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PERCOLATION AND MULTIFRAGMENTATION OF NUCLEI

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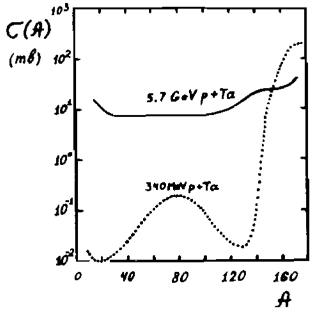
1. Introduction

It is well known (see, e. g. [1]) that transition from low to high energies causes substantial changes in the structure and the number of nuclear fragments created in hadron-nucleus (hA) and nucleus-nucleus (AA) interactions. At low energies the evaporation and fission products of the nuclei, remained after the cascade (fast) stage of the nuclear reaction, dominate among all the fragments (see Fig.1). At high energies the multiple production of the fragments with the masses of the order of 10-20 a.e.m. occurs and the mass spectrum changes in the corresponding way (Fig. 1), i.e. nuclear multifragmentation takes place. Since it is difficult to admit these fragments result from the evaporation and fission processes, an assumption has appeared [3] that in this case we deal with the liquid-to-gas phase transition with the consequent condensation of the nuclear gas to the droplets-fragments. This hypothesis arises from the form of the fragment mass spectrum of the $1/M_f^{\tau}$ type [3,4] which is consistent with the predictions of liquid condensation theory [5] (M_f is the mass of the fragment). However, finality of the nucleon number and the binding energy effects considerably distort this spectrum (see [3,4]). Therefore, to test the hypothesis of this type, it is necessary to consider the "phase transitions" in a system with the finite number of particle. This problems arise and are solved in investigations of the processes united under the name "percolation" [6]. Therefore it seems natural to try to describe the nuclear multifragmentation process in terms of percolation. In particular, in paper [7] it is

Fig. 1. The fragment mass spectra at two energies of the incident proton (Fig. is from [2]).



supposed that in the course of interaction the nucleus is heated and changes to the gase state. At the gas condensation simulated by percolation of free positioned points, the observable fragments appear (approach I). In the series of papers [8] an approach which is inverse, in a sence, was applied: it is supposed that the nucleus is the cubic crystalline structure which is partially destroyed when heated (approach II). At present within the framework of this approach a quite good description of the set of the experimental data is reached (see [8,9]). Unfortunately, this agreement is obtained by introducing quite arbitrary phenomenological parameters and relationships. As a result, the predictive ability of this approach is decreased. Besides, within the framework of this approach it is impossible to put a question about the percolation structure of the "cold" nuclei (i.e. the structure of the nuclei in the ground state), because the structure of the crystalline type is postulated. At the same time this question is quite natural within the framework of the first approach. Below (section 2) we consider one of the possible approaches to clearing up the percolation structure of the "cold" nuclei and suggest a method of building the nuclei as a united (bound) percolation cluster. A possible mechanism of destruction of these clusters in nuclear reactions is given in section 3. In the forth section the fragment mass spectra are described taking into account evaporation of particles from the "hot" excited fragments.

2. Percolation of nuclei

There are many methods of building ("assembling") percolation clusters (see refs. in [6]). For example, the first nucleon can be placed into the origin of the coordinate frame. The second one, linked with it, can be placed at the distance r from the first nucleon with the probability $f(r)$. The third one can be linked with the first or the second one and placed at the distance r_1 or r_2 with the probability $f(r_1)$ or $f(r_2)$, etc. Unfortunately, we have not found the form of the function f(r) to obtain the density of the nucleons in the assembled cluster close to the known nuclear density. Therefore we follow [10] and sample the nucleon coordinates for the nucleus with the mass number A independently of one another in accordance with the distribution

$$\rho(r) = Const/[1 + exp((r-R)/c)],$$
$$R = 1.08 A^{1/3} fm, \quad c = 0.545 fm. \qquad (1)$$

Then one should link the nucleons. Taking into account the short range of nuclear forces, we will assume that the occuring of the links between the closely placed nucleons is more preferable, i. e., the probability $p(r_{ij})$ of the linking of two nucleons i and j with the distance r_{ij} between them is equal to 1 at $r_{ij} \to 0$ and 0 at $r_{ij} \to \infty$. In the papers [7,10] $p(r_{ij})$ was chosen in the form of θ -function:

$$p(r_{ij}) = \begin{cases} 1 & r_{ij} \le r_c \\ 0 & r_{ij} > r_c \end{cases} \qquad (2)$$

Generally speaking, in this case the "cold" nucleus does not represent a single percolation cluster. Besides there are no reasons for choosing any definite value of r_c (see Fig. 2a).

Taking this into account we choose $p(r_{ij})$ in the form:

$$p(r_{ij}) = exp(-r_{ij}/r_c). \qquad (3)$$

The percolation structure of the cold nucleus varies (on the average) depending on r_c. However, at $r_c \sim 1/m_\pi \sim 1.4$ fm all the nuclei are represented by a set of 5-6 percolation clusters which are not connected with one another (see Fig. 2b). Note, that the behaviour of the curves in Fig. 2b allows an assumption that at $r_c \simeq 1.4$ fm in the system of the infinite number of particles a single infinite connected percolation cluster is created.

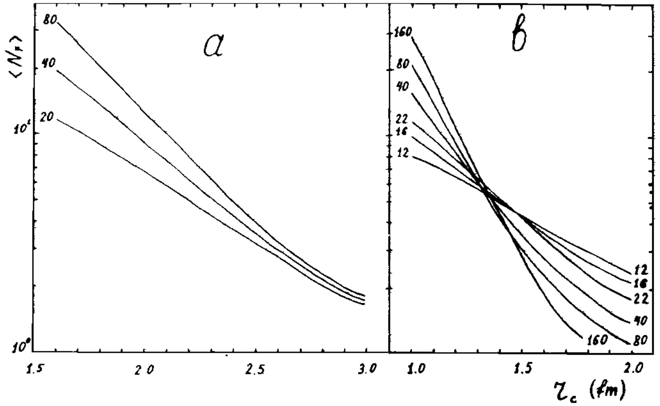


Fig. 2. The average number of the "fragments" of the "cold" nuclei as a function of the parameter r_c calculated using formulae (2) and (3) (Figs. a and b, respectively). The mass numbers of nuclei are given in the figure.

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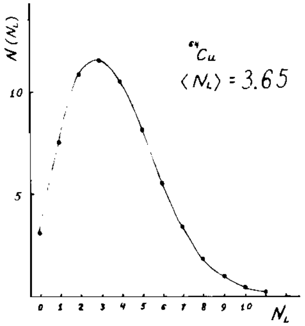
In other words, we suppose that $r_c = 1.4$ fm is a critical value of the percolation parameter. In the following calculations we used this value. Certainly, the fact that our "cold" nuclei do not represent a single percolation cluster is inconvenient. However, the analysis shows that these nuclei mainly consist of a big single connected cluster and 3-4 unlinked nucleons (see, e.g Fig. 3). This seems to be due to imperfection of the assembling method and the use of the noncorrelated distribution of (1). To get rid of these "homeless" nucleons we connect them "by force" to the nearest neighbours belonging to the connected cluster. Thus our cold nucleus is always a single connected cluster.

Fig. 3. The nucleon distribution of the ^{64}Cu "nucleus" over the link number.



A few words about the internal structure of the clusters. In Fig. 3 the nucleon distribution of the ^{64}Cu nucleus over the coordination number N_L (i.e. the number of the neighbours linked with the considered nucleon) is given. As seen, on the average there are about 3 nonlinked nucleons with $N_L=0$. The average number of links $\langle N_L \rangle = 3.56$ *). At $\langle N_L \rangle = 2$ the nucleus would look like a "crumpled" chain or a necklace. If the arbitrary vertex is removed this nucleus would decay into two fragments. At $\langle N_L \rangle = 3$ various structures of nuclei are possible. The simplest one is the binary tree structure which usually decays into three fragments of various

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*) I.e., the nucleus greatly differs from the crystal with the cubic lattice, where $\langle N_L \rangle = 6$.

lengths when any vertex is removed. Adding loops decreases the number of the produced fragments. When the peripheral nucleons are removed, the produced number of fragments can be less than three too. Therefore, the percolation structure of the cluster and its configuration can greatly affect the form of the fragment mass spectra.

According to our calculations $\langle N_L \rangle > 3$, therefore, when an arbitrary nucleon is removed, production of more than three fragments is possible, in principle. The real fragment spectrum is conditioned by removig both central and peripheral nucleons. To estimate the degree of destruction of the nuclei, more accurate additional assumptions are needed.

3. Nuclear multifragmentation

It is experimentally found rather long ago that the products of nuclear reactions at high energies contain the nucleons which cannot be attributed to evaporation particles. Traditionally these particles are supposed to be nucleons-participants of the fast stage of the interaction. In the papers using the photoemulsion methods the charged particles of this type are named g-particles. To describe their yield and destruction of the nucleus one could use any model of inelastic nucleus-nucleus interactions. In this paper the Glauber approximation is used.

According to the Glauber approximation, the probabilities of various inelastic processes in interactions of nuclei A and B at the fixed impact parameter $\vec{b}$ and the given coordinates of the nucleons of the nuclei with the mass numbers A and B in the impact parameter plane ($\{\vec{s}_A\}$ and $\{\vec{\tau}_B\}$ respectively), are specified by different terms of the expression expansion [11,12]:

$$1-\prod_{i=1}^{A}\prod_{j=1}^{B}(1-g_{ij})=\sum_{i=1}^{A}\sum_{j=1}^{B}g_{ij}\prod_{\substack{k=1\\k\neq i}}^{A}\prod_{\substack{l=1\\l\neq j}}^{B}(1-g_{kl})+ \tag{4}$$

$$+\frac{1}{2}\sum_{i=1}^{A}\sum_{\substack{j,k=1\\j\neq k}}^{B}g_{ij}g_{ik}\prod_{\substack{l=1\\l\neq i}}^{A}\prod_{\substack{m=1\\m\neq j,k}}^{B}(1-g_{lm})+$$

$$+\frac{1}{2}\sum_{\substack{i,j=1\\i\neq j}}^{A}\sum_{k=1}^{B}g_{ik}g_{jk}\prod_{\substack{l=1\\l\neq i,j}}^{A}\prod_{\substack{m=1\\m\neq k}}^{B}(1-g_{lm})+\dots$$

$$g_{ij}=\gamma(\vec{b}-\vec{s}_i+\vec{\tau}_j)+\gamma^*(\vec{b}-\vec{s}_i+\vec{\tau}_j)-\gamma(\vec{b}-\vec{s}_i+\vec{\tau}_j)\gamma^*(\vec{b}-\vec{s}_i+\vec{\tau}_j),$$

where $\gamma(\vec{b})$ is the amplitude of elastic NN-scattering in the impact parameter representation. The first expansion term gives the probability that only one nucleon in nuclei A and B will be touched ("wounded"). The second one gives the probability that one nucleon in nucleus A and two nucleons in nucleus B will be

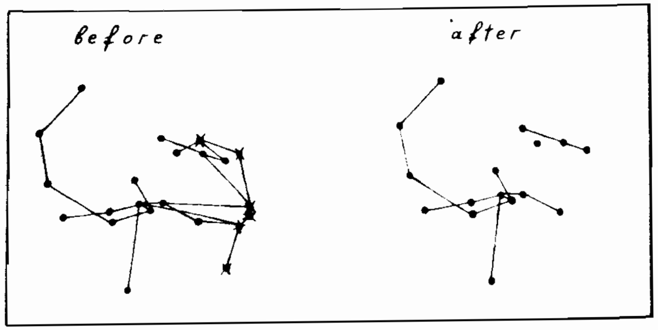

Fig. 4. The pattern of the ^{22}Ne nucleus and its fragments in the impact parameter plane before and after interaction with the ^{80}Br nucleus. The wounded nucleons are marked by crosses.

touched, etc. Removal of the "wounded" nucleons naturally leads to the destruction of the nuclei. In Fig. 4 the ^{22}Ne nucleus before and after interaction with the ^{80}Br nucleus is given as an example. As seen, taking into account the destruction of the percolation structure of the nucleus after the knocking-out of nucleons, one can in principle explain the phenomenon of multifragmentation of the nuclei. Unfortunately, the calculations show that the yield of the moderate mass fragments is strongly suppressed. To describe the experimental data one has to assume an abnormally high degree of excitation of the produced fragments. In this case the yields of light fragments with the masses in the 5-12 a.e.m. region are strongly suppressed. It is not a surprise, because it is known that the Glauber approximation should be supplemented by an additional assumption. In paper [13] it was suggested that every intranuclear collision results in 3-4 additionally knocked nucleons. Since this number is close to the value $\langle N_L \rangle$, we assumed that in the course of the fast stage the wounded nucleon leaves the nucleus together with its nearest neighbours. It considerably improves the results.

To take into account the excitation energy of the primary "hot" fragment we introduced the parameter ε^* into the model. It is the excitation energy per broken link. Obviously, the value of $\varepsilon^* \langle N_L \rangle$ should be close to the nucleon binding energy.

4. Choise of the ε^* value and description of the experimental data

To describe the decay of excited nuclei-fragments we use the evaporation-fission model described in paper [14]. The sampling of inelastic reactions according to the expression (4) was carried out by the Monte Carlo algorithm in the code DIAGEN [15]. The parameters of the NN elastic amplitude were taken from paper [16].

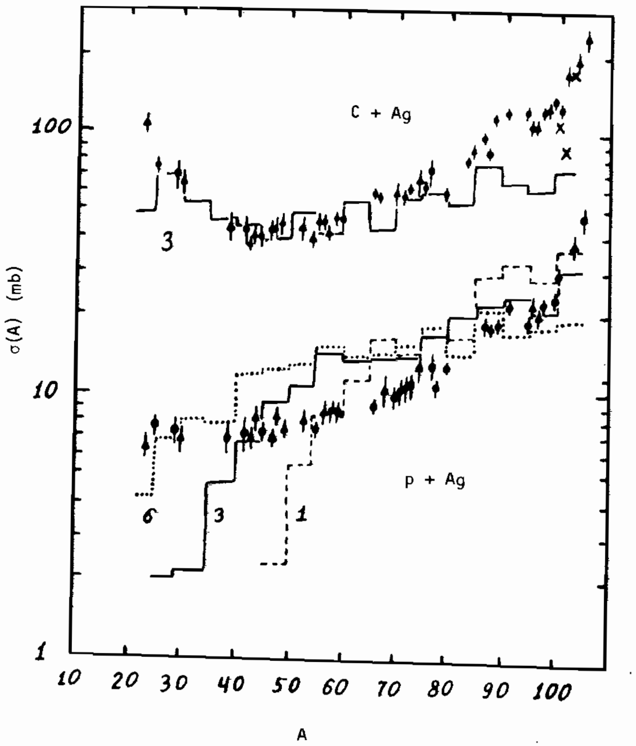

Fig. 5. Mass yield curves in the reactions p + ^{108}Ag (at 300 GeV) and ^{12}C + ^{108}Ag (25.2 GeV) (multiplied by 3). The points are the experimental data [17]. The histograms are our calculations at different values of the parameter ε^* (numbers in the figure).

The results obtained by varying the parameter ε^* are given in Fig. 5. The best description of the reaction p + ^{108}Ag we reached at ε^*=3 MeV is not quite good. At the same time the description of the reactions ^{12}C + ^{108}Ag, p + ^{64}Cu and ^{12}C + ^{64}Cu is quite satisfactory (see Fig. 6). We assume that the more correct consideration of the nuclear structure and the knock out the

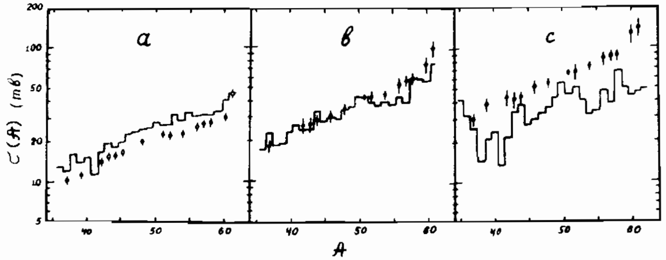

Fig. 6. Mass yield curves in the spallation reactions of ^{64}Cu by 3.9 GeV protons (a), 25 GeV ^{12}C (b) and 80 GeV ^{40}Ar (c). The histograms are our calculations at $\mathcal{E}^*=3$ MeV. The points are the experimental data [18,19].

nucleons from the nuclei (the cascading of the secondary particles) allows great improvement of the results.

5. Conclusion

On the whole the percolation approach is a good basis for the description of nuclear multifragmentation. If the percolation structure of the nucleus and its destruction in the nuclear reaction are taken into account, one can explain the main features of the nuclear multifragmentation process. However, the reaction products strongly depend on the following deexcitation process of the primary "hot" fragments. To describe the multifragmentation process quantitatively, there is a set of ways of the model improvement, e.g. more specific description of the percolation structure of nuclei and the process of deexcitation of "hot" fragments, taking into account the cascading of the secondary particles, etc. It is of great interest to explain the momentum spectra of the produced fragments within the framework of the percolation approach. This work is now in progress.

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Перколяция и мультифрагментация ядер

Предложен способ построения "холодных" ядер как перколяционных кластеров. В рамках определенных предложений о закономерностях разрывов связей между нуклонами в ходе ядерных реакций получено описание процесса мультифрагментации в адрон-ядерных и ядро-ядерных взаимодействиях при высоких энергиях.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Percolation and Multifragmentation of Nuclei

A method to build the "cold" nuclei as percolation clusters is suggested. Within the framework of definite assumptions of the character of nucleon-nucleon link breaking resulting from the nuclear reactions a description of the multifragmentation process in the hadron-nucleus and nucleus-nucleus reactions at high energies is obtained.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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