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PREDICTING STABLE QUARK CORES IN NEUTRON STARS FROM A UNIFIED DESCRIPTION OF QUARK-HADRON MATTER

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1. Introduction

Already shortly after the invention of the notion of quarks, the question has been investigated whether the dense neutron star interior may serve as a place where the phase transition from ordinary hadron matter to quark matter in bulk can occur (see [1] for refs.). Using a two-phase description with bag model type equation of state for the quark phase, and advanced nuclear matter approaches for the hadron phase, most of the previous authors come to the conclusion, that in stable neutron stars the critical density for the deconfinement transition is not reached [2]. Moreover, even if the neutron star is capable of producing a quark core [3], the stability of the resulting object has to be carefully examined [4]. This central issue is often neglected, as well as the possible occurrence of stable stellar objects with quark cores beyond the stable neutron star peak [5] is mostly overlooked in discussing this topic.

The recent observation of an extremely short-pulsed radiation from a possible neutron star born in SN 1987A [6] has renewed the interest in massive quark cores in neutron stars, since only highly compact objects can fulfill the requirements of possessing large enough mass and rotate with subluminal surface velocity.

Here we present an investigation of the above mentioned topic by employing an improved equation of state. While previous authors rely on different parameterizations for the quark and the hadron states of strongly interacting matter, we use here for the first time an equation of state for neutron star matter on the quark level of description, where the nucleons are viewed as three- quark bound states. The basic ingredient of the present approach is the treatment of the many-quark system with confinement interaction, which can be accomplished by generalizing the string flip approach by Lenz et al. [7] to an arbitrary number of quark sorts and by applying the cluster-Hartree-Fock approach [8] within the framework of thermodynamic Greens functions. Details of the resulting approach can be found in ref. [9], where especially the deconfinement transition in isospin-symmetric matter is considered.

2. The cluster-Hartree-Fock approach

The starting point of our consideration is a cluster decomposition for the density which is successfully applied in the cases of particle clustering and phase transitions in nuclear matter (see, e.g., [10] for a review) and which can be used for a many-quark system as well. In what follows we consider non-strange matter with quasi-free quarks and their respective color-neutral nucleonic bound-states n = udd and p = uud. (The strange matter problematic deserves a separate investigation.) Allowing for isospin asymmetry the density of up quarks can be written as

$$\rho_u(T,\mu_u,\mu_d) = \Omega^{-1} \sum_1 f_u(E(1) + \Delta^H(1)) \tag{1}$$
$$+ \Omega^{-1} \sum_{123} \sum_{\nu_n P} f_n(E^0_{\nu_n P} + \Delta^{Pauli}_{\nu_n P}) \; |\Psi_{\nu_n P}(123)|^2$$
$$+ 2\,\Omega^{-1} \sum_{123} \sum_{\nu_p P} f_p(E^0_{\nu_p P} + \Delta^{Pauli}_{\nu_p P}) \; |\Psi_{\nu_p P}(123)|^2 .$$

For the density of down quarks the same formula applies with the replacements $u \Leftrightarrow d$ and $n \Leftrightarrow p$. In eq.(1) the eigenvalues and antisymmetrized wavefunctions of the neutron (proton) are denoted by $E_{\nu_n P}$ ($E_{\nu_p P}$) and $\Psi_{\nu_n P}$ ($\Psi_{\nu_p P}$), respectively, where P is the total momentum and ν_n (ν_p) stands for the internal quantum number. The label numbers 1,2,3 stand for momentum, spin and isospin of the particles 1,2,3. The distribution function for quasi-free quarks

$$f_q(E) = [1 + \exp((E - \mu_q)/T)]^{-1} \tag{2}$$

and for three-quark clusters

$$f_p(E) = [1 + \exp((E - 2\mu_u - \mu_d)/T]^{-1}, \tag{3'}$$
$$f_n(E) = [1 + \exp((E - 2\mu_d - \mu_u)/T]^{-1}, \tag{3''}$$

are determined by the temperature and the quark chemical potentials μ_q, q = u,d.

According to eq.(1) the quarks and the nucleons are described as quasi-particles which obey an energy shift due to the influence of the surrounding matter. For quarks the Hartree shift Δ^H is determined in an adiabatic approximation from the potential energy of a quark configuration, where only nearest neighbors are allowed to interact in order to avoid the unphysical color van der Waals forces (for details cf. [9]).

3. Effects of the Pauli blocking

The Pauli blocking shift Δ^{Pauli}, which is only operative for the clusters, is due to the fact that the occupation of phase space will prevent the formation of bound states at high densities as a consequence of the Pauli exclusion principle. The always positive contribution to the nucleon energy on the quark level of description serves as an explanation of the hard-core NN interaction potential [9,11]. Since the available phase space volume depends very sensitively on the isospin

content, measured e.g., by the proton fraction $x = Z / A = \rho_p / (\rho_n + \rho_p)$, of the nuclear matter, the Pauli shift is responsible for the symmetry energy, thus being of special interest for the consideration of the deconfinement transition in neutron stars.

In order to investigate this subject let us consider the T = 0 limit of eq.(1). The Pauli blocking then is obtained by evaluating the correct antisymmetrization of the six-quark wave functions with respect to one- and two-quark exchanges between the three-quark wavefunctions of the nucleons. Evaluation up to second order in the range parameter b of the three-quark wave packets, taken as Gaussians, yields

$$\Delta^{Pauli}_P = (24\sqrt{3\pi})^{-1} \frac{b}{m} \sum_{\nu'} [15\, a_{\nu\nu'}\, P_F(\nu')^3 +$$
$$\frac{17}{12}\, b_{\nu\nu'}\, b^2\, (P^2 + P_F(\nu')^2) P_F(\nu')^3], \tag{4}$$

where the coefficients $a_{\nu\nu'}$ and $b_{\nu\nu'}$ are obtained from a calculation of the matrix elements of one- and two-quark exchange operators with respect to the spin/flavor/color degrees of freedom. For neutron matter ($\nu' = n_\uparrow, n_\downarrow$) one gets $a_{\nu\nu'} = -\frac{1}{3}$ and $b_{\nu\nu'} = \frac{49}{3}$, whereas for symmetric nuclear matter $a_{\nu\nu'} = -1$ and $b_{\nu\nu'} = 31$.

The Fermi momenta of the nucleons depend on the total baryon density ρ and on the proton fraction x. In the limiting cases of (i) neutron matter ($x = 0$, $\gamma = 2$) and (ii) symmetric nuclear matter ($x = 0.5$, $\gamma = 4$) the Fermi momenta are directly connected with the nucleon densities via

$$P_F(\rho_i) = [(6\pi^2/\gamma)\; \rho_i]^{1/3}, \quad i = n,\, p, \tag{5}$$
$$\rho_p = x\, \rho, \qquad \rho_n = (1-x)\; \rho. \tag{6}$$

Inserting in eq.(4) one arrives at a density dependent Pauli blocking shift for the respective nucleon densities (see also [11,13])

$$\Delta^{Pauli}_\nu(\rho, x=0) = 5\; (24\sqrt{3\pi})^{-1} \frac{b}{m} [-3\pi^2\rho + \tfrac{51646}{6975}\, b^2\, (3\pi^2\rho)^{5/3}], \tag{7}$$

$$\Delta^{Pauli}_\nu(\rho, x=0.5) = 5\; (8\sqrt{3\pi})^{-1} \frac{b}{m} [\,-1.5\pi^2\, \rho + \tfrac{1054}{225}\, b^2\, (1.5\pi^2\, \rho)^{5/3}].$$

These shifts show a strong repulsive behavior, even at subnuclear densities. To obtain the saturation property of symmetric nuclear matter one has to introduce mesonic degrees of freedom which is on the quark level not yet achieved. So we are forced to introduce the one-pion exchange [14] by hand, by adding the Hartree energy of the one-pion exchange potential [15] which contributes as a self-energy shift to the one-particle energy in eq.(1) via $E^o_{\nu_{n,p}P} \Rightarrow E^o_{\nu_{n,p}P} + E^{opep}$,

$$E^{opep} = -\, 4\pi\, f_\pi^{\,2}\, \rho\, \lambda^{-2}, \tag{8}$$

where f_π = 93 MeV is the pion decay constant and λ^{-1} = 1.41 fm is the Compton wavelength for nucleons.

Now we are in the position to calculate the specific energy per baryon which, at T = 0, coincides with the density of the Helmholz free energy and can be obtained from eq.(1) by inverting $\rho(\mu)$ and using

$$E(\rho,x) = \rho^{-1} \sum_{i=n,p} \int_0^{\rho'} d\rho' \; \mu_i(\rho',x) \; c_i(x) \qquad (9)$$

where $c_p = x$, $c_n = (1-x)$. The evaluation of such a formula for arbitrary values of x, x = 0...1, is quite lengthy. Here we want to illustrate the result by considering the symmetry energy E_s defined as

$$E_s(\rho) = E(\rho,x=0) - E(\rho,x=0.5), \qquad (10)$$

where

$$E(\rho,x=0) = M + \frac{3}{10M} (3\pi^2 \rho)^{2/3} - \alpha_1 \rho + \beta_1 \rho^{5/3}, \qquad (11)$$

$$E(\rho,x=0.5) = M + \frac{3}{10M} (1.5\pi^2 \rho)^{2/3} - \alpha_2 \rho + \beta_2 \rho^{5/3},$$

are the energies per particle for neutron matter and symmetric nuclear matter with the parameters α_1 = 1.0 b/m + 6.28 f_π^2/λ^2, β_1 = 53.4 b^3/m, α_2 = 1.51 b/m + 6.28 f_π^2/λ^2, β_2 = 31.9 b^3/m (M = 939 MeV and m = 350 MeV denote the nucleon and the constituent quark masses, respectively). The range parameter of the Gaussian wave function is chosen as b = 0.59 fm [12].

To include the effect of β equilibrium with electrons in charge neutral neutron star matter we insert again eq. (5) in (4) without specifying the value of x but instead using the relations

$$(1 - x)^\alpha + x^\alpha \cong 2^{-\alpha} [2 + \alpha(\alpha - 1)(1 - 2x)^2], \qquad (12)$$

$$x (1 - x) = [1 - (1 - 2x)^2]/4$$

in order to evaluate eq.(4) with respect to the asymmetry ratio $(1-2x)^2$. Finally we arrive at a familiar approximation for the specific energy of neutron-rich matter (cf., e.g., [16,17])

$$E(\rho,x) = E(\rho,x=0.5) + E_s(\rho) \; (1 - 2x)^2, \qquad (13)$$

where the nuclear symmetry energy is obtained only in terms of the parameters of the underlying quark model

$$E_s(\rho) = \frac{1}{6M} (\tfrac{3}{2}\pi^2\rho)^{2/3} + (8\sqrt{3\pi})^{-1} \frac{b}{m} [\frac{5\pi^2}{4}\rho + \frac{17}{3} (1.5\pi^2)^{5/3} b^3 \rho^{5/3}]. \qquad (14)$$

In this way, the present approach can be used to predict properties of bulk nuclear matter by employing a quark potential model which is capable of reproducing the known nuclear structure effects and NN scattering data as well (see, e.g., [11]).

In β equilibrium the proton fraction becomes, of course, density dependent in order to fulfill the requirement of charge neutrality for the nucleon - electron system. In particular, with

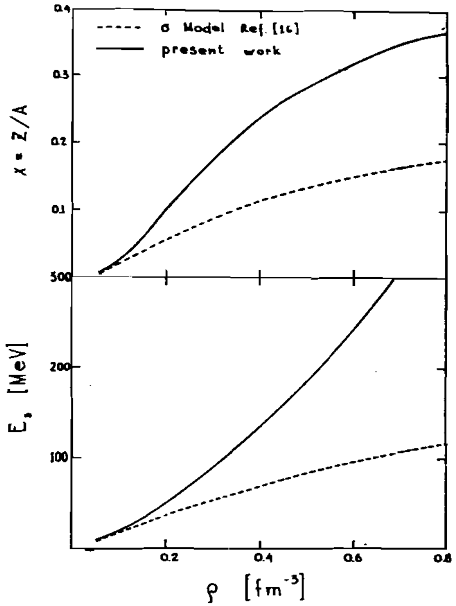

Fig.1. The proton concentration (top) and the symmetry energy E_s (lower part) as function of the baryon density (a- present work relying on Pauli blocking effects; b- σ model calculations of ref. [18]).

$$E_{tot}(\rho,x) = E(\rho,x) + E_{el}(\rho,x) \qquad (15)$$

the function $x(\rho)$ is determined by

$$(\partial E_{tot}/\partial x)_{\rho=const} = 0. \qquad (16)$$

With $E_{el}(\rho,x) = (3/4) \; x^{4/3} \rho^{4/3} (3\pi^2)^{1/3}$ this leads to

$$x^{1/3}/(1 - 2x) = 4E_s(\rho) \; (3\pi^2 \rho)^{-1/3}. \qquad (17)$$

The function $x(\rho)$ is shown in fig.1 together with the symmetry energy. Especially, when comparing with the σ model calculation of ref. [18] which is displayed for comparison, the close correlation between symmetry energy and proton fraction can be seen. Note that a value of x ≅ 1/3 at densities slightly above the nuclear saturation density ρ_0 is favorable to obtain supernova explosions in modeling the stellar collapse [18].

4. The deconfinement transition

At high densities the above made assumption of neglecting the quasi-free quark contribution in eq.(1) is no longer valid. At densities 3 - 5 ρ_0 a phase of deconfined (i.e., quasi-free) quarks is expected to occur being the QCD analogous of the Mott transition in plasma physics (see [8,9]).

The free energy of the quasi-free quark phase, corresponding to the quadratic confinement potential [12], has been evaluated in ref.[9]

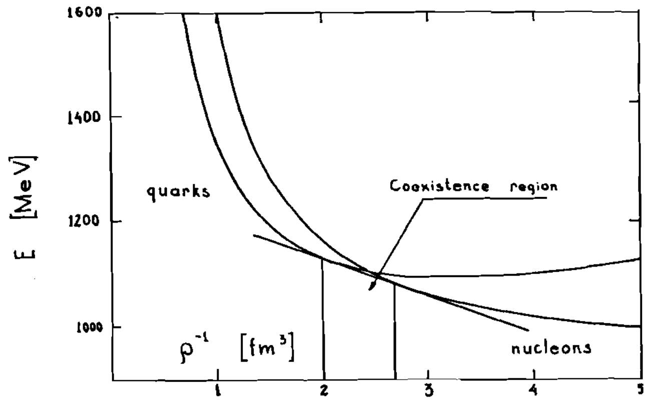

Fig.2. The energy per particle E/A for quark matter and β-stable neutron matter as function of the baryon density. The double-tangent construction is indicated and the resulting coexistence region, as well.

for isospin-symmetric matter. It can be used in the present calculation for β-stable quark matter since an isospin dependence enters only via the different fillings of the up/down quark Fermi seas, which result in negligible effects for the free energy in the phase transition region [13].

The quark - hadron phase transition is obtained from a double-tangent construction for the free energy (see fig.2), and a coexistence region of quark and β-stable neutron-rich matter is obtained for 2.3. $\leq \rho/\rho_0 \leq$ 3.1. Whereas in symmetric nuclear matter, at T = 0, the phase border is not reached until 4...5 ρ_0 [9], in asymmetric matter the phase transition is predicted at about twice nuclear saturation density, so that probably inside neutron stars, where these densities are reached, the deconfinement transition can take place.

5. Stable neutron stars with quark cores

We have integrated the Tolman-Oppenheimer-Volkoff equations [19] with the equation of state for the pressure

$$p(\rho,T=0) = \rho^2 \, (\partial f(\rho,T=0)/\partial\rho) \qquad (18)$$

(for details cf. [13,15]). One important result of the calculations is the mass of the neutron stars as a function of the central density which is displayed in fig.3. The resulting maximum mass can be compared with astronomical observations of neutron star masses which point to a

lower limit of the maximum mass of $M_{max} \cong 1.85\ M_\odot$ [20]. In fig.3 one observes that the effect of the phase transition lowers the maximum mass. We obtain for neutron stars with quark cores a maximum mass of 2.2. $M_\odot$. The above mentioned problem of stability has been analyzed, and it is found that the presented calculations predict stable neutron stars with radii of about 15 km and with quark cores with typical radii in the order of 3 km. These values are compatible with the pulse frequency observed from a possible compact remnant in SN 1987A [6] in that sense that the surface does not rotate with supraluminal velocity.

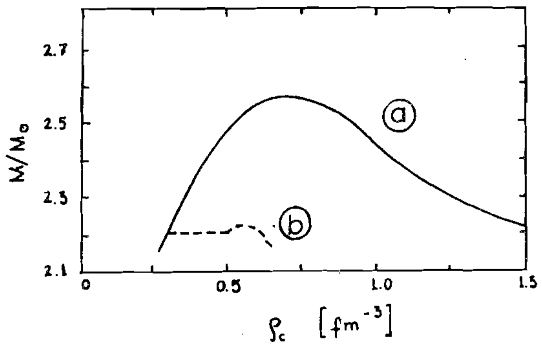

Fig.3. The neutron star mass as function of the central density (a- without deconfinement; b- allowing for deconfinement).

Otherwise, as can be seen in fig.3, the quark-core neutron star peak is rather small, and its persistence may depend on small correction effects not included in the present investigations. At present we are refining our approach along the following lines:
(i) inclusion of strangeness degrees of freedom,
(ii) fully selfconsistent solution of the set of thermodynamic equations in order to discuss the phase diagram in the full thermodynamical parameter space spanned by up, down and strange flavor degrees of freedom,
(iii) checking the sensitivity of the results obtained for different parameterizations of the interquark potential model.

6. Summary

In summary we extend a unified potential model description, wherein the hadron phase and the quark phase are considered on an equal footing, to isospin asymmetric matter. Particular emphasis is devoted to the hadron equation of state. Our approach predicts stable neutron stars with quark cores being compatible with both the observed neutron star masses and the constraints on the neutron star radii by the possibly observed pulsar frequency.

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Блашке Д., Товмасян Т., Кэмпфер Б. E2-89-651

Предсказание стабильных нейтронных звезд в рамках единого описания кварк-адронной материи

Описывается фазовый переход кварки – адроны в ядерном веществе при нулевой температуре в потенциальной модели с помощью кластерного подхода Хартри-Фока. Несжимаемость и энергия асимметрии ядерной материи обусловлены принципом Паули для нуклонных составляющих. Область сосуществования кварк-нейтронного вещества перекрывает значения 2,3-3,1 ρ_0. Путем интегрирования уравнений Толмана-Оппенгеймера-Волкова получена стабильная ветвь нейтронной звезды с максимальной массой 2.20 и радиусом кварковой сердцевины около 3 км.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

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Predicting Stable Quark Cores in Neutron Stars from a Unified Description of Quark-Hadron Matter

The quark-hadron phase transition in isospin-asymmetric nuclear matter at zero temperature is described within a potential model using a cluster-Hartree-Fock approach. Stiffness and asymmetry energy of the nuclear matter are caused by the Pauli exclusion principle for the nucleon constituents. The coexistence region of quark - neutron matter covers the range 2,3-3,1 ρ_0. A stable neutron star branch, with maximum mass of 2.20 $M_{\odot}$ and with quark core radii of about 3 km, is obtained by integration of the Tolman-Oppenheimer-Volkoff equations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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