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ON EQUAL CONSTITUENT QUARK MASSES IN DIFFERENT HEAVY QUARKONIA POTENTIAL MODELS

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1. GENERAL REMARKS

We study the parametrization of a representative set of well-known central nonrelativistic, spin-averaged potential models for heavy quarkonia which flavour-invariantly reproduce excitation energies and decay widths of charmonium and bottonium states with surprising accuracy [1-6].

Especially, we are interested in the role of the constituent quark masses. In each model they are introduced as free parameters to be fitted to the data together with the remaining parameters which directly appear in the potential. This leads to values of m_{bi} and m_{ci} (i - model index) distributed in the intervals $1.2 < m_{ci} < 1.9$ GeV and $4.6 < m_{bi} < 5.3$ GeV. Presently a number of successful potentials exist and, from a more general point of view, one should look for an alternative approach with equal constituent masses for all models. Indeed, if nonrelativistic quantum mechanics is taken seriously as a tool of successful phenomenological description of heavy quarkonia, the constituent quark mass should be a quantity of the physical bound state to be described approximately by different potential ansatzes and therefore should not vary as functional of the potentials $V_i(r)$, whatever its meaning may be in a deeper QCD-like theory. Of course, if the different potential models acquire common masses $m_Q (Q = c,b)$, the freedom of the choice of parameters is restricted and one should study the consequences for data reproduction.

To proceed in this direction, we start with the Schroedinger equations for the $Q\bar{Q}$ levels in different models

$$[-\frac{1}{m_{Qi}}\Delta + 2m_{Qi} + V_i(r)]\,\psi^{(i)}_{n,m_{Qi}}(\vec{r}) = M^{(i)}_{n,m_{Qi}}\,\psi^{(i)}_{n,m_{Qi}}(\vec{r}) \qquad (1)$$

and study the simultaneous transformations

$$V_i'(r) = V_i(r) - V_{0i}\,, \qquad m'_{Qi} = m_{Qi} + \frac{1}{2}V_{0i}\,. \qquad (2)$$

As is known from fit experience [4,6,7], application of eqs.(2) compensates the major effect of a change $m_{Qi} \to m'_{Qj}$ by a constant shift V_{0i} of the potential. But with increasing V_{0i} the

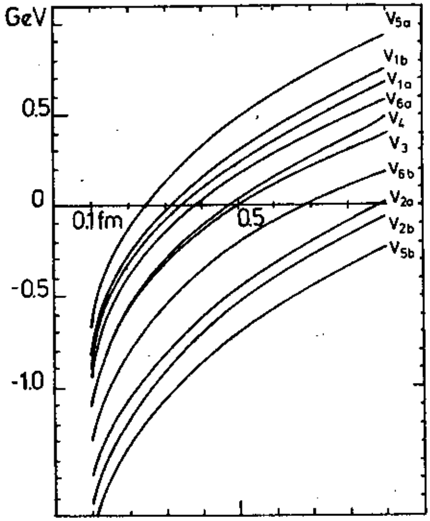

Fig.1. The underlying potentials in the region 0.1 < r < < 1.0 fm.

energy level deviations become larger and must be investigated. It is a known fact that in the relevant region $0.1 < r < 1.0$ fm of interquark distances for $c\bar{c}$ and $b\bar{b}$ physics, the potentials $V_i(r)$ approximately differ from each other only by constant terms (see Fig.1). This property we use to apply eqs. (2) in order to shift together the potential curves between 0.1 and 1.0 fm to study the new masses m'_{Qi}. But first we briefly review the set of potentials and their parameters used in the following.

2. THE UNDERLYING POTENTIALS

We evaluate six well-known potentials in different parametrizations. The corresponding quark masses m_{Qi} are given in the table.

1. Cornell potential [1]

$$V_1(r) = -K/r + ar \tag{3}$$

	k	a(GeV^2)
a)	0.494	0.173 [8]
b)	0.47	0.19 [6]

2. Martin potential [2]

$$V_2(r) = -A + Br^{a} \tag{4}$$

	A(GeV)	B(GeV^{1+a})	a
a)	-6.31	5.22	0.126 [2]
b)	-8.06	6.87	0.100 [9]

3. Richardson potential [3]

$$V_3(r) = -\frac{4}{3}\,\frac{16\pi^2}{b_0}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{\vec{p}^{\,2}\ln(1+\vec{p}^{\,2}/\Lambda^2)} \tag{5}$$

$b_0 = 11 - 2/3 n_f$ $\Lambda = 0.375$ GeV [6]
(n_f - number of effective quarks).

4. Potential of Buchmüller, Grunberg and Tye [4]
Short range part:

$$V_4^{(s)}(r) = -\frac{4}{3}\frac{1}{r}\frac{4\pi}{b_0 F(r)}\left[1 - \frac{b_1}{b_0}\frac{\ln F(r)}{F(r)} + \frac{c}{F(r)} + \dots\right],$$

$$F(r) = \ln(1/(\Lambda r)^2), \tag{6}$$

Long range part:

$$V_4^{(L)}(r) = a' r$$

$$\left(\begin{array}{l} b_1 = 102 - \frac{38}{3} n_f,\ c = \frac{1}{360}(31 - \frac{10}{3} n_f) + 2\gamma_E \\ \Lambda = \Lambda_{\overline{MS}} = 0.5\ \text{GeV},\quad a' \simeq 1\ \text{GeV}^2 \end{array}\right),$$

where $n_f = 3$ and γ_E is Euler's constant.

The intermediate part of this potential is obtained from direct interpolation of the β function between small and large relative quark momenta. No parameters to be fitted to the $c\bar{c}$ and $b\bar{b}$ data (except the constituent masses) are contained in this potential.

5. Kühn-Ono potential [5]

$$V_5(r) = -\frac{4}{3}\frac{1}{r}\frac{4\pi}{b_0 f(r)}\left[1 - \frac{b_1}{b_0}\frac{\ln f(r)}{f(r)} + \frac{c}{f(r)} + \dots\right] + A\sqrt{r} + C, \tag{7}$$

$$f(r) = \ln[1/(\Lambda r)^2 + B]$$

	Λ(GeV)	A($\text{GeV}^{3/2}$)	B	C(GeV)
a)	0.14	0.63	20	-1.39 [5]
b)	0.20	0.67	238	-1.41 [6]

The short range part differs from that of V_4 by introduction of the parameter B avoiding the Landau singularity in the extrapolation to larger r.

6. Potential of Hagiwara et al. [6]

$$V_6(r) = V_6^{(S)}(r) + V_6^{(I)}(r) + V_6^{(L)}(r),$$

$$V_6^{(S)}(r) = -\frac{c_F}{r}\alpha_s\ ,\quad V_6^{(I)}(r) = r(c_1 + c_2 r)e^{-r/r_0}\ ,\quad V_6^{(L)}(r) = c_3 r\ . \qquad (8)$$

	Λ(GeV)	C_1 (GeV^2)	C_2 (GeV^3)	C_3 (GeV^2)	r_0 (GeV^{-1})
a)	0.2	0.22	-1.12	1.19	0.70 /6/
b)	0.4	0.18	-1.35	1.15	0.57 /6/.

$V_6^{(S)}$ mainly agrees with the short range behaviour of V_5. A phenomenological intermediate part $V^{(I)}$ is included to get a soft, flexible interpolation.

3. CHANGE OF THE CONSTITUENT MASSES m_{Qi}

The potentials $V_i(r)$ are shifted to agree in some point $r = r_0$ if in eqs.(2)

$$V_{0i} = V_i(r_0) + V_0\ , \qquad (9)$$

where V_0 appears as arbitrary constant not depending on the model index. We have studied the masses $m'_{Qi}(r_0)$ for each potential of section 2 ($V_0 = 0$) and obtain the two curves given in Fig.2 for the mean values

$$\bar{m}'_{Qi}(r_0) = \frac{1}{12}\sum_{i=1}^{12}\left(m_{Qi} + \frac{1}{2}V_i(r_0)\right)\begin{pmatrix} V_{1a} = V_1 & V_5 = V_6 \\ V_{1b} = V_2 & V_7 = V_8 \\ \vdots & \end{pmatrix}, \qquad (10)$$

where all 10 parametrizations (table) are included with the same weight for each of the six potentials. The surprisingly small mean square errors are indicated. These errors are one order of magnitude smaller than those of the fit quark masses m_{Qi}.

$$\left(\overline{m'^2_{Qi}} - \overline{m'}^2_{Qi}\right)^{1/2} \ll \left(\overline{m^2_{Qi}} - \overline{m}^2_{Qi}\right)^{1/2}\ . \qquad (11)$$

They have minima near 0.5 fm and near 0.3 fm for b and c, respectively, as shown in Fig.3. But how to fix r_0? First we give two phenomenological arguments: a) The best agreement of the potential curves $V_i(r)$ between 0.1 and 1.0 fm is obtained for $r_0 \simeq 0.5$ fm /4,10/ which should correspond to the lowest mean square error of $m'_{Qi}(r_0)$. Figure 3 confirms this if we rely more on the bottonium curve giving the smaller error. b) It

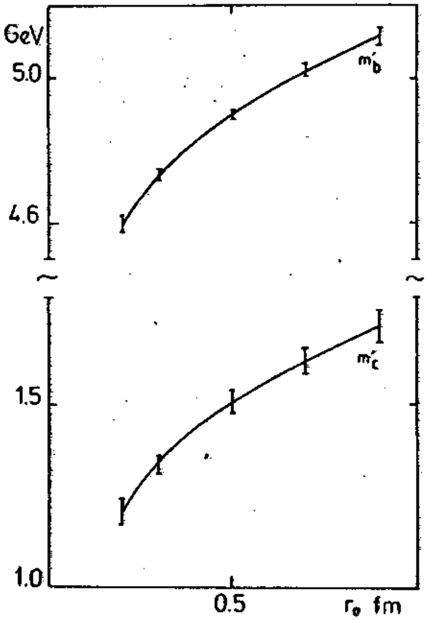

Fig.2. $m'_{ci}(r_0)$ and $m'_{bi}(r_0)$ as functions of the point r_0 in $V'_i(r) = V_i(r) - V_i(r_0)$. The mean square errors are indicated.

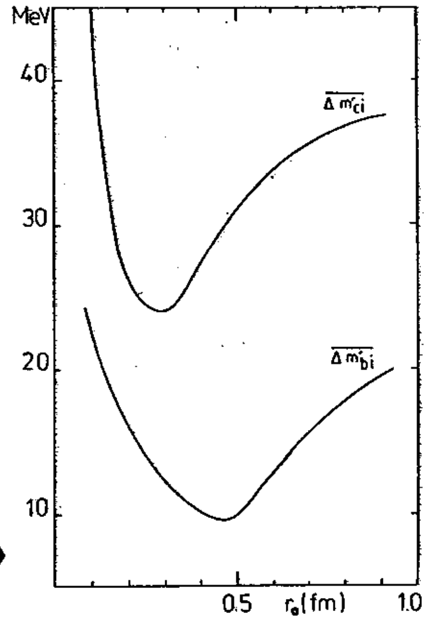

Fig.3. The mean square errors of m'_{bi} and m'_{ci} as functions of r_0.

should be a reasonable assumption that the mean value of the fit quark masses is not too far from $\overline{m'_{Qi}}$. Their approximate equality $\overline{m_{Qi}} \cong \overline{m'_{Qi}}$ entails $\overline{V_i}(r_0) \cong 0$. Using the above set of potentials to calculate $V_i(r_0)$, we obtain $r_0 = 0.52$ fm which is in the expected region. A third more principal argument in favour of $r_0 \cong 0.5$ fm is due to the construction of the potentials V_3 and V_4 which work with a minimum of parameters and do not admit any shift of their fit quark masses. This fact requires the equalities

$$\overline{m'_{ci}}(r_0) \simeq m_{c3} \simeq m_{c4} \,,$$
$$\overline{m'_{bi}}(r_0) \simeq m_{b3} \simeq m_{b4} \,, \qquad (12)$$

which again lead to $r_0 = 0.5$ fm (Fig.2 and the table). We have checked that the point r_0 is not too sensible against exclusion or inclusion of one or the other potential. A subjective error related to our choice of a representative set of potentials should not be larger than ± 0.05 fm. Hence we arrive at $r_0 = 0.50 \pm 0.05$ fm which corresponds to uniform constituent masses (Fig.2)

Table. The values of m_{Qi} and m'_{Qi} ($Q = b,c$) and the difference $m_{bi} - m_{ci}$ for the underlying potentials. Mean values and mean square errors are given

model	potential index	m_{ci}	m_{bi}	$m_{bi}-m_{ci}$	m'_{ci}	m'_{bi}
Cornell	1a	1.35	4.75	3.42	1.47	4.89
	1b	1.32	4.75	3.43	1.47	4.90
Martin	2a	1.76	5.14	3.38	1.54	4.92
	2b	1.80	5.17	3.37	1.54	4.91
Richardson	3	1.50	4.91	3.41	1.50	4.91
Buchmüller et al.	4	1.48	4.88	3.39	1.51	4.90
Kühn-Ono	5a	1.22	4.66	3.44	1.47	4.91
	5b	1.90	5.26	3.36	1.55	4.91
Hagiwara et al.	6a	1.58	4.99	3.41	1.48	4.89
	6b	1.36	4.79	3.43	1.46	4.89
mean values		1.53	4.93	3.40	1.50	4.90
mean square errors		0.21	0.19	0.026	0.030	0.008

$$m_c = 1.50 \pm 0.07 \text{ GeV} \quad m_b = 4.90 \pm 0.05 \text{ GeV} \tag{13}$$

for all potentials of section 2. Uniform masses not agreeing with (13) would exclude the potentials V_3 and V_4 from the analysis. In our opinion, this cannot be admitted because just these potentials based on a minimal number of parameters, are the most interesting ones from a theoretical point of view.

4. THE RELATIVE STABILITY OF $m_{bi} - m_{ci}$

It is a known remarkable fact discussed already 10 years ago[11,12] that the difference of the bottom and charm fit quark masses $m_{bi} - m_{ci}$ approximately appears as potential model independent quantity. Its mean square error is one order of magnitude smaller than the corresponding errors of m_{bi} and m_{ci} (see the table). Using the above result (13) on the relative accuracy and approximate model independence of the primed masses $m'_{Qi} \cong m_Q$ we can simply explain this stability of $m_{bi} - m_{ci}$:
Considering two potential models labelled by the indices i and j one obtains by substraction of the two equations

$$m_{Qi} + \frac{1}{2} V_i(r_0) = m'_{Qi} \cong m_Q ,$$

$$m_{Qj} + \frac{1}{2} V_j(r_0) = m'_{Qj} \cong m_Q ,$$

the relation

$$m_{Qi} - m_{Qj} \cong \frac{1}{2} [V_j(r_0) - V_i(r_0)] ,$$

where the right-hand side carries no flavour index Q which immediately leads to the model independence of the difference $m_{bi} - m_{ci}$:

$$m_{bi} - m_{bj} \cong m_{ci} - m_{cj} \quad \text{or} \quad m_{bi} - m_{ci} \cong m_{bj} - m_{cj} . \tag{14}$$

5. INFLUENCE ON ENERGY LEVELS AND LEPTONIC DECAY WIDTHS

If all potentials are related to the same constituent quark masses m_Q, the number of adaptable parameters is reduced and the effect on data reproduction should be studied. Eqs. (2) transform the Schroedinger equations (1) into

$$[-\frac{1}{m_Q} \Delta + 2m_Q + V'_i(r)] \psi_n^{(i)}(r) = M_n^{(i)} \psi_n^{(i)}(\vec{r}) . \tag{1a}$$

Because of the common mass m_Q now the mass index at the eigenvalues and wave functions is dropped. The new $Q\bar{Q}$ levels

$$M_n^{(i)} = 2m_Q + \langle i,n | [T_i + (V_i - V_{0i})] | n,i \rangle ,$$

are expressed by the old ones

$$M^{(i)}_{n,m_{Qi}} = 2m_{Qi} + \langle i,n,m_{Qi} \,|[T_i + V_i]|\, m_{Qi},n,i \rangle$$

through the relation (H = T+V)

$$M^{(i)}_{n} = M^{(i)}_{n,m_{Qi}} + \langle i,n|H_i|n,i\rangle - \langle i,n,m_{Qi}|H_i|m_{Qi},n,i\rangle . \qquad (15)$$

As an example we have studied the change of S states of the Cornell potential $V_1(r)$ (3) with masses m_{c1} = 1.32 GeV and m_{b1} = 4.75 GeV. Following Eichten et al.[1], we started with the asymptotic dimensionless radial Schroedinger equation

$$\left(\frac{d^2}{d\rho^2} - \rho + \zeta_n\right) u_n(\rho) = 0 , \qquad (16)$$

where

$$r = a_1(m_{Q1}a_1)^{-1/3}\rho,\ E_n = m_{Q1}(m_{Q1}a_1)^{-4/3}\zeta_n,\ R(r) = \left(\frac{m_{Q1}}{a_1^2}\right)^{1/2}\frac{u(\rho)}{\rho} \qquad (17)$$

and $V_1^{(L)}(r) = r/a_1^2$. Eqs.(15)-(17) yield

$$M_n^{(1)} = M^{(1)}_{n,m_{Q1}} + a_1^{-1}(a_1 m_{Q1})^{-1/3}\left[1-\left(1+\frac{V_{01}}{2m_{Q1}}\right)^{-1/3}\right]\zeta_n \qquad (18)$$

and the Coulombic interaction can be taken into account perturbatively in the eigenvalues ζ_n which is sufficient approximately also in the case of bb. Figures 4a,b show the deviations of the lowest three level spacings as functions of V_{01} for the Cornell potential $V_1(r) = -k/r + r/a_1^2$ (Fig.4a), and for the pure asymptotic potential $V_1^{(L)}(r) = ar = r/a_1^2$ (Fig.4b). As is seen from Fig.4, the changes of the level spacings in both cases are roughly similar i.e. they are mainly determined by the long range part of the potential. Thus Fig.4b can give rough information also for other linearly raising potentials.

The result taken from Fig.4a for the Cornell potential with

$$V_{01} = V_1\,(0.5\ \mathrm{fm}) = 2(m_Q - m_{Q1}) \simeq 0.30\ \mathrm{GeV},$$

are corrections $\Delta^{b\bar{b}}_{2S-1S}$, $\Delta^{b\bar{b}}_{3S-2S}$ and $\Delta^{b\bar{b}}_{4S-3S}$ to the S level spacings between 3 and 7 MeV. The corresponding $c\bar{c}$ quantities

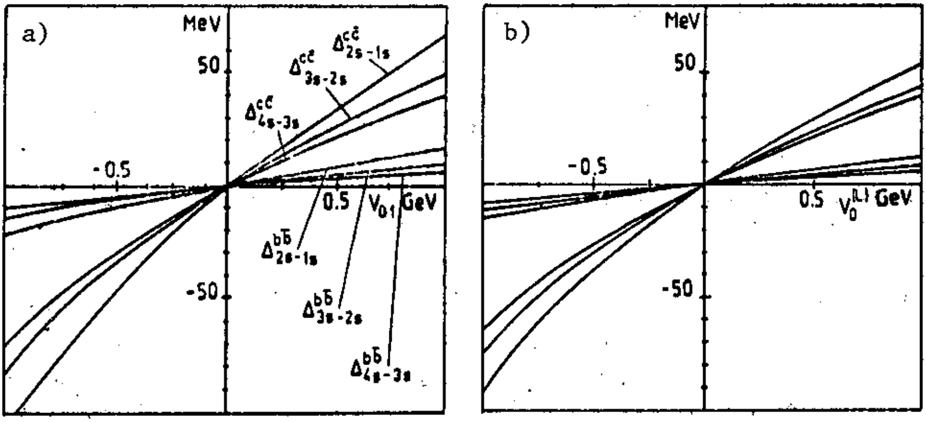

Fig.4. a) The change of the three lowest S level spacings for $c\bar{c}$ and $b\bar{b}$ as function of V_{01} in the Cornell potential model with k = 0.47 and a = 0.19 GeV^2. b) The same functions as in Fig.4a for a pure linearly raising potential $V^{(L)}(r) = ar$.

are 3–4 times higher. The overall shift of the spectrum is near 3 MeV for $b\bar{b}$ and near 14 MeV for $c\bar{c}$.

Generally, it should be noted that these deviations in the spectra can be reduced more or less by adapting the other parameters of the potentials to the common quark masses. This should work the better, the more parameters exist in a given potential model. As example we mention the Kühn-Ono potential (7). It contains four parameters and was fitted to the data with rather different values of these parameters together with correspondingly different quark masses in the regions $1.22 < m_{c5} < 1.90$ GeV and $4.66 < m_{b5} < 5.26$ Gev. One of these fits /6/ works with $m_{c5} = 1.41$ and $m_{b5} = 4.83$ not far from the common masses (13). Hence, in the case of potentials containing three and more parameters, the condition of common, fixed quark masses mainly reduces the possible number of fits with different parameter configurations rather than the quality of data reproduction.

Concerning the relative leptonic decay widths in different potential models, the introduction of equal quark masses should be studied thoroughly. For S states the replacements

$$M^{(i)}_{nS,m_{Qi}} \to M^{(i)}_{nS}\ , \qquad \psi^{(i)}_{nS,m_{Qi}} \to \psi^{(i)}_{nS}$$

are necessary in the expression

$$\frac{\Gamma_{nS}}{\Gamma_{1S}} = \frac{M_{1S}^2}{M_{nS}^2} \int d^3r\, |\psi_{nS}(r)|^2 \frac{dV(r)}{dr} / \int d^3r\, |\psi_{1S}(r)|^2 \frac{dV(r)}{dr} \qquad (19)$$

obtained from the Weisskopf - Van Royen formula [13] and the relation [14,15]

$$|\psi_{nS}(0)|^2 = \frac{m_Q}{4\pi} \int d^3r\, |\psi_{nS}(r)|^2 \frac{dV(r)}{dr}. \qquad (20)$$

As example we have numerically evaluated eq.(19) for the Cornell potential with $m_c = 1.5$ and $m_b = 4.9$ and have calculated $(\Gamma_{nS}/\Gamma_{1S})$ with $n = 2,3,4$. The deviations from the known results [10,16] are smaller than the experimental limits of error for $c\bar{c}$. Practically no deviations appear for $b\bar{b}$. Some authors [2,5] prefer a relatively large $m_c \approx 1.8$ to improve the agreement with experiment for $(\Gamma_{nS}/\Gamma_{1S})_{c\bar{c}}$ with $n \geq 3$. But up to now the relation between these discrepancies and relativistic effects in the $c\bar{c}$ system is not clear.

6. CONCLUSION

Summarizing, first we note that the introduction of common fixed quark masses in the considered potential models, as alternative to the usual point of view, seems to be attractive because the common masses turn out automatically when the potentials are shifted together at $r_0 = 0.5 \pm 0.05$ fm. The values of m_b and m_c from empirical arguments agree with the fit quark masses of the models of Richardson [3] and Buchmüller, Grunberg and Tye [4] where these masses are fixed because of a minimum of parameters. The approximate model independence of the mass difference $m_{bi} - m_{ci}$ can be simply explained. Maximal differences between the fit masses and the common fixed masses appear in potentials containing three and more parameters adaptable to the new conditions. Hence, the change of energy levels should remain small as confirmed for potentials with linearly raising long range behaviour. Especially, bottonium maintains its role as "ideal hydrogen atom of particle physics". But a complete survey on data reproduction of different potential models under the condition of fixed constituent quark masses requires corresponding new fits.

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