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объединенный институт ядерных исследований дубна

E2-89-58

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CAN QCD BE A PERTURBATION THEORY OF HADRONS?

The Talk on IV Adriatic Meeting on Particle Physics (Dubrovnik 12-22 IV, 1989)

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1989

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1. Introduction

QCD as a perturbative theory is applied mainly to describe the deep-inelastic reactions by means of calculations of the imaginary parts of the quark-gluon diagrams.[1,2]

The status of the perturbative theory is uniquely defined by the asymptotic freedom formula and the consideration of quarks and gluons as free particles on their mass-shell (i.e. partons).[3]

In this paper we attempt to construct a perturbative QCD of hadrons in order to give a unique description of the spectra of hadrons and their elastic and inelastic amplitudes.

There are two facts which prevent the construction of a perturbative QCD of hadrons: i) the consideration of quarks and gluons as the partons on their mass-shell, and ii) the generalization of the asymptotical freedom formula to the non-asymptotical low-momentum region (where it becomes a hypothesis). The first fact yields the "gauge independence" illusion meaning that to calculate the S-matrix one can use any gauge and any time-axis of quantization. The second fact leads to another illusion of "strong coupling" which means that the perturbation theory of QCD is not valid outside of the asymptotical region. To remove these illusions, we shall use the theoretical (QED for atoms) and practical (the potential model) experiences.

The paper is organized as follows. In Section 2., the "gauge independence" illusion and its elimination by the minimal quantization method [4-6] are considered. Section 3. is devoted to the second illusion and its removal by means of a rising potential. In Section 4., the perturbative QCD of gluons and their bound states is discussed. Section 5., presents the perturbative theory of quarkonia, where the unification of the potential model with the effective chiral Lagrangians is demonstrated.

2. Removal of the "gauge independence" illusion and the minimal quantization method

2.1. Relativistic covariance of bound states

Bound state computations in QED are carried out mainly in the radiation (Coulomb) gauge.[7] This gauge has a number of advantages. In particular, it provides a natural separation of binding (the instantaneus Coulomb) and perturbating (the transversal-photon exchange) interactions. However, the radiation gauge is not manifestly covariant. The dominating belief is that the relativistic covariance of bound states is realized by transition to the covariant gauge [8,9]

In the literature different definitions of the transition from one gauge to another are used. Here by this transition we understand a modification of the Feynman rules [10]

As well known, the Green functions, S-matrix and all physical results are invariant under the substitution of variables [10]

$$A_i^T = V\,[A^g]\,(\,A_i^g + \partial_i\,)\,V\,[A^g]^{-1},$$

$$\Psi^T = V\,[A^g]\,\Psi^g, \qquad (2.1)$$

where $V\,[A^g]$ transforms the Coulomb gauge fields $(\,A^T,\,\Psi^T\,)$ to covariant gauge fields $(\,A^g,\,\psi^g\,)$. This substitution is equivalent to the modification of the Feynman rules (i.e. the gauge change) and inclusion of spurious diagrams induced by $V[A^g]$ (which do not follow from the initial Lagrangian). Such additional diagrams do not contrbute on the mass-shell, and the invariance under the gauge change takes place. But off the mass-shell the dependence on a gauge takes place. In this case the change of a gauge by the Feynman rule modification is equivalent to the inclusion of the spurious diagrams (owing to the invariance under (2.1)).

There is a set of works[8,9] devoted to the proof of the gauge independence of an atom spectrum. In these treatments, the Coulomb binding interaction is used in the rest frame with the choice of the time-axis $\eta_\mu = (1,0,0,0)$. However, all the authors have not taken into account that the vector η_μ (contained in the Coulomb part of the interaction) indeed can be arbitrary, and that a transition from one vector η_μ to another η'_μ ($\eta'^2_\mu = 1$) is realized by means of a special change of the gauge.

It is easy to check that the usual Lorentz transformation ($P \to P'$) or special gauge change ($\eta \to \eta'$) break the dispersion law (i.e. $P'^2 \neq M^2_B$).[11] The dispersion law is invariant only under a combination of the usual Lorentz and special gauge transformations ($P \to P', \eta_\mu \to \eta'_\mu$) satisfying the paralellism of the time-axis to the total momentum ($\eta_\mu \sim P_\mu$, $\eta'_\mu \sim P'_\mu$) . This combined transformation has been pointed out at first by Heisenberg and Pauli,[12] and the parallelism is equivalent to the Markov-Yukawa description of nonlocal fields,[13] i.e. to the choice of the relative space and time with respect to eigenvectors of the total momentum operator.

Thus, we have seen that the dependence of bound state calculations on a gauge does not only exist, but it is necessary to provide the relativistic covariance. Certainly, if we take into account the spurious diagrams (in accordance to eq.(2.1)) in the transition from one gauge to another, then the bound state calculation results do not change. The radiation gauge is unique which does not demand these spurious diagrams to reproduce the observed Lamb-shifts in atomic spectra.

What is a reason of the peculiarity of the radiation gauge? And why does the relativistic transformation changing a gauge arise (in the off mass-shell case)? These questions get a natural explanation in the minimal quantization scheme[4-6].

2.2. The minimal quantization scheme

The Feynman rules in the radiation gauge and the relativistic transformation accompanied by the gauge change are justified by the minimal quantization scheme of gauge field theories that has been formulated in refs.[4-6] as the following axioms:

i) the energy-momentum tensor is the Belinfante one

$$T_{\mu\nu} = F_{\mu\lambda}(x)F^{\lambda}{}_{\nu}(x) + \bar{\Psi}(x)(i\gamma_{\mu}\nabla_{\nu})\Psi(x) - g_{\mu\nu}L(x) + \frac{i}{4}\partial_{\lambda}[\bar{\Psi}(x)\Gamma^{\lambda}{}_{\mu\nu}\Psi(x)] , \tag{2.2}$$

where $\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2}[\gamma^{\lambda},\gamma_{\mu}]\gamma_{\nu} - g_{\mu\nu}\gamma^{\lambda} - g^{\lambda}_{\nu}\gamma_{\mu}$, $L(x)$ and $F_{\mu\nu}(x)$ are the Lagrangian and the usual tension, respectively, and ∇_{μ} is the covariant derivative;

ii) all the physical quantities (the Hamiltonian and the

Belinfante tensor) are defined on the explicit solution of the Gauss equation for the time component of the gauge field A_o, i.e.

$$\frac{\delta L}{\delta A_o} = 0;$$

iii) the minimal set of only physical variables is quantized by diagonalization of the Belinfante Hamiltonian.

For example, in the case of QED, the substitution of the Gauss equation solution

$$A_o[A] = \frac{1}{\partial^2} (\partial_i \partial_o A_i + j_o),$$

into expression (2.2.) leads to the nonlocal transversal variables (since $F_{oi} = \partial_o A_i - \partial_i A_o[A] \equiv (\delta_{ij} - \partial_i \partial^{-2} \partial_j)\partial_o A_i - -\partial_i \partial^{-2} j_o^T$)

$$\left.\begin{aligned} \hat{A}_i^T[A] &= V[A] (\hat{A}_i + \partial_i) V^{-1}[A] \\ \Psi^T [A] &= V[A] \Psi \end{aligned}\right\} \quad V = \exp(\frac{1}{\partial^2} \partial_i \hat{A}_i), \hat{A} = ieA \; . \tag{2.3}$$

These nonlocal variables are invariant under the gauge transformation of the initial fields (A_i, Ψ). The usual Lorentz transformation of the initial fields with the parameters ε_k ($\delta_L^o A_k = \varepsilon_k A_o + \varepsilon_i (x_i \partial_t - t\partial_k) A_k$), on the solution of the Gauss equation, leads to the following transformation for the transversal variables as the functionals on A and Ψ, i.e.

$$\begin{aligned} A_k^T [A_i + \delta_L^o A_i] - A_k^T [A_i] &= \delta_L^o A_k^T - \partial_k \Lambda \\ \Psi^T [A_i + \delta_L^o A_i , \Psi + \delta_L^o \Psi] - \Psi [A_i, \Psi] &= \delta_L^o \Psi^T + ie\Lambda\Psi^T, \end{aligned} \tag{2.4}$$

where

$$\Lambda = \varepsilon_k \frac{1}{\partial^2} (\partial_o A_k^T + \frac{\partial_k}{\partial^2} j_o)$$

is a special gauge transformation parameters that changes the time-axis η_μ : $\eta_\mu \longrightarrow \eta'_\mu + (\delta^0_L \eta)_\mu$.

The same transformation laws (2.4.) are valid for a usual boost of the quantum fields, for example,

$$i\varepsilon_k [M_{ok}, \Psi^T] = \delta^0_L \Psi^T + i\theta\Lambda\Psi^T ,$$

where $M_{ok} = \int d^3x \, (T_{oo} \, x_k - T_{ok} \, t)$.

It should be noted that the minimal quantization is unique for which the relativistic transformation laws for the classical variables (2.4) and the quantum ones (2.4) coincide.

In the quantum perturbation theory the combined transformation (2.4) leads to additional diagrams with the vertex Λ defined by the Belinfante tensor (2.2) [4,6]. As we have seen in the previous subsection, such additional (spurious) diagrams are equivalent to the change of the gauge.

The consistent minimal quantization scheme yields the result that the radiation gauge is a unique one that appears without gauge fixing as an initial assumption [4,6].

But the minimal quantization scheme leaves a freedom for the choice of the initial time-axis (η). As we have seen, the choice of η must be done under the Markov-Yukawa principle [13] ($\eta_\mu \sim P_\mu$), which provides the relativistic invariance of the dispersion law.

For an interaction of many bound states described by the bilocal fields M(x,y) we suggest to choose the time-axis for a bound state as an eigenvector of its total momentum operator :

$$\hat{P}_\mu \, M(x,y) = \frac{1}{i} \frac{\partial}{\partial X_\mu} M(x,y) \qquad (2.5)$$

where $X = (x + y)/2$ is the total coordinate.

As a result, the spectrum of fermionic bound states and their interactions in the neglect of radiative corrections (due to the transversal-field exchange) can be described by the effective action[14]

$$S_{eff} = \int d^4x \left\{ \bar{\Psi}(x) \; (i\partial\!\!\!/ - m^0 \;) \Psi(x) \right. \tag{2.6a}$$
$$\left. - \frac{1}{2} \int d^4y \; \Psi_{\beta_2}(y)\bar{\Psi}_{\alpha_1}(x)\left[K^{\eta}(x-y)\right]_{\alpha_1\beta_1;\alpha_2\beta_2} \; \Psi_{\beta_1}(x)\Psi_{\alpha_2}(y) \right. ,$$

where the kernel K^{η} has the form

$$\left[K^{\eta} \; (x) \; \right]_{\alpha_1\beta_1;\alpha_2\beta_2} = \eta\!\!\!/_{\alpha_1\beta_1} V(x^{\perp}) \; \delta(x\eta) \; \eta\!\!\!/_{\alpha_2\beta_2} \tag{2.6b}$$

$$\left(x^{\perp}_{\mu} = x_{\mu} - x^{\parallel}_{\mu} \;\; , \;\; x^{\parallel}_{\mu} \; = \; \eta_{\mu}(x\eta), \;\; \eta\!\!\!/ = \gamma^{\mu}\eta_{\mu} \right).$$

$V(x)$ is a potential. This action according to ref.[13] is relativistic invariant.

Such explicit construction of the nonlocal physical variables for the non-Abelian gauge theory has been done in refs.[4-6]. It has been shown that this construction of the variables contains their topological degeneration (due to the homotopy group $\Pi_3(SU(3)_c) = Z$), and leads to a confinement mechanism as a destructive interference of the phase factors of the generation[5]. The minimal quantization axioms (i) and (ii) can explain the quark-hadron duality in the spirit of the t' Hooft mechanism[15].

Generalization of the minimal quantization scheme to the non-Abelian theory requires an additional axiom:

iv) the transition to purely transversal nonlocal variables (like (2.6)) which are singled out by the principle of the correspondence to the perturbative theory.

The minimal quantization can be reduced[4] to the explicit gauge invariant construction of the Schwinger operator quantization of the non-Abelian theory [16,17] with the Hamiltonian

$$H(g^2D) = \int d^3x \left[\frac{1}{2} (E_i^a(x))^2 + \frac{1}{4} (F_{ij}^a(x))^2 + \bar{q}(x)(i\gamma_k\nabla_k + m^0)q(x) + \right. \tag{2.7}$$
$$+ \frac{1}{2}\int d^3x \; d^3y \; J^a_{tot}(x) \left[\; g^2 D^{ab}((x-y|A) \; \right] J^b_{tot}(y) \; + \text{ nonlocal Schwinger terms.}$$

Here $\nabla_k = \partial_k + g\, A^a_k \lambda^a/2$, $F_{ij} = \partial_i A^a_j - \partial_j A^a_i + g\, f^{abc} A^b_i A^c_j$, $J^a_{tot} = q^+(\lambda^a/2)q + f^{abc} E^b_i A^c_i$, $\partial_i A_i = \partial_i E_i = 0$, g is the coupling constant, and the function $D^{ab}((x-y|A)$ satisfies the equation

$$\left[(\nabla_i\partial_o)\frac{1}{\partial^2} (\nabla_i\partial_o) \right]^{ac} D^{cb}((x-y|A) = \delta(x-y)\, \delta^{ab} \tag{2.8}$$

(where $\nabla_i^{ab} = \delta^{ab}\, \partial_i + g\, f^{abc}\, A^c_i$).

As to the Schwinger terms, they are defined from the Lorentz covariance condition[4-6].

It is easy to see from (2.7) and (2.8) that in the case of QED the function $D(x-y|A)$ is the same but not the Coulomb propagator. So, the interaction part of Hamiltonian (2.7) reproduces the Coulomb gauge description.

We use Hamiltonian (2.7) as a starting point for constructing the perturbative QCD of hadrons ($QCD_{hadrons}$).

3. Removal the "strong coupling" illusion or the infrared redefinition of the QCD Hamiltonian

3.1. The infrared divergencies of QCD

From the point of view of the description of atoms in QED, there two types of the interaction: the "static" one that determines the nonperturbative structure of the bound state, and the "dynamic" one that is considered as a perturbative correction. The QCD Hamiltonian (2.7) allows the same separation due to its Coulomb gauge determination, and bears a new problem.

The non-Abelian Hamiltonian (2.7) leads to a new type of the "static" divergencies in corrections to the Coulomb potential (which are absent in QED). Because of such new divergences the Hamiltonian is not a correct mathematical object, unlike the QED one.

For the infrared redefinition of the QCD Hamiltonian the asymptotical freedom formula (which is theoretically correct only for a large momentum[10]) is not enough.

3.2. The rising potential ansatz

The lattice calculations[20] and the potential quark model phenomenology[21] point out the rising potential in the small momentum region. This potential is used to describe the constituent quark masses[12,22], and therefore it can be used to avoid the infrared problem.

As the rising potential does not play a significant role at the small distances (in particular, in the heavy-quark sector) we think that the modification of Hamiltonian (2.7) by the substitution

$$H\,[g^2 \mathcal{D}(r\,|\,A)] \longrightarrow H\,[V_R(r) + g^2 \mathcal{D}(r\,|\,A)] \tag{3.1}$$

(where V_R is the rising potential) is enough to give the correct infrared definition of the perturbative theory for Hamiltonian (2.7) and to take into account the nonperturbative effects like the constituent masses of quarks and gluons as well as their bound states.

The QCD with the "minimal" Hamiltonian (2.7) and the rising potential ansatz (3.1) will be called by us the QCD of hadrons (QCD_h)[14], in contrast to the QCD of partons.

Now, we should consider the constituent gluon mass and the glueball mass spectrum in the lowest order in the coupling constant (g), and estimate the modified running coupling constant in order to be convinced in the validity of new perturbative QCD of hadrons, i.e. QCD_h .

4. The QCD-description of the gluon physics

4.1. The single-gluon energy

In the purely gluonic sector, in the lowest order in g^2 , we obtain the following Hamiltonian

$$H = \int d^3x \left[\frac{1}{2} (E_i^a(x))^2 + \frac{1}{2} (\partial_i A_j^a(x))^2 \right] + \frac{1}{2} f^{abe} f^{ecd} \int d^3x\, d^3y\, E_i^a(x)\, A_i^b(x)\, V_R(x-y)\, E_j^c(y)\, A_j^d(y). \tag{4.1}$$

For simplicity we consider the oscillator potential[14]

$$V_R(r) = V_0\, r^2, \qquad V_0 = (234\ \mathrm{MeV})^3. \tag{4.2}$$

The fields E_i^a and A_i^a have the following decomposition over creation and annihilation operators $a_k^{b(\mp)}$

$$E_j^b = i\int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{\omega(k)}{2}} \left\{ \exp\left[i(\omega(k)t - kx) \right] e_j^r a_r^{b(+)}(k) - \exp\left[-i(\omega(k)t + kx) \right] e_j^r a_r^{b(-)}(k) \right\},$$

$$A_j^b = \int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{1}{2\omega(k)}} \left\{ \exp\left[i(\omega(k)t - kx) \right] e_j^r a_r^{b(+)}(k) + \exp\left[-i(\omega(k)t + kx) \right] e_j^r a_r^{b(-)}(k) \right\}. \tag{4.3}$$

Here $k_j e_j^r = 0$, $e_i^r e_j^r = \delta_{ij} - \frac{k_i k_j}{k^2}$, $k = |\mathbf{k}|$, the operators $a^{(\pm)}$ satisfy the commutator relations

$$\left[a_r^{b(-)}(k), a_{r'}^{c(+)}(q) \right] = \delta^{bc} \delta_{rr'} \delta(k-q),$$
$$\left[a_r^{b(\pm)}(k), a_{r'}^{c(\pm)}(q) \right] = 0,$$

and the single particle energy $\omega(k)$ is defined as the average of the Hamiltonian (4.1) over the one-gluon states $|\, b, r, k >_g$ with the quantum numbers b, r and the momentum k

$$_g< b', r', k' |H| b, r, k >_g = \omega(k)\ \delta(k-k')\delta^{bb'} \tag{4.4}$$

with
$$|b, r, k>_g = a_r^{b(+)}(k)\,|0>.$$

After the substitution of (4.3) into (4.1) expression (4.4) can be rewritten as the following equation for $\omega(k)$

$$\frac{\omega(k)}{2} + \frac{k^2}{2\omega(k)} - V_0 N_c \left\{ \left[\frac{1}{2\omega(k)} \frac{d\omega(k)}{dk} \right]^2 - \frac{1}{k^2} \right\} = \omega(k), \tag{4.5}$$

where the left-hand side corresponds to the three terms of Hamiltonian (4.1) . To obtain the solution of (4.5) two numerical methods are used : the "shooting"[23] and the

Runge-Kutta-Gill methods[24]. Both give similar results (the solution is shown in Fig. 1).

In dimensionless variables the asymptotic behaviour is the following

$$\underline{\omega}(\underline{k}) \xrightarrow[\underline{k} \to 0]{} \frac{2}{\underline{k}^2}$$

$$\text{with} \begin{pmatrix} \underline{\omega} \\ \underline{k} \end{pmatrix} = (N_c V_0)^{-1/3} \begin{pmatrix} \omega \\ k \end{pmatrix} \qquad (4.6)$$

$$\underline{\omega}(\underline{k}) \xrightarrow[\underline{k} \to \infty]{} \underline{k}$$

Thus the gluons effectively acquire the structure mass depending on the momentum ($m_g(k^2) = \sqrt{\omega^2(k) - k^2}$) and such that $m(0) = \infty$.

We see that the increasing potential leads to the appearance of a mass for massless color particles, i.e. it has infrared regularizing properties.

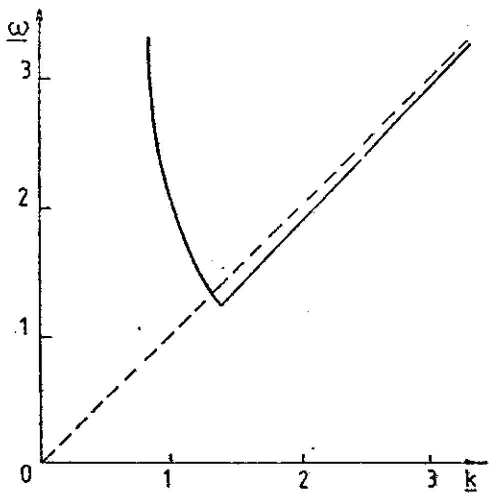

Fig.1. The solution to eq.(4.5) for the gluon spectrum expressed in units of the energy scale $(V_0 N_c)^{1/3}$, ($\underline{\omega} = (V_0 N_c)^{-1/3}\omega$, $\underline{k} = (V_0 N_c)^{-1/3} k$), where the dashed and solid lines correspond to the free and bounded (in a hadron) gluons , respectively.

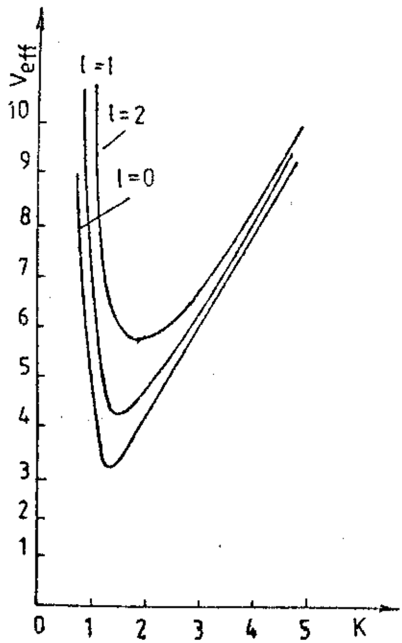

Fig.2. The effective potential for the spinless glueballs where l is the orbital moment characterizing the orbital excitation of a glueball.

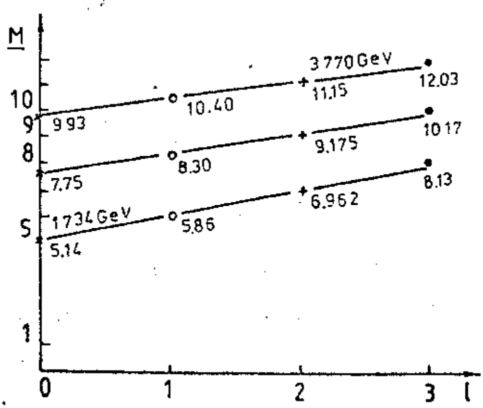

Fig.3. The masses of glueballs with several orbital moments, in units of the energetic scale, $\underline{M} = (V_o N_c)^{-1/3} M$

4.2. The two-gluon (glueball) spectrum

Now let us consider the simplest bound state of two gluons : the scalar (spinless) glueball. They are a linear combination of the two-particle eigenstates of the Hamiltonian

$$H|b,r,\mathbf{k};c,s,-\mathbf{k}>_{gg}=M|b,r,\mathbf{k};c,s,-\mathbf{k}>_{gg} , \tag{4.7}$$

where M is the glueball mass and the state is defined by the action of the creation operators on the vacuum

$$|b,r,\mathbf{k};c,s,-\mathbf{k}>_{gg} = a_r^{b(+)}(\mathbf{k})\ a_s^{c(+)}(-\mathbf{k})\ |0> .$$

For these states the spectral equation has the form

$$M\Phi_G(k_o,\mathbf{k})=2\omega(\mathbf{k})\Phi_G(k_o,\mathbf{k})-\frac{1}{2}N_c V_0\left\{\left[\frac{1}{\omega(\mathbf{k})}\frac{d\omega(\mathbf{k})}{d\mathbf{k}}\right]^2-\frac{4}{\mathbf{k}^2}+2\Delta_{\mathbf{k}}\right\}\Phi_G(k_o,\mathbf{k}) \tag{4.8}$$

with

$$\Delta_{\mathbf{k}} = \frac{\partial^2}{\partial \mathbf{k}^2} , \qquad \Phi_G(k_o,\mathbf{k}) = \sum_{b,r} |b,r,\mathbf{k};b,r,-\mathbf{k}>.$$

Φ_G is the wave function of the glueball G. The substituion of the standard decomposition of Φ_G into (4.8) gives for $f_l = k\ \psi_l(k)$ the "radial" equation ($\Phi_G = \sum_{l,m} f_l(k)\ Y_{lm}(\theta,\varphi)$)

$$\frac{d^2 f_l(k)}{d\, k^2} + \left[\ M - V_l(r)\ \right]\ f_l(k) = 0\ ,\ V_l(k) = \frac{3}{2}\omega(k) + \frac{k^2}{2\omega(k)} + \frac{l(l+1)+1}{k^2} .$$

Here $V_l(k)$ is the effective potential (see Fig. 2) with the quantum number l of the orbital momentum. The masses M and the "radial" wave functions $\psi_l(k)$ were found numerically by the "shooting" method[24] and are shown in Figs. 3 and 4 . One can see that the values for the glueball masses are in the region expected up-to-date[25].

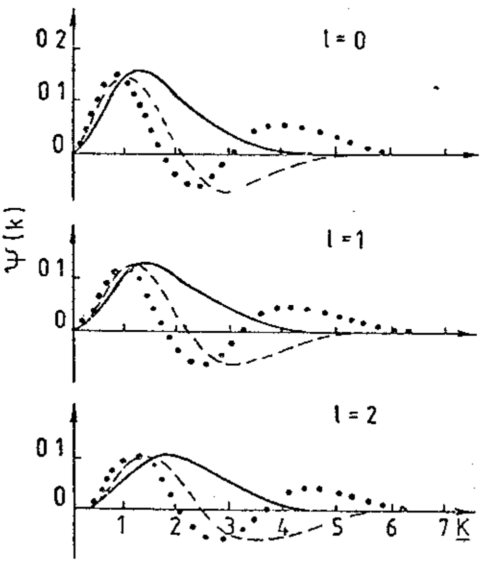

Fig.4. The "radial" wave functions of glueballs with the orbital moments l =0, 1, 2 (where $\underline{k} = (V_o N_c)^{-1/3} k$ is the dimensionless relative momentum).

4.3. The running coupling constant

A new perturbation theory in g^2 has to be formulated in terms of quasiparticles (quarks and gluons with nonzero structure masses). It is easy to see that the Green function of the transversal gluon

$$D^{mod}_{ij}(k_o,k)=(\delta_{ij}-\hat{k}_i\hat{k}_j)\frac{1}{k_o^2-\omega^2(k)-i\varepsilon}\ ,\ \hat{k}_i=\frac{1}{|k|}k_i \qquad (4.9)$$

vanishes in the region of a small k and changes into the standard parton Green function for large momenta k(k ≥ 300 MeV).

The asymptotical behaviour of (4.6) in the propagator of (4.9) eliminates all infrared divergences and modifies the usual formula of the asymptotic freedom in the region of small momenta.

To compare the modified formula with the experimental data, it is enough to estimate it with a one-loop diagram and propagator (4.9). We substitute such one-loop diagram, or a modified polarization operator

$$\Pi^{mod}(Q^2) = \frac{i}{\pi^2} \int d^4q \; D^{mod}(q+Q) \; D^{mod}(q), \qquad D^{mod} = \frac{1}{q_0^2 - \omega^2(q)}$$

into eq. (1.5) instead of $\ln\frac{Q^2}{\Lambda^2}$. In this way we obtain the infrared modified running coupling constant having a finite limit

$$\alpha_s^{mod}(0) = \frac{1}{\beta\,[1 + \ln(\frac{4N_c\,V_0^{1/3}}{\Lambda})^2]} \approx 0.2, \qquad \beta = \frac{11}{4\pi}.$$

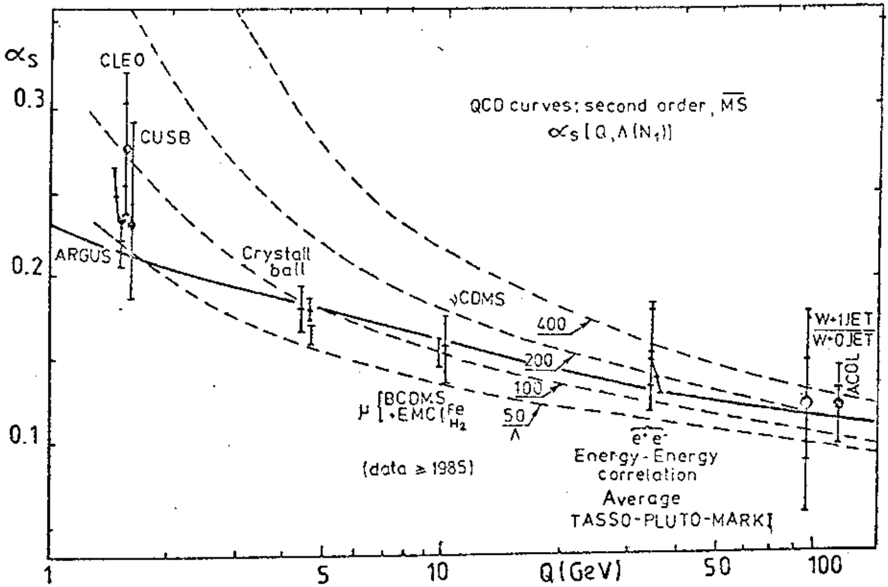

Fig.5. The dependence of α_s on the momentum Q and parameter Λ. Given are the experimental data and theoretical (dashed lines) values obtained by the asymptotical freedom formula. The solid line corresponds to the modified formula with $\alpha_s^{mod}(Q=0) = 0.24$ when $N_f = 0$, $N_c = 3$ and $\Lambda = 110$ MeV.

This modified running constant α_s^{mod} (Q^2) is in the whole region smaller than α_s^{mod} (0) $\lesssim$ 0.2 , and therefore one can use the perturbation theory for all transfer momenta Q^2 (see Fig.5).

In this approach, it seems better to work with the parameters $\alpha_s^{mod}(0)$ and V_0 rather than with the parameter Λ (since the parameter Λ can be expressed through $\alpha_s^{mod}(0)$ and V_0).

5. Unification of the potential model with the effective chiral Lagrangians

5.1. Bilocal Lagrangians and the choice of the time-axis

In describing the quarkonia in the framework of QCD_h one can neglect the transversal fields (A_i^T) in Hamiltonian (2.7) . As a result, one gets the following effective action for the quark sector in the color-singlet channel (the color factor $\lambda^n_{(a)}\lambda^n_{(b)}/4$, in this channel, equals $(4/3\ \delta_{(ab)})$[14,20]:

$$S_{eff} = \int d^4x \Big\{ \bar{q}(x) \Big[G^{-1}_{\tilde{m}^o} \Big] q(x) - \frac{1}{2N_c} \int d^4y\ q_{\beta_2}(y)\bar{q}_{\alpha_1}(x) \Big[K^{\eta}(x-y) \Big]_{\alpha_1\beta_1;\alpha_2\beta_2} * \\ * \ q_{\beta_1}(x)\bar{q}_{\alpha_2}(y) \Big\} . \qquad (5.1)$$

Here $G^{-1}_{\tilde{m}^o} = i\,\partial\!\!\!/ - \tilde{m}^o$ is the Dirac operator for free quarks with the bare masses $\tilde{m}^o = (m^o_1,\ldots,m^o_{n_f})$, α or β are the short notation for the Dirac and flavour indices, K^{η} is the instantaneous interaction (kernel) with the defined time-axis η_μ , i.e.

$$\left[K^{\eta}(x)\right]_{\alpha_1\beta_1;\alpha_2\beta_2} = \eta\!\!\!/_{\alpha_1\beta_1} V(x^{\perp})\,\delta(x\eta)\,\eta\!\!\!/_{\alpha_2\beta_2}\ ,\ \eta^2=1\ , \tag{5.2}$$

and $V(x^{\perp})$ is the sum of the Coulomb and oscillator potentials,

$$V(r) = \frac{4}{3}\left(-\frac{\alpha_s}{r} + V_o r^2 \right), \qquad (r = |x^{\perp}|) \tag{5.3}$$

For the relativistic description of the mass spectrum of mesons and their interactions the Markov–Yukawa principle should be used (as has been stated in Section 2), i.e.

$$\eta \sim \hat{P} = \frac{1}{i}\frac{\partial}{\partial X}\ , \qquad X = \frac{1}{2}(x+y) \tag{5.4}$$

Action (5.1), rewritten shortly[26] as

$$S = \left[(q\bar{q},\ -G^{-1}) - \frac{1}{2N_c}(q\bar{q},\ K^{\eta}\ q\bar{q}) \right],$$

in terms of the Legendre transformation can lead to the linearized (on $q\bar{q}$) form

$$S = \left[q\bar{q},\ (-G^{-1} + M) \right] + \frac{N_c}{2}\left[M,\ [K^{\eta}]^{-1}\ M \right],$$

where $M(x,y)$ is the bilocal field[26,27]. After quantization over the quark fields we get the following effective action[14,26]

$$S_{eff}[M] = N_c\left\{ \frac{1}{2}(M,(K^{\eta})^{-1}M) - i\ \mathrm{Tr}\ \ln\left[-G^{-1}_{\hat{m}^o} + M \right] \right\}, \tag{5.5}$$

where Tr means both the integration over continuous variables and the trace over discrete indices.

Extremum condition for action (5.5), $\delta S_{eff}/\delta M = 0$, coincides with the Schwinger–Dyson equation for the quark mass operator Σ (when $M = \Sigma$)

$$\Sigma(x-y) = \hat{m}^o\delta^4(x-y) + i\ K^{\eta}(x-y)\ G_{\Sigma}(x-y) \tag{5.6}$$

where $G_\Sigma^{-1}(x-y) = i \not\partial \delta^{(4)}(x-y) - \Sigma(x-y)$. This equation defines the spectrum of quarks and, in particular, the spontaneous generation of the dynamical quark mass.

Expansion of action (5.5) around the classical solution (5.6) over fluctuations, $M' = M - \Sigma$, gives the free part of the action

$$S_{free}(M) = \frac{N}{2}c\left\{(M,(K^{\eta})^{-1}M) + i\mathrm{Tr}(G_\Sigma M)^2\right\}, \qquad (5.7)$$

and the term describing the interaction of the bilocal fields has the form

$$S_{int}[M'] = i\sum_{n=3}^{\infty}\frac{1}{n}\,\mathrm{Tr}(G_\Sigma M')^n = i\sum_{n=3}^{\infty}\frac{1}{n}\,\Phi^n = \sum_{n=3}^{\infty}W^{(n)}, \qquad (5.8)$$

where the field Φ

$$\Phi = \Phi(x,y) = \int d^4z\; G_\Sigma(x,z)\, M'(z,y) \;,$$

is introduced for convenience , and Tr is to be understood as

$$\mathrm{Tr}\;\Phi^n = \mathrm{tr}\int d^4x_1 d^4x_2 \ldots d^4x_n\; \Phi(x_1,x_2)\Phi(x_2,x_3)\ldots\Phi(x_n,x_1)\;. \qquad (5.9)$$

Variation of (5.7) over M' leads to the homogeneous Bethe-Salpeter equation in the ladder approximation to the vertex function $\Gamma_{(ab)}(x,y)$ of the bound state

$$\Gamma_{(ab)}(x,y) = -iK^{\eta}(x-y)\int d^4z_1 d^4z_2\, G_{\Sigma(a)}(z_1,z_2)\Gamma_{(ab)}(z_1,z_2)G_{\Sigma(b)}(z_2-y) \qquad (5.10)$$

that must be considered with equation (5.6).

5.2. Constituent quark masses

Let us consider the Schwinger-Dyson equation (5.6) for a quark in the momentum space

$$\Sigma_{(a)}(k^{\perp}) = m^0_{(a)} - i\int\frac{d^4q}{(2\pi)^4}\,\underset{\sim}{V}(k^{\perp}-q^{\perp})\not\eta\, G_{\Sigma(a)}(q)\,\not\eta, \qquad (5.11)$$

where

$$\underset{\sim}{V}(k^{\perp})=\int dx e^{-ikx} V(x^{\perp})\ \delta(x^{\parallel}), \quad G_{\Sigma}(q)=\int dx e^{-iqx} G_{\Sigma}(x); \quad \eta\!\!\!/ = \eta_{\mu}\gamma_{\mu}\ .$$

$$\eta_{\mu} = P_{\mu} / \sqrt{P^2}$$

Separating the integration variable in (5.11) in longitudinal and transversal components $q_{\mu}=(q^{P},q^{\perp})$ and carrying out the integration over q^{P} with

$$i\frac{d^4q}{(2\pi)^4} = i\ \frac{d(q\eta)}{(2\pi)}\ \frac{d^3q^{\perp}}{(2\pi)^3}\quad , \tag{5.12}$$

one can easily see that the mass operator depends only on the transversal momentum $(k^{\perp})$

$$\Sigma_{(a)}(k^{\perp})=k\!\!\!/^{\perp}+\ S^{-2}_{(a)}(k^{\perp})E_{(a)}(k^{\perp});\ S^{2}_{(a)}(k^{\perp})=\exp\left\{-\hat{k\!\!\!/}^{\perp}[\phi(k^{\perp})-\ \frac{\pi}{2}\ ]\right\} \tag{5.13}$$

$$\hat{k}^{\perp}_{\mu} = \frac{1}{|k^{\perp}|}\ k^{\perp}_{\mu}$$

$S(k^{\perp})$ is the transformation matrix of the Foldy–Wouthuysen type. Equation (5.11) after integration over the longitudinal component $(q\eta)$ splits into two equations for $\phi_{(a)}(k^{\perp})$ and $E_{(a)}(k^{\perp})$

$$E_{(a)}(k^{\perp})\sin\phi_{(a)}(k^{\perp})=\ m^{o}_{(a)}-\ \frac{1}{2}\int\frac{d^3q^{\perp}}{(2\pi)^3}\ \underset{\sim}{V}(k^{\perp}-q^{\perp})\ \sin\phi_{(a)}(q^{\perp})$$

$$E_{(a)}(k^{\perp})\cos\phi_{(a)}(k^{\perp})=\ |k^{\perp}|+\ \frac{1}{2}\int\frac{d^3q^{\perp}}{(2\pi)^3}\ \underset{\sim}{V}(k^{\perp}-q^{\perp})(\hat{k}^{\perp}\hat{q}^{\perp})\cos\phi_{(a)}(q^{\perp})\,. \tag{5.14}$$

In the rest frame, where $\eta_{\mu}=(1,0,0,0)$, these equations coincide with the ones of ref.[12] In that work, the numerical solutions, in the case of the oscillator potential and $\hat{m}^{o} = 0$, yielding the spontaneous quark mass have been obtained. Equations (5.14) for the oscillator potential with $\hat{m}^{o}\neq 0$ have been solved numerically in ref[28] where it is shown that the effect of the spontaneous breakdown vanishes when $\hat{m}^{o}\gtrsim (4/3\ V_{o})^{1/3}\simeq$ 300 MeV (see

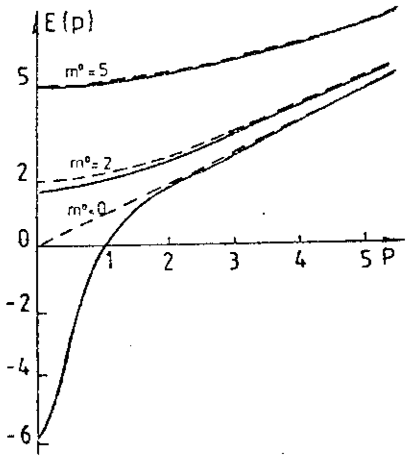

Fig.6. The numerical solution, E(p), to the Schwinger-Dyson equation ((5.19), (5.20)) for different bare quark masses m^0, in units of $(4V_0/3)^{1/3} \approx$ 300 MeV. Here $E_0(p) = (p^2 + (m^0)^2)^{1/2}$.

Fig.6). (Note that the Coulomb potential should yield renormalization of the bare mass, $\hat{m}^0$) It should be noted that the chiral symmetry breakdown has been essentially substantiated in ref[29] where the authors have discovered this phenomenon instead of the confinement suggested due to an increasing potential.(Remember that QCD_h has another confinement mechanism.)

5.3.New three-dimensional relativistic equation for a quarkonium

The spectrum of a quarkonium is described by the Bethe-Salpeter equation (5.10) that in the momentum space is written as

$$\Gamma_{(ab)}(k|P) = -i\int\frac{d^4q}{(2\pi)^4}\underset{\sim}{V}(k^{\perp} - q^{\perp})\eta'\left[G_{\Sigma_{(a)}}(q + \frac{P}{2})\Gamma_{(ab)}(q|P)G_{\Sigma_{(b)}}(q - \frac{P}{2})\right]\eta' . \qquad (5.15)$$

The one-particle Green function $G_{\Sigma_{(a)}}(q)=\left[\not{q}-\Sigma_{(a)}(q^{\perp}) \right]^{-1}$ can be represented in the form of the expansion over the states with positive and negative energies $(q\eta)$

$$G_{\Sigma_{(a)}}(q)= - \left[\frac{\Lambda^{(\eta)}_{+(a)}(q^{\perp})}{\Sigma_{(a)}(q^{\perp})-(q\eta)-i\varepsilon} + \frac{\Lambda^{(\eta)}_{-(a)}(q^{\perp})}{\Sigma_{(a)}(q^{\perp})+(q\eta)-i\varepsilon} \right], \tag{5.16}$$

where $\Lambda^{(\eta)}(q^{\perp})$ are the projectors

$$\Lambda^{(\eta)}_{\pm}(q^{\perp})= S(q^{\perp})\ \Lambda^{(\eta)}_{\pm}(0)\ S(q^{\perp}); \qquad \Lambda^{(\eta)}_{\pm}(0)=(\ 1\ +\ \not{\eta})/2. \tag{5.17}$$

The vertex function $\Gamma(k|P)$ as well as the mass operator $\Sigma(k^{\perp})$ depends only on the transversal momentum (ηP). The integral over the longitudinal momentum is easily performed with the equality $(\eta P)=\sqrt{P^2}$.

We denote this integral by $\psi_P(q^{\perp})$

$$\psi_P(q^{\perp}) \equiv i\int\frac{d(\eta P)}{2\pi}\ G_{\Sigma_{(a)}}(q+\frac{P}{2})\ \Gamma_{(ab)}(q|P)\ G_{\Sigma_{(b)}}(q-\frac{P}{2}) = $$
$$= \frac{\Lambda_{(+)(a)}(q^{\perp})\Gamma_{(ab)}(q^{\perp}|P)\Lambda_{(-)(b)}(q^{\perp})}{E_T(q^{\perp}) - \sqrt{P^2} - i\varepsilon} + \frac{\Lambda_{(-)(a)}(q^{\perp})\Gamma_{(ab)}(q^{\perp}|P)\Lambda_{(+)(b)}(q^{\perp})}{E_T(q^{\perp}) + \sqrt{P^2} - i\varepsilon}, \tag{5.18}$$

where E_T is the sum of the energies of two quarks

$$E_T(q^{\perp}) = E_{(a)}(q^{\perp}) + E_{(b)}(q^{\perp}) \tag{5.19}$$

being the solutions of Schwinger–Dyson equations for each quark.

Then equation (5.15) for the wave function ψ_P takes the form

$$\left[E_T(q^{\perp})-\sqrt{P^2}\right]\psi^{(+)}_P(q^{\perp})+\left[E_T(q^{\perp})+\sqrt{P^2}\right]\psi^{(-)}_P(q^{\perp}) = $$
$$= -\int\frac{d^3k^{\perp}}{(2\pi)^3}\underset{\sim}{V}(q^{\perp}-k^{\perp})\left[\psi^{(+)}_P(k^{\perp})+\psi^{(-)}_P(k^{\perp})\right], \tag{5.20}$$

where $\psi^{(\pm)}$ are the components of the expansion of the function ψ over the projectors

$$\psi_P^{(\pm)}(q^\perp) = \Lambda_\pm^{(\eta)}(q^\perp)\, \psi_P(q^\perp)\, \Lambda_\mp^{(\eta)}(q^\perp) . \tag{5.21}$$

According to eq. (5.18) ψ satisfies the identities

$$\Lambda_+^{(\eta)}(q^\perp)\, \psi\, \Lambda_+^{(\eta)}(q^\perp) = \Lambda_-^{(\eta)}(q^\perp)\, \psi\, \Lambda_-^{(\eta)}(q^\perp) \equiv 0 \tag{5.22}$$

which allow us to define the expansion of $\psi = \psi^{(+)} + \psi^{(-)}$ over the Lorentz structures

$$\psi_{(ab)}^{(\pm)} = S_{(a)}^{-1}(q^\perp) \left\{ \gamma_5\, L_{\pm\,(ab)}(q^\perp) + (\, \gamma_\mu - \eta_\mu\, \not{\eta}\,) N_{\pm\,(ab)}^\mu \right\} \Lambda_\pm^{(\eta)}(0) S_{(b)}^{-1}(q^\perp) \tag{5.23}$$

(The equations for the components $L_\pm = L_1 \pm L_2$; $N_\pm = N_1 \pm N_2$ are given in Appendix A).

Equations (5.20)-(5.23) are the relativistic covariant generalization of the Schrödinger equation with the rising potential for quarkonium.

Up to the one-particle energies and the projectors equation (5.20) coincides in the rest frame (η = (1,0,0,0)) with the Salpeter equation[30] got for the Coulomb potential. In the latter case the Schrödinger equation is a good approximation.

In our case of the sum of the rising potential (oscillator) and the Coulomb one the Schrödinger equation is a good approximation for heavy quarks with masses much larger than the energy scale of the rising potential ($\simeq$ 300 MeV) too.

But when the quark masses are much smaller than the scale, the solutions of equations (5.20)-(5.23), differ not only from the solutions of the Schrödinger equation but also from those of the quasi-potential equations[91]. In the latter case the contribution from the negative energy states is absent and one does not take into account the Schwinger-Dyson equation for the one-particle energy. Just these changes are very important for the proof of the Goldstone theorem.

Let us show that equations (5.14) and (5.20) describe the purely relativistic effect, the Goldstone mode which accompanies the spontaneous chiral symmetry breaking. The latter means that the nontrivial solution ($\sin\phi$) of equation (5.14) exists for the zero current mass $m^{o}=0$.

Comparing equation (5.14) for $m^{o}= 0$ with covariant equation (5.20) we see that the same function

$$\psi = \psi^{(+)} + \psi^{(-)} = \gamma_5 \frac{\sin\phi}{F}; \qquad (S^{-1}_{(a)}\gamma_5 S^{-1}_{(a)} = \gamma_5) \tag{5.24}$$

(where F is a mass scale parameter) is a solution of equation (5.20) with the eigenvalue $\sqrt{P^2} = 0$.

The proof of the Goldstone theorem in ref.[11] where the Salpeter equation is considered only in the rest frame (η=(1,0,0,0)) is not correct as for the Goldstone mode a state in the rest frame does not exist (in addition the Lorentz structures γ_5 and $\gamma_0\gamma_5$ have been confused, therefore the massless pion has been transformed into the fourth component of a vector particle).

Though there are inaccuracies in ref.[11] , the conclusion about the strong mass splitting of $\pi-$ and $\rho-$mesons and a qualitative structure of the light quarkonia spectrum (for the oscillator potential and m^{o})[11] are true and are in fair agreement with the experimental data.

Thus equations (5.14)-(5.20) for the sum of the rising and

Coulomb potentials qualitatively describe the spectroscopy of light and heavy quarkonia.

5.4 Quantization of the bilocal fields

For calculations of matrix elements of meson interactions it is necessary to use the expansion of the bilocal fields $M(x,y)$ over the orthonormalized set of solutions of equations (5.14)–(5.23) with eigenvalues $\sqrt{P^2} = M_H$ and energies $\omega_H = \sqrt{P^2 + M_H^2}$

$$M(x,y) = M(z|X) = \sum_H \int \frac{d^3 P}{(2\pi)^{3/2} \sqrt{2\omega_H(P)}} \int \frac{d^4 q}{(2\pi)^4} e^{iqz} *$$
$$* \left\{ e^{iPX} \Gamma_H(q^\perp|P) a_H^+(P) + e^{-PX} \bar{\Gamma}_H(q^\perp) a_H^-(P) \right\}, \qquad (5.25)$$

where H is a set of hadron quantum numbers

$$\Gamma_H(q^\perp|P) = (E_T - M_H) \psi^{(+)}(q^\perp) + (E_T + M_H) \psi^{(-)}(q^\perp)$$
$$\bar{\Gamma}_H(q^\perp|P) = (E_T - M_H) \psi^{(+)}(q^\perp) + (E_T + M_H) \psi^{(-)}(q^\perp). \qquad (5.26)$$

The annihilation and creation operators for mesons with the quantum numbers H satisfy the relations

$$\left[a_H^-(P), a_{H'}^+(P') \right] = \delta_{HH'} \delta^3(P-P'); \qquad \left[a_H^{\pm}, a_{H'}^{\pm} \right] = 0. \qquad (5.27)$$

The normalization of solutions of (5.21)–(5.23) is defined from the free action (5.9) (like in the local case[17])

$$\frac{N_c}{M_L} \int \frac{d^3 q^\perp}{(2\pi)^3} \left[L_1(q^\perp) L_2^*(q^\perp) + L_2(q^\perp) L_1^*(q^\perp) \right] = 1$$
$$\frac{N_c}{M_N} \int \frac{d^3 q^\perp}{(2\pi)^3} \left[|N_1^\mu(q^\perp) N_{2\mu}^*(q^\perp)| + |N_2^\mu(q^\perp) N_{1\mu}^*(q^\perp)| \right] = 1. \qquad (5.28)$$

For the calculation of (5.28) we use the expansion of (5.9)

$$(E_T \pm M_H) - [\, E_T \pm M_H)^2 / (E_T \pm \sqrt{P^2} \,)] = \frac{\omega_H}{M_H} (P_0 - \omega_H) + \ldots$$

The Green function for the bilocal fields in terms of the vertex function (5.26) has the form

$$G(q^{\perp}, p^{\perp} | P, Q) = (2\pi)^4 \, \delta^4(P-Q) \sum_H \left[\frac{\Gamma_H(q^{\perp}|P)\bar{\Gamma}_H(p^{\perp}|Q)}{(P_0 - \omega_H - i\varepsilon)\, 2\omega_H} - \frac{\Gamma_H(p^{\perp}|P)\bar{\Gamma}_H(q^{\perp}|Q)}{(P_0 + \omega_H - i\varepsilon)\, 2\omega_H} \right].$$

5.5. Matrix elements

Matrix elements for action (5.8) can be written in terms of the field operators $\Phi = G_H \cdot M$

$$\Phi(x,y) = \sum_H \int \frac{d^3 P}{(2\pi)^{3/2} \sqrt{2\omega_H}} \int d^4q \, e^{iqx} \left[e^{iPX} \Phi_H(q^{\perp}|P) a_H^{+}(P) + e^{-iPX} \bar{\Phi}_H(q^{\perp}|P) a_H^{-}(P) \right],$$

where

$$\Phi_H \; (q^{\perp}|P) = G_{\Sigma} \left(q + \frac{P}{2} \right) \Gamma_H \; (q^{\perp}|P)$$

$$\bar{\Phi}_H \; (q^{\perp}|P) = G_{\Sigma} \left(q - \frac{P}{2} \right) \bar{\Gamma}_H \; (q^{\perp}|P).$$

The matrix elements for the interaction $W^{(n)}$ (5.8) between the vacuum and states of n mesons is

$$\langle H_{P_1}, H_{P_2}, \ldots H_{P_n} | W^{(n)} | 0 \rangle = -(2\pi)^4 \, \delta^4\left(\sum_{i=1}^{n} P_i\right) \prod_{j=1}^{n} \left[\frac{1}{(2\pi)^3 \omega_{H_j}} \right]^{1/2} M^{(n)}(P_1 \ldots P_n)$$

$$M^{(n)}(P_1 ... P_n) = \frac{i}{(2\pi)^4} \int d^4q \sum_{\{i_k\}} \frac{1}{n} \Phi_{H_{i_1}}^{a_1 a_2}(q|P_{i_1}) \Phi_{H_{i_2}}^{a_2 a_3}(q - \frac{P_{i_1} + P_{i_2}}{2} | P_{i_2}) *$$

$$* \Phi_{H_{i_3}}^{a_3 a_4}(q - \frac{2P_{i_2} + P_{i_1} + P_{i_3}}{2} | P_{i_3}) ... \Phi_{H_{i_n}}^{a_n a_1}(q - \frac{2(P_{i_2} + ... + P_{i_{n-1}}) + P_{i_1} + P_{i_n}}{2} | P_{i_n}) \cdot \quad (5.30)$$

Expressions (5.25)-(5.30) give the Feynman rules for the quantum field theory in terms of bilocal fields.

5.6. Low-energy limit of bilocal action

As a next step let us consider the bilocal meson interaction in the low-energy limit. At first we define the pion constant F_π of leptonic decays by the formula

$$<l|S_L^{(2)}|\pi_P> = (2\pi)^4 \delta^4(P_\pi - P_l) \frac{(2\pi)^{-3/2}}{\sqrt{2\omega_\pi}} F_\pi \; P^\mu l_\mu(P), \quad (5.31)$$

where l_μ is the matrix element of the leptonic current.

The interaction of a meson with the leptonic current is defined by the redefinition of the mass operators ($\hat{m}^o \longrightarrow \hat{m}^o -$ $- \gamma_5 \; \not{l}(P) \; \exp(iP_L X)$) in the action (5.1). Then the action $S^{(2)}$ in (5.31) takes the form

$$S_L^{(2)} = iN_c \, tr \int d^4x_1 d^4x_2 d^4y_1 \mathcal{M}(x_1, x_2) G_{\Sigma_{(a)}}(x_2 - y_1) \gamma_5 \not{l}(P) \pi^+_{ab} e^{iP_2 y_1} G_{\Sigma_{(b)}}(y_1 - x_2) \cdot$$

$$(5.32)$$

The substitution of expansion (5.25) into eqs. (5.31) and (5.32) leads to the following expression for F_π

$$F_\pi = \frac{2N_c}{M_\pi} \int \frac{d^3q^\perp}{(2\pi)^3} \; L_2(q^\perp) \sin \phi(q^\perp) \cdot \quad (5.33)$$

If a pion corresponds to the Goldstone mode ($L_1=\frac{\sin\phi}{F}$) (see (5.24)), expression (5.33) coincides with the normalization condition for L_1 and L_2, when the equality $F=F_\pi$ takes place.

We can also find the solution of the equation (5.20) for L_2 which has the form

$$M_\pi\ L_2(q^\perp)=2E(q^\perp)L_1+\int\frac{d^3k^\perp}{(2\pi)^3}\underset{\sim}{V}(q^\perp-k^\perp)L_1(k^\perp) \tag{5.34}$$

up to the order $O((m^0)^2)$. If $L_1=\sin\phi\ /F_\pi$, one can see that the right-hand side of eq. (5.34) is equal to $2m^0/F_\pi$ (in accordance with eq. (5.14)). In this case we get $L_2 \simeq 2m^0/\ F_\pi\ m_\pi$.

The substitution of the last expression into the definition of F_π (5.33) and the conventional definition of the vacuum expectation value

$$\langle q\bar{q}\rangle = iN_c\ 2tr\int\frac{d^4q}{(2\pi)^4}\ G_\Sigma(q) = -\ 2N_c\int\frac{d^3q^\perp}{(2\pi)^3}\ \sin\phi$$

gives the well-known low-energy theorem

$$-2m^0\ \langle q\bar{q}\rangle = M_\pi^2\ F_\pi^2\ .$$

Thus, for light quarkonia the elements of the chiral Lagrangians here arise which very slightly depend on the form of the potential.

This independence of the bilocal fields of the potential in the low-energy region can easy be explained if we take into account that the bilocal meson field $\mathcal{M}'(x,y)$ is connected with the normalized wave function $\Psi(x^\perp - y^\perp)$ by the relation

$$\mathcal{M}'(x,y) \sim V(x^\perp - y^\perp)\ \Psi(x^\perp - y^\perp)\ \delta\ (\eta(x-y))\cdot$$

In the low-energy limit the wave function Ψ has a δ-type asymptotics $\Psi \sim \delta^3(x^{\perp} - y^{\perp})$, which is equivalent to the local approximation of the bilocal field

$$M(x,y) \sim \delta^4(x-y)\,.$$

The same limit arises in the case of a local potential or the local Bethe–Salpeter kernel

$$K(x-y) \sim \delta^4(x-y).$$

Just such a potential for the four-fermion interaction was the initial one in the original formulation[32] of the spontaneous chiral symmetry breaking.

Recently it has been shown in ref.[33] that a δ-type potential leads to the phenomenological chiral Lagrangians. In our case this Lagrangian occurs due to the asymptotical behaviour of the quarkonium wave function. Light mesons in the framework of our approach in terms of action (5.1) but with a δ-type potential were studied in ref.[34]

6. Conclusion

We have shown that the S-matrix for atoms and hadrons (unlike the S-matrix for asymptotical free elementary particles) depends on a gauge as the elementary particles are off mass-shell in the bound states.

The S-matrix for bound states should be constructed by the projection of the Belinfante energy-momentum tensor on the Gauss equation solution for the time component $A_0=(\eta A)$ (we call this a "minimal quantization"). The minimal quantization leads to the Feynman rules in the Coulomb gauge with the Heisenberg–Pauli transformation group that changes the gauge for a new relativistic frame. In QCD this quantization can explain the confinement as the destructive interference of phases of the topological degeneration of the physical variables constructed by the explicit solution of the Gauss equation[45].

The time component A_0 or the time-axis of quantization η is chosen in according to the Markov-Yukawa description of the nonlocal object in the quantum field theory: the time-axis is an eigenvector of the bound state total momentum operator.

In QED the Lamb shift calculation does not contradict these principles but rather confirms them.

We have shown that the QCD Hamiltonian determined in the infrared region by the rising potential ansatz, besides the parton model in the specific gauge, contains the nonrelativistic potential model for heavy quarkonia, the chiral Lagrangians for light quarkonia with their spectrum, the glueball physics and the modified asymptotical freedom formula with the small effective coupling constant in the whole region of transversal momenta.

The QCD_h can be applied to the description of decays of heavy quarkonia into light mesons.

Acknowledgements.

The authors thank Profs. B. De Witt, D.Ebert, V.A.Efremov, V.G.Kadyshevsky, M.Müller-Preußker, H.Nielsen and G.A.Vilkovyski for useful discussions.

Appendix A.

Let us consider the equation (5.20) in the terms of (5.23) in the rest frame $\eta = (1,0,0,0)$, $N^{\mu\perp} = (0,N^{i})$

$$M_L L_{2(1)}(k^{\perp}) = E_T L_{1(2)}(k^{\perp}) -$$

$$- \int \frac{d^3 q^{\perp}}{(2\pi)^3} V(k^{\perp}-q^{\perp}) \Big[A_{(a)} A_{(b)} + C^i_{(a)} C^i_{(b)} + (-) B^i_{(a)} B^i_{(b)}) \Big] L_{1(2)}(q^{\perp}) \tag{A.1}$$

$$M_N N^k_{2(1)}(k^{\perp}) = E_T N^k_{1(2)}(k^{\perp}) -$$

$$- \int \frac{d^3 q^{\perp}}{(2\pi)^3} V(k^{\perp}-q^{\perp}) \Big[(A_{(a)} A_{(b)} - C^i_{(a)} C^i_{(b)} + (-) + (-) B^i_{(a)} B^i_{(b)})\delta^{kl} +$$

$$+ C^k_{(a)} C^l_{(b)} + C^l_{(a)} C^k_{(b)} - (+) \; - (+) B^k_{(a)} B^l_{(b)} - (+) B^l_{(a)} B^k_{(b)} +$$

$$+ i\varepsilon^{kli} (C^i_{(a)} A_{(b)} + C^i_{(b)} A_{(a)}) \Big] N^l_{1(2)}(q^{\perp}) ,$$

where A, B^i, C^i are defined with the help of the matrix (5.13)

$$S^{-1}_{(a)}(q^{\perp}) S_{(a)}(k^{\perp}) = A_{(a)}(k^{\perp},q^{\perp}) + B^i_{(a)}(k^{\perp},q^{\perp})\gamma^i + C^i_{(a)}(k^{\perp},q^{\perp})(i\gamma_5\gamma_0\gamma^i)$$

$$A_{(a)} = A_{(a)}(k^{\perp},q^{\perp}) = c_{(a)}(k^{\perp}) c_{(a)}(q^{\perp}) + (\hat{k}^{\perp}\hat{q}^{\perp}) \, s_{(a)}(k^{\perp}) s_{(a)}(q^{\perp})$$

$$B^i_{(a)} = B^i_{(a)}(k^{\perp},q^{\perp}) = \hat{q}^{\perp}_i \, s_{(a)}(q^{\perp}) c_{(a)}(k^{\perp}) - \hat{k}^{\perp}_i \, s_{(a)}(k^{\perp}) c_{(a)}(q^{\perp})$$

$$C^k_{(a)} = C^k_{(a)}(k^{\perp},q^{\perp}) = -\varepsilon^{ijk} \, \hat{k}^{\perp}_i \, \hat{q}^{\perp}_j \, s_{(a)}(k^{\perp}) s_{(a)}(q^{\perp})$$

$$s_{(a)}(q^{\perp}) = \sin(\tfrac{1}{2}\tilde{\phi}_{(a)}(q^{\perp})) \, , \; c_{(a)}(q^{\perp}) = \cos(\tfrac{1}{2}\tilde{\phi}_{(a)}(q^{\perp})) \, , \; \tilde{\phi} = \phi - \frac{\pi}{2}$$

In ref's [11] the following errors have been done : i) the structures of (N_1, L_1) and (N_2, L_2) are confused, ii) the equation for $N^i_{1(2)}$ is true up to certain omitted terms.

The systems (A.1) , describe the spectrum of bound states with arbitrary potential [20].

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Received by Publishing Department
on January 30, 1989.