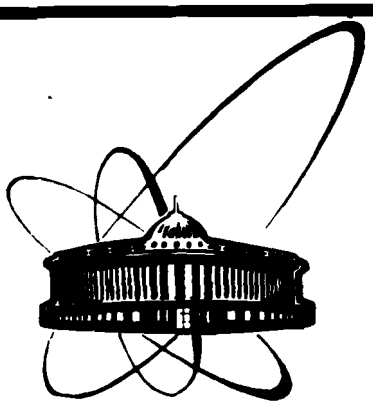

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THE VOLTERRA MODEL AND QUARK HADRONIZATION INTO MULTICOMPONENT HADRON SYSTEM

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1. - Introduction

The Problem of the parton (quark and gluon) hadronization is not still finally solved in the framework of the Quantum Chromodynamics (QCD) (see, for instance, ([1,2])). Especially multicomponent product (resonances) emission needs explanation.

In the previous article ([3]) we have proposed one more model, and as a result, the problem of the famous "mystery" for negative binomial distribution involving the KNO-scaling violation (see ([4])) was explained.

The KNO relation is well described by gamma(Γ)-distribution in hadron-hadron scattering processes

$$(1) \qquad \langle n_c \rangle \cdot \sigma_{n_c} / \sigma \equiv \psi(z_c) = \frac{a^a}{\Gamma(a)} z_c^{a-1} \exp(-a\, z_c),$$

where the parameter a must be fixed (a=4) and defined by the Wroblewski rule ([5])

$$(1a) \qquad D_c^2 = \frac{1}{a} (\langle n_c \rangle - \varepsilon_c)^2 .$$

However, this parameter is "mysteriously" changed ([5]) with changing experimental conditions, particularly, in limited intervals of pseudorapidity distribution $|\eta| < \eta_c$.

Solution of this problem consists in the generalization of the Γ-distribution (1) for generating a few (ν) components (resonances) to the following expression:

$$(2) \qquad \prod_{i=1}^{\nu} \langle n_i \rangle \cdot \sigma(n_1,\ldots,n_\nu)/\sigma = f(z_1,\ldots,z_\nu) \Rightarrow t^{a-\nu} \exp[-(a/\nu)t],$$

where $t = \sum_{i=1}^{\nu} z_i$, $z_i = n_i/\langle n_i \rangle$, $\sigma(n_1,\ldots,n_\nu)$ and σ are exclusive and inclusive cross-sections, respectively, for the reaction with $n_1, n_2, \ldots, n_\nu$ hadron multiplicities of the ν-type.

Generally speaking, the shape of the Γ-distribution (2) is strongly modified by averaging over the $(\nu-1)$-multiplicities $n_2,\ldots,n_\nu$ ([6]). We must get rid of this modification assuming that under certain experimental conditions (for instance, in different intervals of pseudorapidity η) a few (ν_1)

components are generated so that they are distributed as the Kronecker $\delta(z_i-\alpha_i)$ function. Here α_i are resonance decay parameters with $i=2,\ldots,\nu_1$. Then, the KNO-function (1) for charged particles is modified as follows:

$$(3)\qquad \Psi(z_c) \Rightarrow (z_c+\alpha)^{\nu_1(a_e-1)} \exp[-a_e(z_c+\alpha)]$$

where $\alpha \cong \sum_{i=1}^{\nu} \alpha_i$.

The comparison of formula (3) with the experimental data ([4]) confirms our hypothesis about a few number of correlated components at hadronization of quarks (see the Table).

Below (sect. 2) we shall give the meaning to this hypothesis in the framework of the model of competing species for the same food. Now we should like to stress that if we use only the Volterra and other nonlinear models of interacting species in biological population ([7,8]), then discovery of the dependence of function (3) and other characteristics on the number ν of interacting components is a new phenomenon and though these models are long ago known it has not been learned until present time. The present status of the problem of confinement and colour forces does not contradict, for instance, an uncertainty between the number ν of components and the effective correlation intensity a_e measured by formula (3) that their production will be constant:

$$\nu_1 . a_e = a \Rightarrow 4 \ .$$

We are glad to note that now we can explain analogous enigma for other relations ([9]). As an example, we discuss the relation between an average number $\langle n_i(n_1)\rangle$ of particles of the i-th type and the number $n_c = n_1$ of charged particles associated with them

$$(4)\qquad \langle n_i(n_1)\rangle = \sum_{n_2 \ldots n_\nu} n_i \sigma_{n_1 \ldots n_\nu} \Big/ \sum_{n_2 \ldots n_\nu} \sigma_{n_1 \ldots n_\nu} \ .$$

It is difficult to enumerate all 15 year attempts to explain the fact that the relevant experimental data (see, for instance, ([10])) can be fitted by the linear function

$$(5)\qquad \langle n_i(n_c)\rangle = A_i n_c + B_i$$

and that the parameters A_i and B_i depend on the total energy $s^{1/2}$ of colliding particles.

This natural change of the parameters was explained by using the multi-dimensional KNO-function (2) in formula (4), which led to an increased number of correlated components ([6,9,11]).

It is surprising that the parameter a has a tendency to decrease as energy increases. This may be caused by the above-mentioned generation of a few (ν_1) components with local or instantaneous (Kronecker delta) contribution to averaging when a large number ($\nu_2 \gg 1$) of components with multiplicity lags (nonlocal contribution) is also present. Thus, $\nu=\nu_1+\nu_2$ components are generated but only ν_2 take place in multiplicity averaging. Then, relation (4) is

$$(6)\qquad \frac{\langle n_i(n_1)\rangle}{\langle n_i\rangle} = z_1 \frac{\Psi(\nu_2, a-\nu_1+1, \frac{a}{\nu_1+\nu_2} z_1)}{\Psi(\nu_2-1, a-\nu_1, \frac{a}{\nu_1+\nu_2} z_1)}$$

The saturation takes place in the high energy limit when $\nu_2 \gg 1$, i.e. the right-hand side of (6)

$$\left\{\frac{z_1}{\bar{a}}\right\}^{1/2} \frac{K_{a-\nu_1}(2\sqrt{\bar{a} z_1})}{K_{a-\nu_1-1}(2\sqrt{\bar{a}\, z_1})}$$

depends only on the effective constant $\bar{a}_e = a-\nu_1$, which is the explanation of its changing as a result of uncertainty principle: $\bar{a}_e+\nu_1 = a \Rightarrow 4$. Here Ψ and K are confluent hypergeometric and modified Bessel functions, respectively.

It is interesting to note that the dependence on $\bar{a}_e$ and ν appears also in other relations. Exactly, it was discussed in ([9]) that the behaviour of the normalized semi-inclusive spectrum density $\frac{1}{\sigma_n}\frac{d\sigma_n}{d\eta}$ can be explained at $s^{1/2}=540$ GeV. The necessary condition for this is $\bar{a}_e = a-\nu < 1$ allowing the description of the dip-effect observed at small multiplicity values.

Earlier ([12]), intensification of the famous "sea-gull" effect was also observed as the number ν of correlated components was growing.

The main purpose of this article is the investigation of an analogous phenomenon in the framework of the set of differential(D) equations ([3])

$$(7)\qquad \frac{d}{d\tau}\langle n_i\rangle = -\sum_{k=1}^{\nu} N_k D_{ik}, \qquad i=1,2,\ldots,\nu,$$

where N_i is the quark number in i-th type hadron, $D_{ik} = \langle n_i n_k\rangle - \langle n_i\rangle\langle n_k\rangle$ form the matrix of correlation between multiplicities of hadrons of the same and different type. The parameter τ depends either on a given momentum p of an inclusively separated hadron in the case of associated multiplicities ($\tau = \ln(pp_0/p_0^2)$) or on the total energy in the case of mean multiplicities ($\tau = \ln s/2m_p^2$).

It should be noted, that the mean (associated) multiplicities have already been described by the Volterra-type integral equation obtained from the Altarelli-Parisi formalism ([13]) (see ([1,2])).

Without a detailed analysis we can say that the set of eqs. (7) may turn out to be more admissible and convenient in the next sense: firstly, it has no relation to such non-perturbative objects as a parton distribution (fragmentation) functions and has no links with such statements as the theorem about factorization of hard and soft parts of the considered processes ([14]) and so on. Secondly, as under hadronization we understand mainly the division of the total number of available quarks (n>>1) between a few species of hadrons

$$(8)\qquad n = \sum_{i=1}^{\nu} N_i n_i,$$

we do not load our model with preconfinement ([15]) or colourless string ([16]) and others.

In sect. 2 we discuss an analogy between the system of eqs. (7) and the Volterra and other nonlinear models ([7]) under some special parametrization of the correlation matrix $D^2=(D_{ik})$. Further, in sect.3 the simple algorithm is given for analytical calculation of a high (ν) order D-eq. as a resultant of the system (7). In sect. 4, the methods for solving the obtained equations describing associated multiplicities are discussed.

2. - Quark hadronization as the Volterra model of competing species

The present article is stimulated by surprising analogy between D-equation (7) and the Volterra and other nonlinear models of interacting species for biological population([7,8]).

In order to make this analogy deeper we take some concepts from this model. Thus, if we are speaking about "quark food" divided among the competing hadron components by formula (8), then "quark- (instead of bio-) mass" would be

$$M = \sum_{i=1}^{\nu} N_i\langle n_i\rangle.$$

It is not difficult also to recognize the equations. The cross section σ_n for semi-inclusive generation of n-partons at large n >>1 obeys the well-known Malthus law ([17])

$$(9)\qquad \frac{d}{d\tau}\sigma_n = -\gamma\cdot n\cdot\sigma_n ,$$

where γ is the loss (growth) rate constant and it is equal either to anomalous dimension of the field ([18]) or to the constant number([3,19]).

Following ([3]) the Malthus equation is also fulfilled for the cross section of generation of ν-component hadron set [I)]

$$(10)\qquad -\frac{d}{d\tau}\sigma(n_1,\ldots,n_\nu) = -\gamma\Big(\sum_{i=1}^{\nu} N_i n_i\Big)\sigma(n_1,\ldots,n_\nu).$$

Averaging over $n_2,\ldots,n_\nu$ leads to the following equation for the cross section of semi-inclusive production of $n_1 = n_c$ charged particles

$$(9a)\qquad -\frac{d}{d\tau}\sigma_{n_1} = -\gamma N_1 n_1\sigma_{n_1} - \gamma\sum_{i=2}^{\nu} N_i \sum_{n_2\ldots n_\nu} n_1\sigma(n_1,\ldots,n_\nu),$$

I) In fact, we consider these equations as characteristic ones corresponding to the first order partial differential equation. Thus, we can go into the physical sense of time parameter and set the multi-dimensional KNO-function (2) as an initial condition ([3]).

where the second term represents the operator of the Volterra type for multiplicity (but not time) lags

$$(9b)\qquad \sum_{n_2..n_\nu} n_1\sigma(n_1,\dots,n_\nu) \Rightarrow \int_{z_1}^{\infty} g(t-z_1) f_\nu(t)\,dt\ .$$

Here f_ν is the function (2) and

$$g(t-z_1) = (t-z_1)^{\nu-2}$$

is the retardation function.
By using formula (4) the loss rate constant for (9a) takes the following form

$$(9c)\qquad K = -\gamma\Big\{ N_1 n_1 + \sum_{i=2}^{\nu} N_i \langle n_i(n_1)\rangle\Big\}.$$

It is interesting to note that after averaging over the next n_1 multiplicity averaging lags disappear but the many-component effect remains in the corresponding relation for the cross-section of the inclusive processes

$$(10a)\qquad \frac{d}{d\tau}\sigma = -\gamma\Big(\sum_{i=1}^{\nu} N_i\langle n_i\rangle\Big)\sigma,$$

This analogy can be continued if we take into account a certain hypothesis about the correlation matrix $D^2=\{D_{ik}\}$ parametrization.

Thus, the single component case with the Wroblewski law (1a), when ε_c^2 is neglected, leads to the Verhulst equation ([20]) (so called logistical equation([8]))

$$(7a)\qquad \frac{d}{d\tau}\langle n_c\rangle = K\langle n_c\rangle(s-\langle n_c\rangle)/s,$$

where $s=2\gamma\varepsilon_c$ is the saturation level (i.e. $\langle n_c\rangle \Rightarrow s$ when $\tau \Rightarrow \infty$), $K= 2\gamma\varepsilon_c N_c/a$.

Let diagonal elements also have the form (1a) in the many component case and others are represented according to the Vollterra "encounter" method

$$(11)\qquad D^2 = \begin{Vmatrix} a_{11}(\langle n_1\rangle-\varepsilon_1)^2,\dots,a_{1\nu}\langle n_1\rangle\langle n_\nu\rangle \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{\nu 1}\langle n_\nu\rangle\langle n_1\rangle\ ,\dots,a_{\nu\nu}(\langle n_\nu\rangle-\varepsilon_\nu)^2 \end{Vmatrix},$$

Then, from (7) we get

$$(12)\qquad \frac{d\langle n_i\rangle}{d\tau} = \langle n_i\rangle\Big(\bar{\varepsilon}_i - \sum_{k=1}^{\nu} \bar{a}_{ik}\langle n_k\rangle\Big),$$

Here $\bar{\varepsilon}_i=2\gamma\varepsilon_i a_{ii}$ represents the natural growth rate of the i-th component and $\bar{a}_{ik}=\gamma N_k a_{ik}$ is proportional to rapidity of the growth (loss) $\langle n_i\rangle$ at the binary encounter with $\langle n_j\rangle$.

This is the usual Volterra model of a few interacting species for dynamical description of biological population. Consideration of this model as quark hadronization into a multicomponent set of hadrons allows one to make further simplications.

In the case of saturation with the same elements .

$$(11a)\qquad a_{ik} = a_{ki} = 1/a,$$

the relation of "pray and predator" is excluded and we are led to the model for ν species which are competing for the same resource (quark food):

$$(13)\qquad \frac{d\langle n_i\rangle}{d\tau} = (\hat{\varepsilon}_i - \gamma_i F(\langle n_1\rangle,\dots,\langle n_\nu\rangle))\langle n_i\rangle.$$

Here

$$(14)\qquad F(\dots) = M \equiv \sum_{i=1}^{\nu} N_i\langle n_i\rangle,\ \gamma_i = \gamma/a,\ \hat{\varepsilon}_i = 2\gamma\,\varepsilon_i/a.$$

It is easy to show that it is also dissipative. Multiply (7) by N_i, and summing we have the quark-mass equation

$$(15)\qquad \frac{dM}{d\tau} = -\gamma\, D^2(M),$$

where

$$(16)\qquad D^2(M) = \Big\langle\Big(\sum_{i=1}^{\nu} N_i n_i - M\Big)^2\Big\rangle = \sum_{i=1}^{\nu} N_i^2 D_{ii} + 2\sum_{i>j=1}^{\nu} N_i N_j D_{ij}.$$

We consider that one of the terms is nonequal to zero and all $D_{ik} \geq 0$, thus $D^2(M) > 0$. This means by the Volterra classification that we have a dissipative set of components. Thus, the model is more realizable in nature than the conservative at $D^2(M) =0$.

Note that eq.(15) allows one to overcome a difficult problem of solving system (7). For instance, in order to obtain the Γ-distribution (2) we must close the set of equations (10),(10a) and (15) by the condition

$$(17)\qquad D^2(M) = \frac{1}{a}\langle M\rangle^2.$$

However, the solution of (7), as we think, is an attractive task aimed at getting important information about assoc-

iated multiplicities. As Montroll et al.([8]) admit, the analytical study of any generalized Volterra model is difficult and the numerical study is expensive. However, we hope to advance in analytical calculation by the method of computer algebra and programming system REDUCE-3 ([21]).

3. - The differential equation for the associated multiplicity. Algorithms

In experiment, multiplicity of a single component is usually measured instead of all the considered ones. In order to describe its averaged values, it is necessary to obtain D-equation of the ν-th degree from set (7) as a differential resultant. This is a hard task in high orders and it may quickly be solved by the programming system REDUCE, for instance, on the PC IBM-AT. For illustration, the suitable program is given for the case $\nu = 3$ and $\varepsilon_1 = 0$. Then, according to (12) we have

$$
\begin{aligned}
\dot{n_1} &= -a_1 n_1^2 - a_2 n_1 n_2 - a_3 n_1 n_3, \\
\dot{n_2} &= -b_1 n_1 n_2 - b_2 n_2^2 - b_3 n_2 n_3, \\
\dot{n_3} &= -c_1 n_1 n_3 - c_2 n_2 n_3 - c_3 n_3^2 .
\end{aligned} \tag{18}
$$

Here

$n_i = \langle n_i \rangle$, $a_i = \gamma a_{1i} N_i$, $b_i = \gamma a_{2i} N_i$, $c_i = \gamma a_{3i} N_i$, $i=1,2,3$.

The set (18) has the following programming form

```
Operator n,a,b,c; Array a(3),b(3),c(3);
For i:=1:3 do <<depend n(i),tm>>;
Df(n(1),tm):=-a(1)*n(1)**2-a(2)*n(1)*n(2)-a(3)*n(1)*n(3)$
Df(n(2),tm):=-b(1)*n(1)*n(2)-b(2)*n(2)**2-b(3)*n(2)*n(3)$
Df(n(3),tm):=-c(1)*n(1)*n(3)-c(2)*n(2)*n(3)-c(3)*n(3)**2$
```

Instead of the method of elimination we shall use the next prescription. Let d_i ($i=1,\ldots,15$) be arbitrary (uncertainty) constants. Make the following substitutions:

$x_1 = \dot{n_1}$, $x_2 = \ddot{n_1}$, $x_3 = n_1^2$, $x_4 = n_1^3$, $x_5 = n_1^4$, $x_6 = n_1^5$,
$x_7 = n_1\dot{n_1}$, $x_8 = n_1\ddot{n_1}$, $x_9 = \dot{n_1}^2$, $x_{10} = n_1^2\dot{n_1}$, $x_{11} = n_1^3\dot{n_1}$,
$x_{12} = n_1^2\ddot{n_1}$, $x_{13} = n_1\dot{n_1}^2$, $x_{14} = n_1\dddot{n_1}$, $x_{15} = \dot{n_1}\ddot{n_1}$.

Then, the desired equation as a resultant of set (I8) will be

$$
\sum_{i=1}^{15} d_i x_i = 0 \tag{19}
$$

and this problem is reduced to finding a coefficient d_i.

In the programming language we have

```
Operator d,x; Array d(15), x(15);
x(1):=df(n(1),tm)$ x(2):=df(n(1),2,tm)$ x(3):=n(1)**2$....$
x(15):=x(1)*x(2)$ s15:= for i:=1:15 sum d(i)*x(i)$ Clear X;
```

The substitutions for x_i with (18) reproduce the left-hand side of equality (19) into the sum of independent monomials $n_1^\alpha n_2^\beta n_3^\gamma$. The arising coefficients must be equal to zero in order to fulfil eq.(19).

We multiply this expression s15 by function sn(0,0,0) and make LET substitution for degrees α,β,γ. Thus we can use COEFF statement ([22]) :

```
Order a,b,c,d,n(1),n(2),n(3),sn; On nero,div; Off nat;
s15:=s15*(n(1)*n(2)*n(3))**3*sn(0,0,0)$
for all b3 let n(3)**b3*sn(0,0,0)=sn(0,0,b3);
for all b2,b3 let n(2)**b2*sn(0,0,b3)=sn(0,b2,b3);
for all b1,b2,b3 let n(1)**b1*sn(0,b2,b3)=sn(b1,b2,b3);
s15:=s15$ array cc(1);
For b1:=3:8 do for b2:=3:8 do for B3:=3:8 do write
c1(b1,b2,b3):= if coeff(s15,sn(b1,b2,b3),cc1) neq 0
then cc(1) else 0;
```

It is interesting to note that such an algorithm allows one to work on PC IBM-AT with limited memory (600 KB) at $\nu \geq 4$. If necessary, one should vary only the first cycle index.

Finally, we must take C1 equal to zero and define d_i. By a simple example of saturated correlation (14), when $a_i = b_i = c_i$, this procedure looks like

```
For i:=1:3 do <<b(i):=c(i):=a(i)>>;
For b1:=3:8 do for b2:=3:8 do for b3:=3:8 do write
C1(b1,b2,b3):=c1(b1,b2,b3);
and the system's answer has the form
c1(4,3,4):=-a(3)*d(1)$
c1(4,3,5):=2*a(3)**2*d(2)$
c1(5,3,3):=-a(1)*d(1)+d(3)$
```

c1(5,3,4):=-a(3)*(-4*a(1)*d(2)+d(7))$

c1(5,4,4):=2*a(3)*a(2)*(d(9)+2*d(8))$ and so on.

The corresponding values of d_i are:

$d_i=0$ for i=I,2,3,4,5,6,7,10,11 and

$d_8 = -d_9/2$, $d_{12} = -d_{13}/2$, $d_{14} = -d_{15}/3$.

As a result, (19) becomes

$$d_9(-\tfrac{1}{2} n_1 n_1'' + n_1'^2) + d_{13}(-\tfrac{1}{2} n_1^2 n_1'' + n_1 n_1'^2) + d_{15}(-\tfrac{1}{3} n_1 n_1''' + n_1'' n_1') = 0.$$

Due to arbitrariness d_i we get two eqs., but only the first is independent

(20) $$n_1 n_1'' - 2n_1'^2 = 0$$

and its derivative leads to the second one.

As is easily seen, for arbitrary coefficient $a_1 \neq b_1$ from (18) we get a nonlinear autonomic equation

(21) $$a_2 n_1 n_1'' - (a_2 + b_2) n_1'^2 - (2a_1 b_2 - a_1 a_2 - a_2 b_1) n_1^2 n_1' + a_1(a_2 b_1 - a_1 b_2) n_1^4 = 0.$$

It is not difficult also to introduce either the rate constant or other complication into consideration.

4. - Solution of the equation for the associated multiplicity

Unfortunately, we are not informed of any experiment for expounding the component structure in multiparticle processes, and we can understand only from mathematical point of view why the obtained formulae show just the value of ν in comparing with the available experimental data. For example, as it is seen from the Table, charge particle emission is similar to the single-component case for full pseudorapidity intervals . This is in agreement with the uncertainly principle for ν=1 and $a_e = a \Rightarrow 4$. Let us prove that "disappearance" of the component structure is equivalent to the change of multiplicity lags by the time ones in eqs. (9a,b).

In our model, a number of quarks n transforms into the rate constants (8) or (9b), if we measure many- or one-variable cross-sections, respectively.

As has been mentioned in Introduction, the quantity (4) in (9c) contains multiplicity n_1 lags (9b) and depends on ν.

Table. Results of fitting the KNO-function (3) compared to the charged particle multiplicity distribution in limited (and full) intervals of pseudorapidity distribution $|\eta|<\eta_c$ (*) at $\sqrt{s}$ = 200 and 900 GeV centre of mass energies ([4]) . The values of the correlated component number ν, the parameter α and the normalization constant A are given at a=4.

η_c	ν	α	A	χ^2 / NDF
		200 GeV		
0.5	2.77 ± 0.18	0.86 ± 0.07	2.60 ± 0.28	5/11
1.5	1.72 ± 0.06	0.45 ± 0.03	6.23 ± 0.58	6/30
3.0	1.30 ± 0.03	0.20 ± 0.02	15.45 ± 1.39	9/50
5.0	1.11 ± 0.02	0.07 ± 0.02	27.91 ± 2. 38	5/55
*	1.04 ± 0.02	0.03 ± 0.02	72.55 ± 6.31	5/28
		900 GeV		
0.5	2.63 ± 0.13	0.82 ± 0.06	2.72 ± 0.21	6/20
1.5	1.87 ± 0.06	0.59 ± 0.04	5.34 ± 0.41	24/49
3.0	1.51 ± 0.03	0.35 ± 0.02	9.50 ± 0.68	46/80
5.0	1.22 ± 0.02	0.15 ± 0.01	19.34 ± 1.29	49/98
*	1.11 ± 0.02	0.08 ± 0.01	55.42 ± 3.94	36/51

When it is linearised as relation (5), then coefficient (9c) becomes

$$K = An_1 + B,$$

where the parameters

$$A = \gamma(N_1 + \sum_{i=2}^{\nu} A_i N_i),\ B = \gamma \sum_{i=2}^{\nu} B_i$$

conceal multi-component structure of ν and depend on the total energy (time τ).

As this is a rough method for cutting coupled equations, the average multiplicity for charged particles obeys Riccati's equation

$$\langle n_c \rangle' = \alpha_1 \langle n_c \rangle^2 + \alpha_2 \langle n_c \rangle + \alpha_3. \qquad (22)$$

In other words, it contains both the Verhulst self-interraction and the time lags in the sense of dependence of the a_i coefficients on τ. We shall not examine it any more but from methodical point of view we should note that it was establi - shed and analysed ([18]) in the framework of the renormalization group ([23]), and the algorithms ([24]) of computer algebra can also be used for this purpose.

Consequently, in our model (7) of competing components we can get equation for associated multiplicity. Its solution depends either on momentum p of an inclusive particle or on transfer momentum Q through the time variable τ (it is possible - on ν). In the model of saturating correlation (11a), (14) when the natural growth rate is zero, $\varepsilon_1 = 0$, we have

$$\langle n_1\rangle' = -\gamma_1 M \langle n_1\rangle,$$

where $\gamma_1=\gamma/a$ and M is quark-mass .

The anzats (17) leads to the solution

$$\langle n_1\rangle = \langle n_1\rangle_0 / (1 + \gamma_1 M_0 \tau).$$

Here $\langle n\rangle_0$ and M_0 are quantities at $\tau=0$. If we want to see ν dependence, we must recall that $M_0 = \nu N_1 \langle n_1\rangle_0$ ([3]).

Finally, we will briefly discuss the model with the partial saturation (18) with $\nu=2$. The nonlinear eq.(21) can be solved, for example, by the Berkovic ([25]) method. We rewrite it as follows:

$$n'' + f(n)\, n'^2 + \beta_1 \varphi(n)\, n\, n' + A(n) = 0, \tag{23}$$

where

$n=\langle n\rangle$, $f(n)=-(a_2+b_2)/(a_2 n)$, $\varphi(n)=n$, $\beta_1=-(2a_1b_2-a_1a_2-a_2b_1)$,

$A(n) = a_1(a_2b_1-a_1b_2)n^3/a_2$.

Then ([25]) eq.(23) reduces to the linear one with fixed coefficient

$$X_t'' + \beta_1 X_t' + \beta_0 X_t + \alpha = 0 \tag{24}$$

if the next condition is fulfilled

$$A(n)=\varphi(n)\exp(-\int f(n)dn)[\beta_0\int\varphi(n)\exp(\int f(n)dn)dn + \alpha]\cdot \tag{24a}$$

It takes place when

$\alpha=0$ и $\beta = a_1(a_2b_1 - a_1b_2)[2 - (a_2+b_2)/a_2]/a_2$.

and, in general,

$$X(n)=\int\varphi(n)\exp(\int f(n)dn)dn\ ,\quad dt=\varphi(n)d\tau. \tag{25}$$

Note also that in the limit of saturation $a_i \Rightarrow b_i$

$X(n)=\ln n$, $dt=n\,d\tau$, $X_t''=0$ and $n \sim -\frac{1}{\tau}$.

Thus, decrease in associated multiplicity is caused by the limited resource.

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Модель Вольтерра и адронизация кварков в многокомпонентную систему адронов

Рассмотрены примеры зависимости характеристик множественного процесса от числа нескольких коррелированных компонент адронов. Обсуждается возможность приведения системы дифференциальных уравнений, полученной ранее при адронизации n-кваркового процесса в многокомпонентную систему, к диссипативной модели Вольтерра для конкурирующих за одну пищу видов биологической популяции. Дан алгоритм для аналитического расчета дифференциального уравнения высокого порядка как результанта возникающей системы. Приведены примеры линеаризации и решения этих уравнений, описывающих ассоциативные множественности заряженных частиц.

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The Volterra Model and Quark Hadronization into Multicomponent Hadron System

The examples of the multiparticle process characteristic dependence on the number of a low correlated components are considered. The possibility for reducing the differential equation system, which was obtained earlier, to a dissipative type Volterra model of competing biological species for the same food is discussed. An algorithm for the analytical computation of the high order differential equation as a resultant of the arising system is given. The examples of linearization and solution of these equations describing the associated multiplicities of charged particles are represented.

The investigation has been performed at the Laboratory of Computing Technique and Automation, JINR.

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