of a model-independent definition of the QCD scale parameter[5-7].

This highly optimistic theoretical viewpoint is changed nowadays towards a more realistic one. During the last decade different groups have also tried to make measurements of $F_2^{\gamma}(x,Q^2)$ but the existing data, supporting the QCD idea about the growth of $F_2^{\gamma}(x,Q^2)$ at large Q^2, are not sufficiently accurate to make quantitative conclusions.

So, the photon structure functions $F_2^{\gamma}(x,Q^2)$ still remain a very interesting object for experimental and theoretical studies. Their experimental study at LEP energies will give a nice possibility for checking one of the most transparent QCD prediction, the growth of $F_2^{\gamma}(x,Q^2)$ with increasing Q^2, which together with the measurement and analysis of the logarithmic decrease of the hadron SF at large Q^2 will complete the QCD-check in deep inelastic processes.

The present paper aimed at further developing the mathematical tool of the QCD-analysis of the photon SF's $F^{\gamma}(x,Q^2)$. As mentioned before, the theoretical attitude to the role of the point-like part of $F_2^{\gamma}(x,Q^2)$ has changed. Namely, the existence of nonphysical singularities in asymptotical $F_2^{\gamma,PL}(x,Q^2)$, collected in the region of small x[7], leads the people to study the $F_2^{\gamma}(x,Q^2)$ as a whole, and especially the interplay of point-like and hadronic parts of $F_2^{\gamma}(x,Q^2)$[8,9].

It is obvious that the necessity of the consideration of $F_2^{\gamma,HAD}(x,Q^2)$ together with $F_2^{\gamma,PL}(x,Q^2)$ leads to the increase of the number of parameters. In this case, Λ will not be a single parameter, but there will appear other phenomenological parameters that describe the distribution of quarks and gluons inside a photon. Then there arises the question concerning the sensitivity of the analysis to Λ considered among other parameters. So, the situation as a whole after including of $F_2^{\gamma,HAD}$ into analysis becomes closer to the one that holds in the QCD-analysis of the nucleon SF. In what follows we shall apply to the QCD-analysis of $F_2^{\gamma}(x,Q^2)$ the mathematical apparatus developed earlier for lepton-hadron deep inelastic processes in [10] and applied in [11].

The paper is organized as follows. In sect.2 we shall present all the formulae needed for QCD-analysis for $F_2^{\gamma}(x,Q^2)$. Then in sect. 3 we shall discuss the main features of the method of QCD-analysis[10] based on the expansion of SF in orthogonal polynomials, and compare it with other approaches. In sect. 4 we shall introduce a new class of polynomials that, for instance, generalize the Jacobi polynomials used in [10,11]. In sect. 5 the efficiency of this method will be demonstrated by applying it to particular functions used in the QCD-analysis of $F^{\gamma}(x,Q^2)$: a) the parton model function for $F^{\gamma}(x,Q^2)$, and b) the interpolation function for $F_2^{\gamma}(x,Q^2)$ obtained recently in [12]. The last sixth sect. is devoted to the application of the developed method to the QCD-analysis of some experimental data. It has only the illustrative character because the present data on $F^{\gamma}(x,Q^2)$ have large statistical errors. So the quantitative check of QCD with $F_2^{\gamma}(x,Q^2)$ will become meaningful only after getting new precise data at higher Q^2.

2. QCD-PREDICTIONS FOR THE PHOTON STRUCTURE FUNCTION

The structure function (SF) of a real photon is measured in the process, shown in fig. 1. In the range of applicability of the QCD perturbation theory, i.e., at large Q^2, it can be

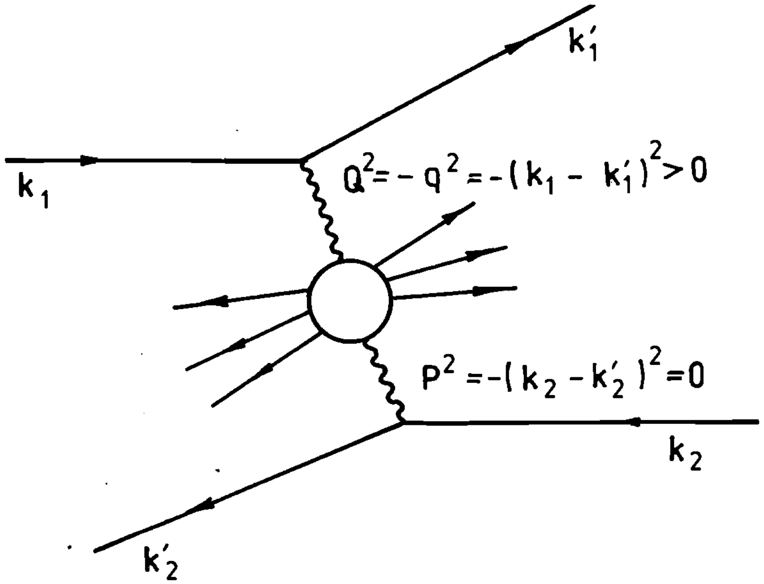

Fig. 1. The diagram of the process of the electron-positron scattering via the photon-photon interaction ($k_1(k_1')$ and $k_2(k_2')$ are the 4-momenta of the electron (positron) in initial and final states). The square other virtual photon 4-momenta $q = k_1 - k_1'$ is negative: $q^2 = (k_1 - k_1')^2 < 0$, while that of the 4-momenta of the second photon emitted by the positron is close to zero: $(k_2 - k_2')^2 = 0$.

represented as series in the strong coupling constant (in what follows we shall use the $\overline{MS}$-scheme)

$$\frac{\bar{\alpha}_S^2(Q^2)}{4\pi} \equiv \frac{\bar{g}^2(Q^2)}{16\pi^2} = \frac{\bar{g}_0^2(Q^2)}{16\pi^2} \cdot \left[1 - \frac{\bar{g}_0^2(Q^2)}{16\pi^2} \cdot \frac{\beta_1}{\beta_0} \cdot \ln\ln\frac{Q^2}{\Lambda^2}\right],$$

$$\beta_1 = 102 - \frac{38}{3} \cdot f, \qquad (2.1)$$

$$\bar{g}_0^2(Q^2) = \frac{16\pi^2}{\beta_0 \cdot \ln\left(\frac{Q^2}{\Lambda^2}\right)}; \quad \beta_0 = 11 - \frac{2}{3} f. \qquad (2.2)$$

To the leading order (LO) in $\alpha_S(Q^2)$ the SF has the form [8]

$$F_2^{\gamma}(x,Q^2) = x \cdot q_{NS}^{\gamma}(x,Q^2) + \langle e^2 \rangle \cdot x \cdot \Sigma^{\gamma}(x,Q^2), \qquad (2.3)$$

where the functions $q_{NS}^{\gamma}(x,Q^2)$ and $\Sigma^{\gamma}(x,Q^2)$ describe the non-singlet (NS) and singlet (S) parton distributions inside a photon, and the symbol $\langle e^k \rangle$ denotes the quantity $\langle e^k \rangle \equiv f^{-1} \sum_f e_q^k$.

The function $q_{NS}^{\gamma}(x,Q^2)$ obeys the following inhomogeneous Altarelli-Parisi equation [13] $(i = NS)$ (we shall follow the notation of ref. [8,9] and [6] *

$$\frac{d q_i^{\gamma}(x,Q^2)}{d \ln Q^2} = \frac{\alpha}{2\pi} K_i^{(0)}(x) + \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \cdot P_i^{(0)}\left(\frac{x}{y}\right) \cdot q_i^{\gamma}(y,Q^2), \qquad (2.4)$$

where the inhomogeneous term $K_{NS}^{(0)}(x)$ is given by

$$K_{NS}^{(0)}(x) = 3f \cdot (\langle e^4 \rangle - \langle e^2 \rangle^2) \cdot 2[x^2 + (1-x)^2]. \qquad (2.5)$$

The function $P_{NS}^{(0)}(x) \equiv P_{qq}^{(0)}(x)$ is a standard splitting function of the Altarelli-Parisi equation.

In the singlet case $(i = s)$ equation (2.4) becomes a matrix equation (it is a system of two coupled equations) with

$$q_s^{\gamma} \equiv \begin{bmatrix} \Sigma^{\gamma} \\ G^{\gamma} \end{bmatrix}; \quad K_S^{(0)} = \begin{bmatrix} K_q^{(0)} \\ 0 \end{bmatrix}; \quad P_S^{(0)} \equiv \begin{bmatrix} P_{qq}^{(0)} & P_{qq}^{(0)} \\ P_{qq}^{(0)} & P_{qq}^{(0)} \end{bmatrix} \qquad (2.6)$$

and

$$K_q^{(0)}(x) = 3 \cdot f \cdot \langle e^2 \rangle \cdot 2[x^2 + (1-x)^2]. \qquad (2.7)$$

The functions $K_i^{(0)}(x)$ stem from the parton function

$$F_{2,PM}^{\gamma}(x,Q^2) = \frac{\alpha}{\pi} \cdot 3f \cdot \langle e^4 \rangle \cdot x[x^2 + (1-x)^2] \cdot \ln\frac{Q}{\mu^2} \qquad (2.8)$$

defined by a box-type diagram that describes the direct $\gamma \to q\bar{q}$ transition [15,16].

Explicit solutions in an analytical form for the QCD Altarelli-Parisi equation are not yet found. Analytical expressions are derived only for the moments of the SF's:

* We remind that the SF's in [8], as compared with the definition in [8,9,14], contain an additional factor $e^2 = 4\pi\alpha$.

$$F_2^{\gamma}(n,Q^2) \equiv \int_0^1 dx \cdot x^{n-1} \cdot \frac{1}{x} \cdot F_2^{\gamma}(x,Q^2)\,, \quad n = 2,3\,... \tag{2.9}$$

that in the leading order (LO) can be represented in the form ($\delta_{NS} = 1$; $\delta_S = \langle e^2 \rangle$) [8] :

$$F_2^{\gamma}(n,Q^2) = \delta_{NS} \cdot q_{NS}^{\gamma}(n,Q^2) + \delta_S \cdot \Sigma^{\gamma}(n,Q^2)\,, \tag{2.10}$$

where the moments of the parton distributions are defined in the following way ($i = NS, S$) :

$$q_i^{\gamma}(n,Q^2) = \int_0^1 dx \cdot x^{n-1} \cdot q_i^{\gamma}(x,Q^2)\,. \tag{2.11}$$

The following formula has been found by the renormalization group method [8,9] :

$$q_i^{\gamma}(n,Q^2) = \frac{4\pi}{\alpha_S(Q^2)} \cdot a_i(n) \cdot \left\{1 - \left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right]^{1-\frac{2P_i^{(0)}(n)}{\beta_0}}\right\} + \left\{\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right\}^{-\frac{2P_i^{(0)}(n)}{\beta_0}} \cdot q_i^{\gamma}(n,Q_0^2)\,, \tag{2.12}$$

where

$$a_i(n) = \frac{\alpha}{2\pi\beta_0} \cdot \frac{1}{1 - 2P_i^{(0)}(n)/\beta_0} \cdot K_i^{(0)}(n)\,. \tag{2.13}$$

The moments of the splitting functions $P_i^{(0)}(n)$ ($i = NS, S$) are connected with standard anomalous dimensions γ_{ij}^{n} (used in [6,7,17]) by the relation $P_i^{(0)}(n) = -\frac{1}{4}\gamma_i^{(0),n}$ (see [8]) and the functions $K_i^{(0)}$ are

$$K_{NS}^{(0)}(n) = 3f \cdot (\langle e^4 \rangle - \langle e^2 \rangle^2) \cdot 2 \cdot \frac{n^2+n+1}{n(n+1)(n+2)}\,, \tag{2.14}$$

$$K_q^{(0)}(n) = 3f \cdot \langle e^2 \rangle \cdot 2\, \frac{n^2+n+1}{n(n+1)(n+2)}\,, \tag{2.15}$$

($K_i^{(0)}(n) = \frac{1}{4} K_i^{0,n}$, where $K_i^{0,n}$ are the functions used in [6]).

The functions $q_i^{\gamma}(n,Q_0^2)$ are the moments of the functions of quark distributions $q_i^{\gamma}(x,Q_0^2)$ ($i = NS, S$) , that give the boundary conditions for the Altarelli-Parisi equation (2.4) at some reference point Q_0^2.

It is useful to compare (2.14) and (2.15) with the formula for the moments of parton function (2.8) [6]

$$\int_0^1 dx \cdot x^{n-2} \cdot F_2^{\gamma}(x,Q^2)\big|_{PM} = \frac{\alpha}{\pi} \cdot 3 \cdot f \cdot \langle e^4 \rangle \cdot \frac{n^2+n+2}{n(n+1)(n+2)} \ln \frac{Q}{\mu^2}\,. \tag{2.16}$$

In the limit $Q^2 \gg \Lambda^2, \mu^2$ and after neglecting all terms proportional to $[\alpha_S(Q^2)/\alpha_S(Q_0^2)]$, from (2.12) we get

$$q_i^{\gamma}(n,Q^2)\big|_{Q^2 \gg \Lambda^2, \mu^2} = \frac{4\pi}{\alpha_S(Q^2)} \cdot \frac{\alpha}{2\pi\beta_0} \times (1 + \gamma_i^{(0),n} | 2\beta_0)^{-1} \cdot K_i^{(0)}(n) \cdot \ln \frac{Q^2}{\Lambda^2}\,. \tag{2.17}$$

In this limit it is easy to see that QCD-expression (2.10) for the moments $F_2^{\gamma}(n,Q^2)$ will transform into parton expression (2.16) (up to some constant factors) if one puts to zero all for anomalous dimansions $P_i^{(0)}(n) = -\frac{1}{4}\gamma_i^{(0),n}$. In this way the correspondence between parton and QCD pictures is established.

Asymptotic expression (2.17) for many years, starting from [5], served for attracting interest to study the photon SF, because on its ground the conclusion that the QCD-perturbation theory allows determination of the Q^2-and x-dependence as well as the absolute normalization of $F_2^{\gamma}(x,Q^2)$ at large Q^2 was made.

Due to this reason and to the fact of the presence of a single parameter, the scale parameter of QCD-Λ, the asymptotic expression for $F_2^{\gamma}(x,Q^2)$*

---

* Analogous behaviour of the moments of F_2^{γ} in the asymptotics was established by the method of summation of diagrams in ref. [18].

$$F^{\gamma}_{2\,asym.}(n,Q^2) = \alpha\cdot\frac{4\pi}{\alpha_S(Q^2)}\cdot\frac{1}{\pi\beta_0}\times$$
$$\times\left[\frac{K^0_{NS}(n)}{1+\lambda^n_{NS}/2\beta_0} + \langle e^2\rangle\cdot\frac{K^0_q(n)\cdot(1+\gamma^{0,n}_{GG}/2\beta_0)}{(1+\lambda^n_-/2\beta_0)\cdot(1+\lambda^n_+/2\beta_0)}\right], \tag{2.18}$$

(where $\lambda^n_{NS}=\gamma^{0,n}_{NS}$ and $\lambda^n_{\pm}$ are defined according to (A.7)) was considered to be the most perspective for the model-independent (i.e., independent of the form of the parton distribution functions $q_i(x,Q_0^2)$ that define the boundary conditions at Q_0^2) check of QCD. This asymptotic expression (2.18) as well as (2.12), has a unique property: it grows with increasing Q^2 as $4\pi\alpha_S^{-1}(Q^2)=\beta_0\cdot\ln\frac{Q^2}{\Lambda^2}$. An expression analogous to (2.18) for $F^{\gamma,PL}_{2,asym.}$ was derived in the second order of perturbation theory in [6] (see also [7]).

This view point suffered an essential change because it was found that the asymptotic function $F^{\gamma,PL}_{2,asym.}(n,Q^2)$ being transformed into the x-space by an inverse Mellin transformation

$$F(x,Q^2) = \frac{1}{2\pi i}\cdot\int_{c-i\infty}^{c+i\infty} dn\cdot x^{-n+1}\cdot M(n,Q^2), \tag{2.19}$$

acquires the singularities at small x. These singularities occur due to poles in variable n in the denominators $(1+\lambda\ /2\beta\ )$ in (2.18) $(i = NS\pm)$. Really, the expressions $d_i(n)=\lambda_i/2\beta_0$ have the following properties [9]: $d_{NS}(n = 0.3099) = -1$, $d_-(n=1.596) = -1$. Accordingly, $F(x,Q^2)$ has in the x-space poles $(1/x)^{0.3099}$ and $(1/x)^{1.596}$ [9].
In the second order in $\alpha_S(Q^2)$ the formula for the moment is [8,9,14]

$$q_i^{\gamma,SO}(n,Q^2) \doteq \frac{4\pi}{\alpha_S(Q^2)}\cdot\left\{1-\left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right]^{1-\frac{2P_i^{(0)}(n)}{\beta_0}}\right\}\cdot a_i(n) +$$
$$+\left\{1-\left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right]^{-\frac{2P_i^{(0)}(n)}{\beta_0}}\right\}\cdot b_i(n) + \tag{2.20}$$

$$+\left\{1-\frac{\alpha_S(Q^2)-\alpha_S(Q_0^2)}{\pi\beta_0}\cdot R(n)\right\}\cdot\left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right]^{-\frac{2P_i^{(0)}(n)}{2\beta_0}}\times q_i(n,Q_0^2),$$

where the coefficient $b_i(n)$ has the form

$$b_i(n) = -\frac{1}{P_i^{(0)}(n)}\left\{2\left[P_i^{(1)}(n)-\frac{\beta_1}{2\beta_0}P_i^{(0)}(n)\right]\cdot a_i(n) + \right.$$
$$\left. + \frac{\alpha}{2\pi}\left[K_i^{(1)}(n)-\frac{\beta_1}{2\beta_0}\cdot K_i^{(0)}(n)\right]\right\}. \tag{2.21}$$

From (2.20) it follows that in the limit $Q^2\to\infty$

$$q^{\gamma,SO}_{i,asym.}(n,Q^2) = \frac{4\pi}{\alpha_S(Q^2)}\cdot a_i(n) + b_i(n). \tag{2.22}$$

This expression contains the single parameter Λ, but its application for definition of the form of $F^{\gamma,SO}_{2,asym}(x,Q^2)$ in the second order of perturbation theory meets, as in the case of (2.17), difficulties connected with the singular behaviour of the coefficients $a_i(n)$ and $b_i(n)$. Apart of the above-mentioned pole singularities of the coefficients $a_i(n)$, the coefficients $b_i(n)$ that appear in the second order do contain the anomalous dimensions $P_i^{(0)}(n) = -\frac{1}{4}\gamma_i^{0,n}$ in their denominators. These anomalous dimensions due to the following properties: $d_{NS}(n=1)=0$ and $d_-(n=2) = 0$ make the SO coefficients $b_i(n)$ to be singular just for the integer numbers, one of which $d_-(n=2) = 0$ takes part for the physical value $n=2$ that appears in the expansion of the SF $F_2^{\gamma}(x,Q^2)$ in its moments.

As shown in [14], these singularities do not occur in the case of the SF of a virtual photon, for which $P^2\neq 0$. On the basis of this result it was suggested in [19] that the singularities of the asymptotical "point-like" part of $F_2^{\gamma,PL}(x,Q^2)$ should be cancelled with the singularities of the hadronic part of $F_2^{\gamma,HAD}$ that contains the dependence on Q_0^2 (see also the discussion in [20]).

A subsequent realization of this viewpoint was performed in [21] (see also [22,23]) where the regularization procedure based on introduction of only one new free parameter, was proposed. This procedure was applied in a number of programs of the QCD-analysis of the data on the photon SF's /24/.

But it is not necessary to introduce a further regularization procedure if, following /8,9/, explicit solutions (2.12) and (2.20) (that contain the boundary conditions at Q_0^2) are used instead of asymptotic expressions (2.17), (2.18) and (2.22) or, following /25/, the procedure of separating of hadronic and point-like parts based on studying the p_T-behaviour of jets in the final hadronic states. Let us discuss the first possibility.

Really, the denominators that lead to the pole singularities $(1-2P_i^{(0)}(n)/\beta_0)=(1+\gamma_i^{0,n}/2\beta_0)=\epsilon\to 0$ and $P_i^{(0)}(n)=-\frac{1}{4}\gamma^{0,n}=\epsilon\to 0$ in the functions $a_i(n)$ and $b_i(n)$ appear in formulae (2.12) and (2.20) in the combinations

$$\left(1-\frac{2P_i^{(0)}(n)}{\beta_0}\right)^{-1}\cdot\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{\left(1-\frac{2P_i^{(0)}(n)}{\beta_0}\right)}\right\} \tag{2.23}$$

and

$$\frac{1}{P_i^{(0)}(n)}\cdot\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{-\frac{2P_i^{(0)}(n)}{\beta_0}}\right\}, \tag{2.24}$$

that allow us to apply the formula

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\cdot(1-a^{\epsilon})=-\ln a. \tag{2.25}$$

These formulae demonstrate the important regularization role of the boundary conditions at the point Q_0^2 in general solutions (2.12) and (2.20). As mentioned in /8/, general solutions of evolution equations (2.3) in the n-space, given by (2.12) and (2.20), automatically contain in themselves the regularization procedure in the same way as in the case of the virtual photon SF /14/, where the role of the reference point Q_0^2 was played by p^2 (see fig. 1). That is why the use of the general solution (i.e. without artificial separation into point-like and hadronic parts and neglect of the last one) containing the boundary conditions at point Q_0^2 allows us to despense with the regularization procedure based on the introduction of a new free parameter λ that has no direct meaning /21/.

It is also useful to mention that the situation with the discussion of the role of the boundary conditions in the formulae used for expressing the solutions of the renormalization group differential equation for the moments of $F_2^{\gamma}(x,Q^2)$ completely repeats the discussion that has taken place previously in /26/ in connection with the solutions of the renormalization equations for the moments of the SF of lepton-hadron deep inelastic scattering (see alsp review /17/, p.254). There (in /26/) for the solution of the renormalization-group equation that does not depend on the reference point $\mu^2=Q_0^2$ the singularity was found in the SO in $\alpha_s(Q^2)$. This singularity appears in the coefficient $\bar{R}_{2,n}^{(-)}$ at the point $d_+^n=d_-^n+1$. The authors of /26,17/ have mentioned that to avoid this singularity, one needs to introduce the functions depending on a reference point Q_0^2 that will serve for defining of boundary conditions.

3. APPLICATION OF THE ORTHOGONAL POLYNOMIALS FOR THE QCD-ANALYSIS OF STRUCTURE FUNCTIONS

The essence of the problems one meets in the QCD-analysis of the data on SF's consists in the fact that the scale parameter Λ enters into the QCD theoretical formulae under the sign of logarithm. This leads, as shown, for example, in /10/, to the situation when large enough changes of Λ in QCD-formulae (by 50 , for example) result in a very small variation of the SF's (about 1%) in the region $x=0.1\div 0.7$.

Then it following /10/ that the application of the theoretical methods of analysis that do not guarantee finding the solutions with accuracy better than 1% do not allow us to obtain reliable conclusions on the found values of Λ. For this reason the question of the mathematical precision of the method of the analysis becomes very important.

Another aspect of the problem of the QCD-analysis consists, as mentioned above, in the fact that the exact analytic solutions of the QCD Altarelli-Parisi equations are not yet found. The analytic form is found only for the solutions of evolutions of moments of the SF

$$M_n(Q^2)=\int_0^1 dx\cdot x^{n-2}\cdot F(x,Q^2) \tag{3.1}$$

(see (2.12) for the LO and (2.20) for SO).

But explicit analytic formulae for the moments in the QCD-analysis did not receive a wide use because the recalculation of the experimental data from $F(x,Q^2)$ to $M_n(Q^2)$ is connected with the extrapolation of $F(x,Q^2)$ into the region $x \simeq 0$ and $x \simeq 1$, where the data usually do not exist (see for discussion [27]). So such an extrapolation leads to the increase in the error of Λ, i.e., it leads to the decrease in the sensetivity of the method to the scale parameter Λ.

During the last years the number of high precise and reliable programs for a numerical solution of the Altarelli-Parisi equations was created (see, for example [27,28]). Nevertheless, one should keep in mind a very essential remark, done in ref. [27], that the application of the method of the QCD-analysis, based on the Altarelli-Parisi equation, does not overcome the main difficulty of the moment method connected with the extrapolation of the SF into the experimentally non-observed region $x = 0.75 \div 1.0$.

Really, as it is seen from (2.4), the integration of the SF in the right-hand side of the Altarelli-Parisi equation is performed up to the value $x = 1$, while the most of the data is contained in the region $x \leq 0.6 \div 0.75$. The authors [27] of [27] using the typical quark model distributions have given a theoretical estimate of the error introduced by the extrapolation of SF into the integration region $x = 0.65 \div 1$. They have shown that the uncertainty introduced by this extrapolation can be quite sizeable.

Unfortunately, in the most of publications devoted to the QCD-analysis with the help of the Altarelly-Parisi equations this inevitably introduced error connected with the extrapolation is not presented and discussed. (Probably it is included, as in the moment method, into the statistical error $\pm\Delta\Lambda$). It is also not attainable for analysis, by a non-involved reader, of the influence of a precision of the method of numerical integration of the Altarelli-Parisi equation on the found value of Λ or its error.

Apart from these two methods developed historically first, i.e. the moment method and the Altarelli-Parisi integro-differential equation method, there exists a third method of the QCD-analysis developed in the last ten years starting from paper [29] (see also [30-32]).

This method is based on the expansion of SF's in series of the polynomials. The Laguerre polynumials in $y = \ln 1/x$ variable, was applied in [33]. The method of the QCD-analysis, based on the results of [33] was realized in the program of the QCD-analysis used by the CHARM collaboration [34,35].

But in both these methods ([30,31] and [33]) the expansion coefficients of $F(x,Q^2)$ into polynomials are infinite series. So these methods as applied require truncation of these series, which necessitates further study (usually not presented in the papers on the QCD-analysis) of the influence of that truncation on the precision of the method (apart of the study of the influence of the truncation of the original series in polynomials themselves). This situation arises, for example, in the method based on the expansion into the Laguerre polynomials in $y = \ln 1/x$ taken as an argument [33]. Really, in this case the polynomials could not be represented as finite series in x-variable. As a result, the coefficients of an expansion in the polynomials $L_n(\ln 1/x)$ could not be represented as combinations of moments of SF's as in the method proposed by Parisi and Sourlas [36].

The Parisi-Sourlas method is based on the expansion of the SF into the Jacobi polynomials in x-variable:

$$\theta_k^{\alpha\beta}(x) = \sum_{n=0}^{k} C_j^k(\alpha,\beta) \cdot x^n , \tag{3.2}$$

where $C_j^k(\alpha,\beta)$ are known coefficients [36-38]. These polynomials are orthogonal in the interval $x \in [0.1]$ with the weight function $w^{\alpha\beta}(x) = x^\alpha \cdot (1-x)^\beta$

$$\int_0^1 dx \cdot w^{\alpha\beta}(x) \cdot \theta_k^{\alpha\beta}(x) \cdot \theta_n^{\alpha\beta}(x) = \delta_{k,n} . \tag{3.3}$$

The expansion coefficients of $F(x,Q^2)$

$$F(x,Q^2) = w^{\alpha\beta}(x) \cdot \sum_{k=0}^{\infty} a_k(Q^2)\, \theta_k^{\alpha\beta}(x) \tag{3.4}$$

can be expressed with (3.3) and (3.2) as finite combinations of the moments of the SF

$$M_n(Q^2) \equiv F(n,Q^2) = \int_0^1 dx \cdot x^{n-2} \cdot F(x,Q^2) , \quad n = 2,3 \ldots$$

by the formula

$$a_k(Q^2) = \int_0^1 dx\, \theta_k^{\alpha\beta}(x) \cdot F(x,Q^2) = \sum_{n=0}^{k} C_j^k(\alpha,\beta)\, M(n+2,Q^2) . \tag{3.5}$$

So, the SF $F(x,Q^2)$ can be represented by the following explicit formula /36/

$$F(x,Q^2) = w^{\alpha\beta}(x) \cdot \sum_{k=0}^{\infty} \theta_k^{\alpha\beta}(x) \cdot \sum_{n=0}^{k} C_j^k(\alpha,\beta) \cdot M(n+2,Q^2), \qquad (3.6)$$

in which for the application of the QCD-analysis $M(n,Q^2)$ are substituted by analytical QCD-expressions of (2.12) and (2.20) type. In this case the influence of the truncation of the series in k at some upper limit $k = N_{MAX}$, naturally needed for practical application of (3.6), can be easily studied, as it was done in /10/, by varying N_{MAX} or expanding some test functions in series of the Jacoby polynomials and studying the approximation precision by series (3.4) for different N_{MAX} in dependence of parameters of the weight function $w^{\alpha\beta}(x)$.

The Jacobi polynomials are very general polynomials and include a large class of well-known polynomials. Thus, fixing the parameters of the weight function $\alpha = \beta = -1/2$ we get the well-known Chebyshev polynomials and with $w(x) = (1-x)^{\alpha-0.5}$ we come to the Gegenbauer polynomials $C_n^\alpha(x)$.

The expansion in the Jacobi polynomials $\theta_k^{\alpha\beta}(x)$ was firstly applied for the QCD-analysis of nonsinglet structure functions in /39/ and the singlet ones in /10/ and /11/.

One of the main properties of the expansion in the Jacobi polynomials is the presence of the weight function $w^{\alpha\beta}(x)$ that can accumulate the part of the x-dependence of the SF. Those weight functions that define the Jacobi polynomials, i.e.,

$$w^{\alpha\beta}(x) = x^\alpha \cdot (1-x)^\beta, \qquad (3.7)$$

appear to be very useful in the analysis of hadronic structure functions because they, first, are similar in from to the distribution of the valence quarks in a nucleon and, second, they allow us to take into account the boundary conditions for the hadronic SF, i.e. vanishing at $x \to 1$.

But as it is known from the parton model, the photon SF has an opposite tendency: it increases as $x \to 1$. For this reason it will be useful to consider a more general class of polynomials for which the weight function can be of a more general form than (3.7).

4. GENERALIZED ORTHOGONAL POLYNOMIALS DEFINED BY THE WEIGHT FUNCTION OF ARBITRARY FORM

The generalization of the Jacobi polynomials considered in the interval $x \in [a,b]$ is defined by the weight function $w(x,\alpha) \equiv w(x,\{\alpha_1, \alpha_2 \ldots \alpha_n\})$ that obeys the following conditions:

1) the $w(x,\alpha) \equiv w(x,\{\alpha_1,\alpha_2\ldots\alpha_n\})$ are positive in the interval $x \in [a,b]$;

2) the values of the moments of the weight function

$$h_n^\alpha = \int_a^b dx \cdot x^n \cdot w(x,\alpha) \qquad n = 0,1,2\ldots \qquad (4.1)$$

are finite.

According to the theory of the classical polynomials, the set of the orthogonal polynomials $P_k^\alpha(x)$, considered in the interval $[a,b]$, is defined unambiguously by the form of the weight function (that satisfied conditions 1) and 2)) with which they are orthogonal to each other in the interval

$$\int_a^b w(x,\alpha) \cdot P_k^\alpha(x) \cdot P_n^\alpha(x)\, dx = \delta_{k,n}. \qquad (4.2)$$

These polynomials are explicitly represented through the moments of the weight function h_n according to the following formulae (in what follows we shall omit the indices $\alpha = \{\alpha_1, \ldots \alpha_n\}$

$$P_0(x) = \frac{1}{\sqrt{h_0}}; \quad P_1(x) = \frac{h_0 x - h_1}{\sqrt{h_0(h_0 h_2 - h_1^2)}}, \qquad (4.3)$$

$$P_k(x) = \sum_{n=0}^{k} x^n \cdot A_n^k = \frac{1}{\sqrt{\Delta_{k-1}\cdot\Delta_k}} \begin{vmatrix} h_0 & h_1 & \ldots & h_k \\ h_1 & h_2 & \ldots & h_{k+1} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ h_{k-1} & h_k & \ldots & h_{2k-1} \\ 1 & x & x^2 & \ldots x^k \end{vmatrix} \qquad (4.4)$$

where Δ_k is the following determinant:

$$\Delta_k = \begin{vmatrix} h_0 & h_1 & \cdots\cdots & h_k \\ h_1 & h_2 & \cdots\cdots & h_{k+1} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ h_k & h_{k+1} & \cdots\cdots & h_{2k} \end{vmatrix} . \qquad (4.5)$$

The expansion of the function $f(x)$ into the generalized polynomials has the form, analogous to (3.4):

$$f(x) = w(x,\alpha) \cdot \sum_{k=0}^{\infty} C_k(\alpha) \cdot P_k^{\alpha}(x) , \qquad (4.6)$$

where for the coefficients $C_k(\alpha)$, using (4.2), we find

$$C_k(\alpha) = \int_a^b dx \cdot f(x) \cdot P_k^{\alpha}(x) . \qquad (4.7)$$

The substitution of (4.3) into (4.7) allows us, in analogy with (3.5), to express the coefficient $C_k(\alpha)$ through a finite sum of the moments $M(n)$ of the expanded function $M(n) = \int_0^1 dx \cdot x^{n-2} \cdot f(x)$:

$$C_k(\alpha) = \int^b dx \cdot f(x) \cdot \sum_{n=0}^{k} x^{(n+2)-2} A_n^k(\alpha) = \sum_{n=0}^{k} A_n^k(\alpha)\, M(n+2) . \qquad (4.8)$$

As a result, we come to the expansion of $f(x)$ into generalized polynomials

$$f(x) = w(x,\alpha) \cdot \sum_{k=0}^{\infty} P_k^{\alpha}(x) \cdot \sum_{n=0}^{k} A_n^k(\alpha) \cdot M(n+2) . \qquad (4.9)$$

This formula in a particular case of $w(x,\alpha) = w(x,\{\alpha_1,\ldots\alpha_n\})$ becomes the expansion into Jacobi polynomials. In what follows we shell use for practical reasons the QCD-analysis of the finite part of the series (4.10)

$$f(x) = w(x,\alpha) \cdot \sum_{k=0}^{N_{MAX}} P_k^{\alpha}(x) \cdot \sum_{n=0}^{k} A_n^k(\alpha) \cdot M(n+2) . \qquad (4.10)$$

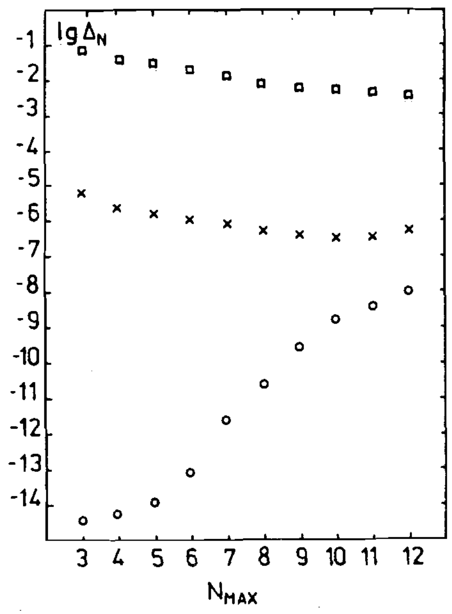

Fig. 2. The dependence of

$$\Delta_{N_{MAX}} = \int_0^1 dx \,|\, f(x) - f_{N_{MAX}}(x) \,| \, (4.11)$$

on N_{MAX} in the case of expansion of the parton model function $f_1^{PM}(x) = x[x^2 + (1-x)^2]$ with the sets of orthogonal polynomial defined by 3 different weight functions:
1) o – $\omega(x) = f_1^{PM}(x)$,
2) × – $\omega(x) = x^{0,99} \cdot (1-x)^{5\times 10^{-5}}$
3) □ – $\omega(x) = x^{-0,5} \cdot (1-x)^{-0,5}$.

Let us now study some particular applications of formulae (3.4) and (4.10). Here we shall consider some test functions that are characteristic of the photon SF.

For a quantitative description of the accuracy of the approximation of the test functions we introduce the next integral characterictics

$$\Delta \equiv \Delta_{N_{MAX}} = \int_0^1 dx \cdot |f(x) - f_{N_{MAX}}(x)| , \qquad (4.11)$$

where the right-hand side of formula (4.10) for $f_{N_{MAX}}(x)$ stands, i.e., the reconstruction of the function $f(x)$ with the help of a finite series of its expansion in generalized polynomials $P_k^{\alpha}(x)$ with the coefficients of this expansion expressed through the moments $M(n)$ of the function $f(x)$ itself.

Figure 2 shows the behaviour of $\lg\Delta_{N_{MAX}}$ as a function of N_{MAX} when the function $f_1^{PM}(x) = x \cdot [x^2 + (1-x)^2]$ (that enters the parton function (2.8)) is expanded in three different types of polynomials. These polynomials are defined by three different weight functions: First, by the weight function, equal to the expanded function, i.e. $w_1(x) = x \cdot [x^2 + (1-x)^2]$ which allows us to estimate the precision level provided by the computer itself; second, by the weight function $w_2(x) = x^{-0,5} \cdot (1-x)^{-0,5}$ that defines the Chebyshev polynomials for which the convergence theorem for the series in these polynomials is proved; and third, by Jacoby polynomials $\theta_k^{\alpha\beta}(x)$.

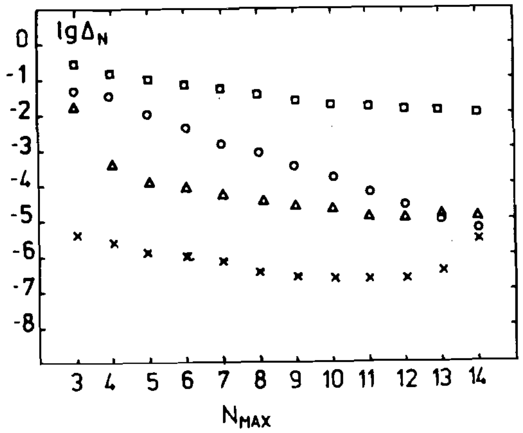

Fig. 3. The dependence of

$$\Delta_{N_{MAX}} = \int_0^1 dx \,| f(x) - f_{N_{MAX}}(x)| \quad (4.11)$$

on N_{MAX} in the case of expansion of the complete parton model function

$$f_2^{PM}(x) = x[x^2+(1-x)^2]\ln\frac{Q^2}{\Lambda^2_{PM}} + 8x^2(1-x) - x$$

with the sets of orthogonal polynomials defined by different weight functions:

1) □ - Chebyshev polynomials

2) ○ – $\omega(x) = x[x^2+(1-x^2)]$

3) Δ – $\omega(x) = x^{-0,1}\,(1-x)^{0,7\cdot 10^{-4}} - 10,4x^{10}\,(1-x)^{5,76}$

4) × – $\omega(x) = x^{0,99}\,(1-x)^{-0,14\cdot 10^{-5}}$

It can be seen by comparing the presented curves that in the case of the expansion of $f_1^{PM}(x)$ in Chebyshev polynomials this convergence with the growth of N_{MAX} (guaranteed mathematically) is more slow in practice than that one for Jacobi polynomials defined by the weight function $w^{\alpha\beta}(x)$ with arbitrary parameters α,β (found to be, $\alpha = 0.99$, $\beta = 5\cdot 10^{-5}$). At the same time the curve of N_{MAX} dependence of Δ_N for the case of $w(x,\alpha) = w_1(x)$ tells us that the computer error that is accumulated with enlarging of N_{MAX} cancels at large N_{MAX} the advantage of such an ideal weight function as compared with Jacoby polynomials.

Below, the dependence on N_{MAX} of the accuracy of the reconstruction of different functions used in the QCD-analysis of $F_2^{\gamma}(x,Q^2)$ will be shown. This reconstruction is performed with different types of polynomials defined by different weight functions. In Fig. 3 this dependence is shown for the complete parton function [15,16]

$$f_2^{PM}(x) = x\cdot[x^2+(1-x)^2]\cdot\ln\frac{Q^2}{\Lambda^2_{PM}} + 8x^2(1-x) - x\,, \qquad (4.12)$$

at $Q^2 = 10\ \mathrm{GeV}^2$ and $\Lambda_{PM} = 0.2$ GeV. The same is shown in Fig.4 for the approximate solution of the Altarelli-Parisi equation for the photon structure function found in [12] ($Y = \ln\frac{Q^2}{\Lambda^2}$)

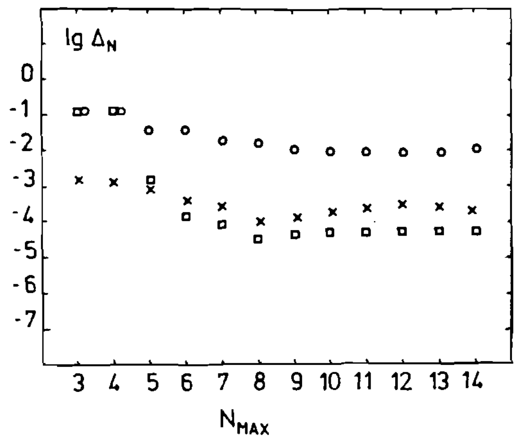

Fig. 4. The dependence of

$$\Delta_{N_{MAX}} = \int_0^1 dx\,| f(x) - f_{N_{MAX}}(x)| \quad (4.11)$$

on N_{MAX} in the case of expansion of the approximate solution of the Altarelli-Parisi equation $xq_2^{\gamma}(x,y)$ (4.13), found in [12] with the sets of orthogonal polynomials defined by 3 different weight functions:

1) 0 - Chebyshev polynomials

2) × – $\omega(x) = x^{6\cdot 10^{-4}}\cdot(1-x)^{0,067}$

3) □ – $\omega(x) = x^{-0,99}\cdot(1-x)^{-0,98}$

$$xq_1^{\gamma}(x,Y) = \frac{Bif_1^{PM}(x)}{1+cf_8(x)}\cdot Y\cdot\left(1-\left[\frac{\ln(Q_0^2)/\Lambda^2}{\ln(Q^2/\Lambda^2)}\right]^{1+f_3(x)}\right), \qquad (4.13)$$

$$Bi = 3\cdot\frac{\alpha}{2\pi}\cdot e_{qi}^2\,;\quad c = 8/b;\quad b = 33-2N_f\,;\quad f_3(x) = -2\ln(1-x) - x\frac{x^2}{2}\,;$$

at $Q^2 = 20\ \mathrm{GeV}^2$, $Q_0^2 = 5\ \mathrm{GeV}^2$ and $\Lambda = 0.2$ GeV.

From figs. 2, 3 and 4 it is seen that a high precision of the reconstruction can be achieved with a limited number of terms of the expansion in these polynomials if, first, the corresponding weight function would be luckily found or, second, the Jacobi polynomials would be used. It is also seen that the dependence of the precision of the reconstruction on the form of the weight function diminishes with the growth of N_{MAX} which agrees with the general theory (see also [10]).

In fig. 5 there is presented, on the background of experimrntal points [2], the behaviour of the photon structure function $\alpha^{-1}\cdot F_2^{\gamma}(x,Q^2)$ (as a function of x- and Q^2-variables) reconstructed from its moments (2.10) by using expansions (3.6) and (4.10) at $N_{MAX} = 7$ and fixed value of $\Lambda = 0.2$ GeV.

The full line corresponds to the part of the structure function that stems from the asymptotical part of the pointlike contribution, i.e. (2.18). The dashed and dashed-dotted curves correspond to the contribution of the complete pointlike function, i.e., to the sum of expressions (2.12) at $N_{MAX} = 7$, at different Q_0^2: $Q_0^2 = 3\ \mathrm{GeV}^2$ and $Q_0^2 = 5\ \mathrm{GeV}^2$.

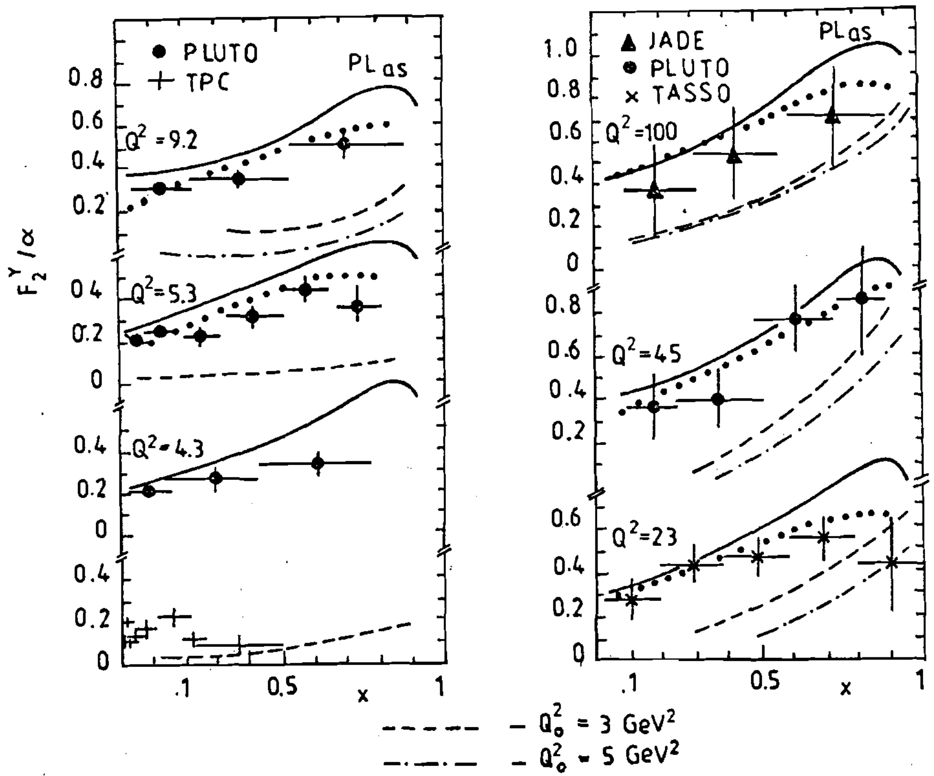

Fig. 5. Theoretical curves for $1/\alpha\, F_2^{\gamma}(x,Q^2)$ (for fixed Λ 200 MeV) shown on the background of the experimental points /2/. a) Full line corresponds to the asymptotics of the point-like part of $1/\alpha\, F_2^{\gamma}(x,Q^2)$ that stems from $F^{\gamma}_{2\,asym}(n,Q^2)$(2.18). b) Dashed and dashed-dotted lines show the behaviour of $1/\alpha\, F_2^{\gamma}(x,Q^2)$ corresponding to the point-like part of the full solution (2.12) (the fiest term in (2.12)) for different choices of $Q_0^2 = 3\ GeV^2$ and $Q_0^2 = 5\ GeV^2$, respectively. c) Dotted line shows the behaviour of $1/\alpha\, F_2^{\gamma}(x,Q^2)$ def. by full solution (2.12) with $q^{\gamma}_{NS}(x, Q_0^2 = 2\ GeV^2) = 0.1\, x^{0.01}\,(1-x)^{0.74}\,(1+x)$; $q^{\gamma}_{S}(x, Q_0^2 = 2\ GeV^2) = 3.3\, x^{2.6}\,(1-x)^{1.22}$; $q^{\gamma}_{G}(x, Q_0^2 = 2\ GeV^2) = 0.8\,(1-x)^{0.21}$ and fixed value of $\Lambda = 0.22\ GeV^2$ ($\chi^2_{d.f.} = 1.05$).

These curves are obtained in the reconstruction with the help of Jacobi polynomials with the parameters $\alpha = -0.95$; $\beta = -0.96$ and $N_{MAX} = 7$. It should be mentioned that the dependence of $F_2^{\gamma}(x,Q^2)$ on the parameters of the weight function is very weak. The structure function varies with their varying in the limit less than one per cent, which is quite enough at the present level of precision of the existing data on F_2^{γ} (to be better called the uncertaint instead of precision).

In the same fig.5 for illustration of the work of the proposed method, we show by the dotted line the structure function obtained by the reconstruction with formulae (3.6) and (4.11) with $N_{MAX} = 7$ and the complete expression for the moment (2.21), i.e., containing the nonzero hadronic part with $q_i^{\gamma}(n,Q^2) \neq 0$.

The moments $q^{\gamma}_i(n,Q_0^2)$ have been chosen as in /10/ in the form of the following integrals of quark and gluon distributions at $Q_0^2 = 3\ GeV^2$:

$$q_i^{\gamma}(n,Q_0^2) = \int_0^1 dx \cdot x^{n-1} \cdot q_i^{\gamma}(x,Q_0^2)\,, \qquad (4.14)$$

where the function $(i = NS, S, G)$

$$q_i^{\gamma}(x,Q_0^2) = A_1^i \cdot x^{A_2^i} \cdot (1-x)^{A_3^i}\,(1 + A_4^i x)\,, \qquad (4.15)$$

is defined by 4 free parameters A^i.

CONCLUSION

From the review given in the present paper we see that the photon structure function $F_2^{\gamma}(x,Q^2)$ for which QCD, due to the presence of the point-like part, predicts a specific growth with Q^2 is a very interesting object for the experimental study. Though the present data, in principle on the qualitative level, support this QCD prediction, nevertheless, they have such large errors and uncertainties (for example, in the position of experimental points on the x-axis (see fig. 5) that it is too early to speek about any qualitative check of QCD with these data. Especially, it is right because to have a possibility of speaking definitely about the value of extracted Λ, one has to use the second order QCD formulae which have a doubtful practical meaning with the present level of precision of experimental data. That is why new measurements of the photon structure function $F_2^{\gamma}(x,Q^2)$, especially at large Q^2, avaliable at LEP, would be of great importance for the QCD development.

In the present paper we have proposed a new method of QCD-analysis of the photon structure function based on the method used previously in QCD-analysis of lepton-hadron structure functions /10/, and particularly, of BCDMS data /11/. This method allows us to extract the values of Λ and of parameters of quark and gluon distributions with a high accuracy.

The advantage of the proposed method consists in the possibility of working, as in the method of analysis based on the Altarelly-Parisi equation, directly with the structure functions in the x-space. And at the same time in this method one works not with the numerical solutions of this equation but uses the explicit analytic solutions of renormalization-group evolution equations for the moments. The use of the analytical solutions allows one to explicitly study the influence of different parameters, used in QCD-analysis, on the form and behaviour of the photon structure functions. A separate publication will be devoted to this subject.

The authors thank P.N.Bogolubov and V.G.Kadyshevsky for interest in the work and valuable discussions.

REFERENCES

1. Isgur N., Llewellyn Smith C.H. - CERN-TH.5013/88, CERN, Geneve, 1988.
2. Aihava H. et al. (TPC Collab.) - Phys. Rev. Lett., 1987, 58, p.97; Z. Phys., 1987, C34, p.1.
   Berger Ch. et al. (PLUTO Collab.) - Nucl. Phys., 1987, B281, p.365.
   Althoff M. et al. (TASSO Collab.) - Z. Phys., 1986, C31, p.527.
   Bartel W. et al. (JADE Collab.) - Z. Phys., 1984, C24, p.231.
3. Berger Ch., Wagner W. - Phys. Rep., 1987, 146, p.1.
   Olsson J. - 1987 Int. Symp. on Lepton and Photon Inter. at High Energies, Hamburg (DESY 87-136).
4. Physics at LEP, v.2 (Edited by J.Ellis and R.Peccei) CERN 86-02, 1986.
   Proc. of ECFA Workshop on LEP 200, v.1 (Edited by A.Bohm and W.Hoogland) CERN 87-08; ECFA 87/108, 1987.
5. Witten E. - Nucl. Phys., 1977, B120, p.189.
6. Bardeen W.A., Buras A.J. - Phys. Rev., 1979, D20, p.166.
7. Duke D.W., Owens J.F. - Phys. Rev., 1980, D22, p.2280.
8. Gluck M., Reya E. - Phys. Rev., 1983, D28, p.2749.
   Gluck M., Grassie K., Reya E. - Phys. Rev., 1984, D30, p.1447.
9. Rossi G. - Phys. Rev., 1984, D29, p.852.
10. Krivokhizin V.G. et al. - Z. Phys., 1987, C36, p.51; JINR Preprint E2-86-564, Dubna, 1986.
11. Benvenutti A.C. et al. (BCDMS Collab.) - Phys. Lett., 1988, 195, p.97.
    Benvenutti A.C. et al. (BCDMS Collab.) - CERN-EP-89-07, Geneva, 1989.
12. Kapusta F. - LPNHEP 88-05, Paris, 1988.
13. Witt R.J. et al. - Phys. Rev., 1979, D19, p.2046.
14. Uematsu T., Walsh T.F. - Nucl. Phys., 1982, B199, p.93.
15. Brodsky S.J., Kinoshota T., Terazawa - Phys. Lett., 1971, 27, p.280.
    Walsh T.F. - Phys. Lett., 1971, 36B, p.121.
    Carlson C.E., Tung W. - Phys. Rev., 1971, D4, p.2873.
    Walsh T.F., Zerwas P. - Nucl. Phys., 1972, B41, p.551.
    Terazawa H. - Rev. Mod. Phys., 1973, 43, p,615.
    Kingsley R.L. - Nucl. Phys., 1973, B60, p.45.
    Walsh T.F., Zerwas P. - Phys. Lett., 1973, B44, 195.
    Ahmed M.A., Ross G.G. - Phys. Lett., 1975, B59, p.369.
    Budnev V.M. et al. - Phys. Rep., 1975, C15, p.182.
16. Hill Ch.T., Ross G.G. - Nucl. Phys., 1979, B148, p.373.
17. Buras A.J. - Rev. Mod. Phys., 1980, 52, p.199.
18. Llewellyn C.H. - Smith - Phys. Lett., 1978, B79, p.83.
    Frazer W.R., Gunion J.F. - Phys. Rev., 1979, D20, p.147.
19. Bardeen W.A. - Proc. 1981 Int. Symp. on Lepton and Photon Inter. at High Energies, Bonn, p.432.
20. Bardeen W.A. - VI Int. Workshop on Photon-Photon Collisions (Lake Takoe, 1984); World Sciencific, Singapore, 1985; Fermilab-Conference 84/122-1, 1984.
    Zerwas P.M. - XV Int. Symp. on Multiparticle Dynamics, Lund, 1984.
21. Antoniadis I., Grunberg - Nucl. Phys., 1983, B213, p.445.
22. Antoniadis I.,Baulieu L., Lacaze R. - Phys. Rev., 1983, B125, p.92.
    Grunber G. - VII Int. Workshop on Photon-Photon Collisions, Paris, 1986, World Scientific, Singapore, 1987.
    Antoniadis I., Marleau L. - Phys. Lett., 1985, B161, p.163.
23. Marleau L. - Phys. Rev., 1987, D36, p.2001.
24. Steinman J.S. (TPC/2 Collab.) - UCLA-HEP-88-004).
    Berger Ch. et al. (PLUTO Collab.) - see ref.
25. Field J.H., Kapusta F., Poggioli L. - Phys. Lett., 1986, B181, p.362; Z. Phys., 1987, C36, p.121.
26. Bardeen W.A., Buras A.J. - Phys. Lett., 1979, B86, p.61.
27. Devoto A. et al. - Phys. Rev., 1983, D27, p.508.
28. Virchaux M., Ouraou A. - Preprint DPhPE 87-15, 1987.
29. Yndurain F.J. - Phys. Lett., 1978, B74, p.68.
30. Gonzales-Arroyo A., Lopez C., Yndurain F.J. - Nucl. Phys., 1979, B153, p.161; ibid. 1979, B159, p.512; ibid. 1980, B174, p.474.
31. Gonzales-Arroyo A., Lopez C. - Nucl. Phys., 1980, B166, p.429.

32. Aubert J.J. et al. - Phys. Lett., 1982, B114, p.291.
33. Furmansky W., Retronzio R. - Nucl. Phys., 1982, B195, p.237.
34. Bergsma F. et al. (CHARM Collab.) - Phys. Lett., 1983, B123, p.269.
35. Allaby J.V. et al. (CHARM Collab.) - Phys. Lett., 1987, p.381.
36. Parisi G., Sourlas N. - Nucl. Phys., 1979, B151, p.241.
37. Abramoviz H., Stegun I. - Handbook of Math. Functions, Doner, New York, 1963.
38. Barker I.S., Langensiepen C.S., Shaw G. - Nucl. Phys., 198 , p.61.
39. Barker I.S., Martin B.R., Shaw G. - Z. Phys., 1983, C19, p.147.
Barker I.S., Martin B.R. - Z. Phys., 1984, C24, p.255.
Martin B.R., Shaw G. - Z. Phys., 1986, C33, p.99.

Received by Publishing Department
on September 14, 1989.

Курлович С.П., Сидоров А.В., Скачков Н.Б. E2-89-655

Фотонная структурная функция и ее КХД-анализ с помощью разложения по обобщенным ортогональным полиномам

Дается обзор современного состояния описания структурной функции фотона в рамках КХД и обсуждаются перспективы его исследования в связи с пуском ускорителя LEP. Предложен новый подход к этой задаче, основанный на методе разложения структурной функции по ортогональным полиномам Якоби. Рассматривается новый класс ортогональных полиномов, являющихся обобщением полиномов Якоби, который вместе с полиномами Якоби применен для реконструкции КХД-выражений для структурных функций, исходя из полученных в рамках КХД аналитических выражений для их моментов. Работа метода проиллюстрирована на примере КХД-анализа в лидирующем порядке теории возмущений существующих в настоящее время экспериментальных данных.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1989

Kurlovich S.P., Sidorov A.V., Skachkov N.B. E2-89-655

Photon Structure Function and its QCD-Analysis by Expansion in Generalized Orthogonal Polynomials

We review a modern state of the description of the photon structure function in the framework of QCD and discuss its further investigation in relation with the future start of the LEP accelerator. A new approach to this problem based on the expansion of the structure function into the Jacobi polynomials is proposed. The new class of orthogonal polynomials, which is a generalization of Jacobi polynomials, is applied together with the Jacobi polynomials to reconstruction of the QCD-structure function using analytical expressions for their moments calculated in QCD. The efficiency of this method is demonstrated in the leading order to the QCD-analysis of some experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1989