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ON THE TEMPERATURE OF THE BOSON CONDENSATE EVAPORATION AND THE BARYON ASYMMETRY OF THE UNIVERSE IN THE AFFLECK-DINE SCENARIO

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The baryogenesis scenario based on the Affleck and Dine model /1-5/ has at least two appealing features. First, it permits to realize baryosynthesis at relatively low temperature and, second, it is well compatible with the Universe inflation. However, one of the statements of the original proposal, namely, the result that the relative baryon number density $\beta = B/N_\gamma$ can be much larger than unity seems rather disturbing.

In this connection we are going to reanalyze the thermodynamics of the cosmological plasma in the presence of the boson condensate with a nonvanishing baryon charge. The thermodynamical equilibrium state of particles, produced after the decay of the scalar field χ with a given baryon charge B and energy density ρ, is considered. The dependence of the equilibrium state characteristics (such as, the plasma temperature, the chemical potentials of the particles, the baryon charge of the fermions, the boson condensate density) on the initial values of B and ρ is studied.

It is shown, that for sufficiently large values of **B** and ρ, χ condensate cannot decay completely and the thermodynamical equilibrium is established in the presence of the boson condensate. At small values of ρ the complete decay of the condensate is possible but the produced equilibrium plasma has a low temperature, $T < m_\chi$.

In the models /1-5/ based on the Affleck and Dine mechanism the baryon excess, generated during the inflationary stage, is stored in the condensate of the scalar field χ. The latter is produced at the inflationary epoch as a result of the growth of the χ field quantum fluctuations. According to the model χ field has a non zero baryon charge, that is not conserved at great values of the field amplitude, e.g. the terms of the type $\lambda(\chi^4 + \bar{\chi}^4)$ in χ self-interaction are essential. As a result, during inflation quantum field fluctuations create a baryon charge density of the order of H_I^3. (Here H_I is the Hubble parameter of the inflationary epoch). Note that in case of B-conservation, the baryon charge density would have been decreasing like **exp** $(-3H_I t)$ due to expansion. At small values of χ when the term $m_\chi^2 |\chi|^2$ dominates in the potential energy, the nonconservation of **B** becomes negligible. At that stage χ can be characterized by a baryon chrage $B = \varkappa m_\chi \chi_0^2$ where $\varkappa$ indicates the value of the baryon charge contained in the χ condensate. $\varkappa$ specifies the relative value of the condensate of χ_0 with $B>0$ and $\dot{\chi}_0$ with $B<0$. The field energy density is defined as $\rho_\chi = m_\chi^2 \chi_0^2$. It is usually assumed that χ is the scalar superpartner of quark ($\chi = \tilde{q}$) and it decays predominantly into the channel $\chi \to q \ell \ell$. Consequently, the baryon charge contained in χ goes into the baryon charge of quarks or, in other words, a charge nonsymmetric plasma, consisting of quarks leptons, photons, χ field quanta and χ condensate, it produced.

In the models discussed in literature, the temperature of the plasma, created by the decay of the condensate was estimated as

$$T \sim [\rho_\chi (t_{decay})]^{1/4} \tag{1}$$

For the natural values of the parameters this gives $T \sim 10^2 m_\chi$ and correspondingly large baryon asymmetry: $\beta \sim 10^2 B/N_\gamma$. However large baryon charge means large chemical potentials of the particles in the plasma and in this case the naive estimate (I) of the temperature is not valid. Particularly, for big energy density $\rho/m_\chi^4 >> 1$ and baryon charge $\varkappa \sim 1$ (like those proposed in the original A-D model) the estimates of the temperature $T > m_\chi$ and the baryon charge density of fermions $\beta_f = \frac{B_f}{N_\gamma} >> 1$ seem to be overestimated. This is due to the fact that in these models the considerable chemical potentials have not been accounted for.

Large baryon charge density, $\beta >> 1$, originated from the decay of the condensate, was questioned in paper /3/. It was claimed there that in the frameworks of the model β should be at most of the order of unity and the temperature of the equilibrium plasma $T \simeq m_\chi$. Our results are essentially the same but we think that the mechanism proposed in ref. 3 of the high temperature suppression of the χ-decays is not generally operative, whereas simple thermodynamical considerations presented below are always valid.

In what follows we will determine the characteristics of the plasma, produced by χ-condensate evaporation, for a wide range of the values of the initial parameters ρ and $\varkappa$. The plasma temperature will be calculated with an account for the nonzero chemical potentials, corresponding to the nonzero baryon charge. It will be shown that the system has different behaviour depending on the baryon charge density value (or on the $\varkappa$ parameter). At small $\varkappa$ χ-condensate completely decays when $T >> m_\chi$. This is in a good accordance with the estimate (I) but the resulting baryon asymmetry is small. At great $\varkappa$ the temperature is less than the mass. For big baryon charge densities, i.e. for $\varkappa \simeq 1$, and big energy

densities $\rho/m_\chi^4 >> 1$ thermodynamical equilibrium is established with nonvanishing χ-condensate.

As the Universe expands the energy density decreases and for some $\rho \leq m_\chi^4$ the complete evaporation of the condensate becomes possible. But, the account for the χ-condensate presence gives new estimates for the termalization temperature, $T < m_\chi$, and for the baryon asymmetry.

2. A qualitative discussion

Let us consider a system of particles in thermal equilibrium. The particle number density in the phase space is defined by the well-known formulae of the Fermi statistics and Boze statistics

$$dN_f(p,\mu_f) = g_f d\tau / [\exp(\tfrac{E_f-\mu_f}{T}) + 1] = g_f n_f d\tau$$
$$dN_b(p,\mu) = g_b d\tau / [\exp(\tfrac{E_b-\mu_b}{T}) - 1] = g_b n_b d\tau \tag{2}$$

where g_i stands for the number degrees of freedom and $d\tau = d^3p/(2\pi)^3$ is the phase volume. For antiparticles chemical potentials differ by sign.

In order to have positive definite particle densities, $N_b(p,\mu)$ and $\bar{N}_b(p,\mu)$, boson chamical potentials are bounded by the condition $|\mu_b| \leq m_b$ where m_b is the boson mass.

The limiting value $|\mu_b| = m_b$ corresponds to the case of bosonic condensate. In the presence of a condensate the bosonic number density is given by:

$$dN_b(p,\mu_b) = (2\pi)^3 N_c\, \delta(p)\, d\tau + g_b d\tau / [\exp(\tfrac{E_b-\mu_b}{T}) - 1] \tag{3}$$

where $\mu_b = m_b$, and N_c denotes the particle number density in the condensate.

Indeed, the kinetic equations for the bosons look like:

$$\frac{\partial n_b}{\partial t} = -\frac{(2\pi)^4}{2E} \sum_Y \int d\tau_Y\, \delta^4(p_b - p_Y) \Big[|A(b\to Y)|^2 n_b \prod_{Y_f} (1-n_{Y_f}) \prod_{Y_b} (1+n_{Y_b}) - |A(Y\to b)|^2 \prod_Y n_Y (1+n_b) \Big] \tag{4}$$

$$d\tau_i = \prod_i \frac{d^3p}{2E(2\pi)^3}$$

$$n_{\gamma_f} = \left[exp\left(\frac{E_{\gamma_f} - \mu_{\gamma_f}}{T} \right) + 1 \right] \quad , \quad n_{\gamma_b} = \left[exp\left(\frac{E_{\gamma_b} - \mu_{\gamma_b}}{T} \right) - 1 \right]$$

In equilibrium $\dot{n}_b = 0$. Substituting the distribution function (3) into the kinetic equation (4) for the bosons, one can see that if $N_c \neq 0$ the collision integral vanishes only when $\mu_b = m_b$.

In the presence of the condensate, given the densities of all conserving charges B_i and the energy density ρ , we can determine the chemical potentials and the plasma temperature, using the equilibrium êquations.

When in addition a boson condensate exists the plasma equilibrium state is also completely defined due to the fact that in that case the additional condition $\mu_b = m_b$ must be fulfilled. Thus the additional parameter N_c determining the condensate density is also defined.

Symbolically the equations for the equilibrium plasma with a given energy density ρ and a baryon-charge density B , can be written as follows:

$$\rho = \sum_i \rho_i \, (T, \mu_i)$$
$$B = \sum_B N_B \, (T, \mu_B) - \sum_{\bar{B}} N_{\bar{B}} \, (T, \mu_{\bar{B}}) \tag{5}$$

where $B < B_{crit}$, $N_B = (2\pi)^{-3} \int d^3p \; n_B \left(\frac{p}{T}, \frac{\mu_B}{T} \right)$ is the baryon number density. The sum is taken over all the components of the system. For small charges and in case of relativistic particles $\rho \sim T^4 (1 + f(\mu/T))$, $f(\mu/T) < 1$ and the temperature is approximately given by $T \sim \rho^{1/4}$ (i.e. estimate (I) is correct).

At a fixed temperature the increase of the initial value of B leads to an increase of μ . But μ_b for bosons is limited , $|\mu_b| \leq m_b$ (See fig.1). So there exists a critical value for the baryon charge $B_{crit}(\mu_b = m_b)$ of the equilibrium system without a condensate. When $B > B_{crit}$ $(\varkappa > \varkappa_{crit})$ the equilibrium at a fixed temperature can be maintained only by a condensate which contains the excess baryon charge $B_c = B - B_{crit}$.

Then ,at $B \geq B_{crit}$

$$\rho = \rho_c + \sum_i \rho_i$$
$$B = B_c + N_B \, (\mu_b = m_b) - N_{\bar{B}} \, (\mu_b = m_b) \tag{6}$$

It is obvious, that one and the same value of B at high temperatures needs an existence of the condensate while at low temperatures the existence **of a condensate may not be necessary as** $B \sim \mu/T$. The critical value B_{crit} is inversely proportional to the temperature.

On the other hand with increasing B and correspondingly μ the part of the energy density contained in the condensate increases. With the increase of μ $f(\frac{\mu}{T})$ increases, i.e. the plasma temperature decreases (for fixed energy density) with the increase of B. The estimate $T \sim \rho^{1/4}$ is no longer correct, in fact, $T < \rho^{1/4}$. When the condensate is formed, the chemical potential reaches its maximum value $\mu_b = m_b$ and stays constant. So, further increase of B needs an increase of the condensate number density N_c . The excess charge $B_c = B - B_{crit} = B - [N_B(m/T) - N_{\bar{B}}(-m/T)]$ is accumulated in the condensate. The existence of the condensate at a given energy density leads to the decrease of the energy density of the relativistic particles, since $\rho = \rho_c + \rho(T)$, and consequently to the decrease of the temperature. With an increase of B ρ_c increases, and the temperature decreases. In the case considered below, the temperature becomes less than the boson mass if $B \gtrsim 0.2\rho/m$ In other words, for big charge and energy density the equilibrium is established at low temperatures.

The evaporation of the condensate at big charges becomes possible when the energy density decreases by several orders of magnitude. This is connected with the already mentioned dependence of **Bcrit** on the temperature: $B_{crit} \sim T^{-1} \sim \rho$.

Note that in case of big baryon charge the evaporation temperature becomes less than the boson mass, in contrast to the case of small charge, when $T > m_b$.

In what follows we will calculate the critical value of the baryon charge, for which the boson condensate is formed (at a given energy), and the equilibrium temperature of the plasma as a function of B and ρ .

3. Basic assumptions, calculations and results

Let us determine the temperature of the thermalized plasma, obtained after the (partial) decay of the χ-condensate knowing the energy density ρ and the baryon charge density B of the plasma before the decay.

We assume that the thermalization rate is much bigger than the expansion rate of the Universe. In the case $B-L=0$, where L is the lepton number of the χ field, the chemical potential of the quarks, μ_q equals that of the leptons μ_ℓ.
In thermal equilibrium $\mu_\ell = -\mu_{\tilde\ell}$, $\mu_q = -\mu_{\tilde q}$, ets.
The direct and the inverse decays $\chi \rightleftarrows 3q + \ell + \tilde\ell$ in equilibrium result in the condition: $\mu_\chi = 3\mu_q + \mu_\ell + \mu_{\tilde\ell}$. It is assumed that $m_{\tilde\ell} = 0$ and correspondingly $\mu_{\tilde\ell} = 0$. Then, having in mind the connection between μ_ℓ and μ_q we obtain $\mu_\chi = 4\mu_q$.

The state of the plasma formed by the decay of the χ-condensate, depends on the value of the parameter $\varkappa$, characterizing the baryon charge, stored in the condensate $B \sim \varkappa \rho/m_\chi$, and the energy density of the condensate ρ at the time of its decay $t_{dec.} \sim \Gamma_\chi^{-1}$: $\rho = m_\chi^2 |\chi^2(t_{dec})|$.

For simplicity we will assume that the fermion creation rate is small and χ field, after inflation is over, starts harmonic oscillations with slowly decreasing amplitude due to the Universe expansion. Then $\chi \sim \chi_o$, where χ_o is the value of the χ field at the end of inflation. In the following we will discuss the case $\rho \gg m_\chi^4$.

A. Let us determine the conditions under which the equilibrium plasma without condensate can be realized. The equations for the energy density and the baryon charge density of the system, obtained after the χ condensate decay and the establishment of the equilibrium, have the form:

$$\rho = \rho_\chi + \rho_f + \rho_\gamma + \rho_{\tilde\ell} + \rho_\nu = \int d\tau\, g_\chi E_\chi (n_\chi + n_{\bar\chi}) + \sum_{f=q,\ell}\int d\tau\, g_f E_f (n_f + n_{\bar f}) +$$
$$+ g_o \pi^2 T^4/30 \quad , \qquad g_o = g_\gamma + g_{\tilde\ell} + \tfrac{7}{8} g_\nu \tag{7}$$
$$B = \rho\varkappa/m_\chi = N_\chi - N_{\bar\chi} + B_q (N_q - N_{\bar q}) =$$
$$= g_\chi \int d\tau\, (n_\chi - n_{\bar\chi}) + B_q g_q \int d\tau\, (n_q - n_{\bar q})$$

where f stands for fermions q, ℓ. $(f = q, \ell)$. $B_q = 1/3$.

The calculation of the integrals from distributions (2) is described, for example, in refs. 6 and 7.

From now on we assume that $m_q = m_\ell = 0$. $m_\chi \equiv m$.

A.1. A case of nonrelativistic χ.

If $E_\chi \simeq m + p^2/2m$ we have

$$\rho = g_\chi \frac{\Gamma(\frac{5}{2})}{2\pi^2}\left(\frac{2mT}{4m}\right)^{5/2} \sum_{n=1}^{\infty} 2ch\left(\frac{n\mu}{T}\right)\frac{\exp(-\frac{nm}{T})}{n^{5/2}} + g_\chi \frac{\Gamma(\frac{3}{2})}{2\pi^2}\left(\frac{2mT}{2}\right)^{3/2} m \sum_{n=1}^{\infty} 2ch\left(\frac{n\mu}{T}\right)\frac{\exp(-\frac{nm}{T})}{n^{3/2}} +$$
$$+ \sum_{i=q,\ell} \frac{g_i}{2\pi^2}\left(\frac{\mu_i^4}{4} + \frac{\pi^2\mu_i^2 T^2}{2} + \frac{7}{60}\pi^4 T^4\right) + g_o \frac{\pi^2}{30} T^4 \tag{8}$$
$$B = g_\chi \frac{\Gamma(\frac{3}{2})}{2\pi^2}\left(\frac{2mT}{2}\right)^{3/2} \sum_{n=1}^{\infty} 2sh\left(\frac{n\mu}{T}\right)\frac{\exp(-\frac{nm}{T})}{n^{3/2}} + \frac{g_q}{2\pi^2} B_q \left(\frac{\mu_q^3}{3} + \frac{\pi^2}{3}\mu_q T^2\right)$$

It is obvious that if $\rho/m^4 \gg 1$ and $m > T$ equations (8) are incompatible.

A.2. Now we consider the opposite case $m < T$.
When χ are relativistic then $\rho_\chi \sim 2 g_\chi \Gamma(4)\zeta(3) T^4/2\pi^2$ and $B_\chi \sim g_\chi \mu_\chi T^2/3$. The expression for the energy densities and the charges of the rest of plasma components are unchanged. Then:

$$\rho = g_\chi \frac{2\Gamma(4)\zeta(3)}{2\pi^2} T^4 + \sum_{i=q,\ell}\frac{g_i}{2\pi^2}\left(\frac{\mu_i^4}{4} + \frac{\pi^2\mu_i^2 T^2}{2} + \frac{7}{60}\pi^4 T^4\right) + g_o \frac{\pi^2 T^4}{30}$$
$$B = \frac{1}{3} g_\chi \mu_\chi T^2 + \frac{1}{2\pi^2} g_q B_q \left(\frac{\mu_q^3}{3} + \frac{\pi^2 \mu_q}{3} T^2\right) \tag{9}$$

The equations are not consistent for large ρ and $\varkappa \sim O(1)$. So the numerical values used in refs. 1-3, i.e. $\varkappa \simeq 1$ and $T \gg m$ cannot be realized.

The system has a solution without a condensate $\left(y=\frac{\mu}{m}<1\right)$ only when $\varkappa$ is small : $\varkappa < 1.6/\sqrt{\rho/m^4}$ и $\rho/m^4 > 62$.

$$\left(\frac{T}{m}\right)^2 \sim \frac{g_q \mu_q/\mu_\chi + 6 g_\chi}{18\left[0.7 g_\chi + \frac{\pi^2}{15}\left(g_\gamma + g_{\tilde e} + \frac{7}{8} g_f\right)\right]} \cdot \frac{y}{\varkappa} = \frac{\sqrt{\rho/m^4}}{\left[0.7 g_\chi + \frac{\pi^2}{15}\left(g_\gamma + g_{\tilde e} + \frac{7}{8} g_f\right)\right]^{1/2}}$$

$$\frac{\mu}{m} \sim \frac{\left[0.7 g_\chi + \frac{\pi^2}{15}\left(g_\gamma + g_{\tilde e} + \frac{7}{8} g_f\right)\right]^{1/2}}{\frac{1}{18} g\, \mu_q/\mu_\chi + \frac{1}{3} g_\chi} \cdot \sqrt{\rho/m^4}, \quad g_f = g_q + g_e$$

For $g_q = 36$, $g_f = 48$ $T \sim 0.4\rho^{1/4}$, $y = 0.6\varkappa\sqrt{\rho/m^4}$.
(See fig.1 and 2).

So, we have shown that only in case of small $\varkappa$ χ-condensate may completely decay and as a result there appears a hot plasma, whith the temperature which is higher than the chemical potentials and the χ boson mass: $\mu < m < T$ (The estimate of the temperature $T \sim \rho^{1/4}$ (I) used in literature in case of large energy density, $\rho/m^4 \gg 1$ and charges, $\varkappa \sim 1$ is not correct).

The baryon asymmetry of the plasma is equal to

$$\beta \sim \frac{B}{N_{tot}} \lesssim \frac{\varkappa\rho}{mT^3} \sim 0.7\left(\frac{m}{T}\right) y \ll 1$$

$$N_{tot} = N_\chi + N_{\tilde\chi} + N_{\tilde e} + N_e + N_{\tilde e} + N_q + N_{\tilde q} + N_\gamma \sim 19\, T^3$$

The baryon asymmetry of quarks is less than one half of the total one: $\beta_q < 0.03\, m/T$. The baryon asymmetry obtained is considerably less than 1, in contrast to the rough estimates that can be found in lòterature. Note that the smallness of β is a result of the smallness of $\varkappa$.

The observations give $\beta_{obs} \sim 10^{-9} \div 10^{-10}$ hence either an accidental smallness of $\varkappa$ (see e.g. ref. 5) or a considerable release of entropy at the later stages, diluting β is necessary.

In case, when $\varkappa$ is not small, the set of equations describing the thermodynamical equilibrium state of the particles without a condensate, cannot be solved. The complete decay of the χ -condensate at $\varkappa \gtrsim \frac{1.6}{\sqrt{\rho/m^4}}$ is not possible.

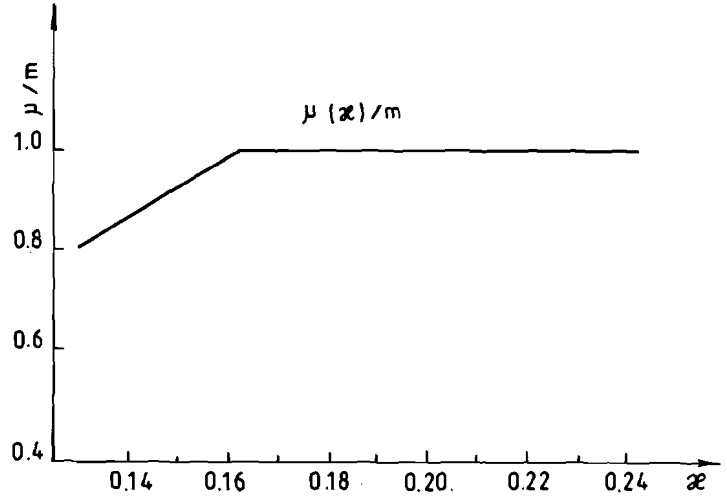

Fig.1. Chemical potential of the χ particles μ as a function of the baryon charge value: $\mu = \mu(\varkappa)$ for different cases described in the text. $\rho/m^4 = 100$. $\varkappa < 0.16$ corresponds to the case without a condensate, $\varkappa > 0.16$ corresponds to a nonzero condensate.

It is necessary to add into the equations a term, allowing for the presence of the condensate.

B. Now let us consider the case of thermalized plasma after the partial decay of the condensate. Using distribution (3) for the χ particles we will have the following equations for the energy density and the charge of the system:

$$\rho = N_c m + \int d\tau\, g_\chi E_\chi \left(n_\chi(\mu_\chi = m) + n_{\tilde\chi}(\mu_\chi = m)\right) + \rho_q + \rho_e + \rho_{\tilde e} + \rho_\gamma + \rho_\nu$$

$$B = N_c \varkappa + \int d\tau\, g_\chi \left(n_\chi - n_{\tilde\chi}\right) + \frac{B_q g_q}{2\pi^3} \cdot \frac{1}{3}\left(\frac{m^3}{4^3} + \frac{\pi^2 m T^2}{4}\right) \qquad (10)$$

The equilibrium densities of the χ particles are calculated at $\mu_\chi = m$, i.e. in case of nonvanishing χ-condensate. In these expressions $\rho_c \sim N_c m$ is the condensate energy density, $B_c = N_c \cdot \varkappa$ is the condensate baryon charge. The expressions for the energy densities and the charges of the fermions have not changed their form but $\mu_\chi = m$ have been substituted in them.

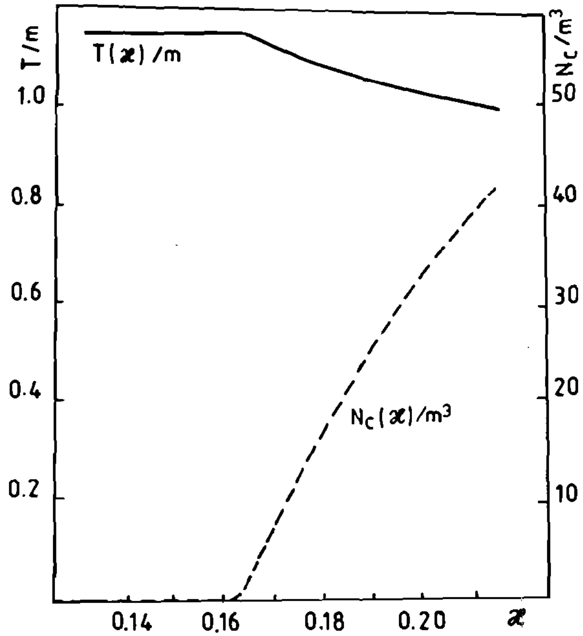

Fig.2. The solid line gives the dependence of the plasma temperature on the initial baryon charge of the system: $T(\varkappa)$
The dashed line gives the dependence of the condensate particle number density N_c on $\varkappa$: $N_c(\varkappa)$. $\mathcal{X}$ particles are relativistic. $\varkappa < 0.16$ corresponds to the case without a condensate (A.2). $0.2 \geq \varkappa \geq 0.16$ corresponds to a nonzero condensate (B.1).

B.1. Analogously to A.2 case we obtain from equations (IO) for $T > m$ the expressions:

$$\rho = N_c m + g_x \frac{2\Gamma(4)\xi(3)}{2\pi^2} T^4 + \sum_{i=q,e} \frac{g_i}{2\pi^2}\left(\frac{m^4}{4^5} + \frac{\pi^2}{2.4^2} m^2T^2 + \frac{7}{60}\pi^4 T^4\right) + g_0 \frac{\pi^2 T^4}{30}$$

$$B = N_c \varkappa + \frac{g_x}{3} m T^2 + \frac{B_q g_q}{2\pi^2} \cdot \frac{1}{3}\left(\frac{m^3}{4^3} + \frac{\pi^2 m T^2}{4}\right)$$

The set of equations can be resolved for $T > m$ only if $\varkappa$ is sufficiently small, i.e. $T > m$ if $\varkappa \lesssim 0.2$ and $\rho/m^4 > 62$.

The temperature is approximately given by:

$$T \sim 0.5\, m / \sqrt{\varkappa}$$

As can be seen from the equations, the plasma temperature does not depend on the energy density, but is determined by the value of the baryon charge (i.e. by the $\varkappa$ parameter).

The particle number density of the condensate is given by the expression:

$$N_c = \frac{\rho}{m} - \frac{2.5}{\varkappa^2} \cdot m^3$$

Curves $T(\varkappa)$ and $N_c(\varkappa)$ are plotted on fig.2.

It is straightforward to show that the baryon asymmetry of the plasma, excluding that of the condensate, is less than one.

The existence of the nonzero condensate density imposes an additional condition: $\varkappa > 1.6/\sqrt{\rho/m^4}$ $N_c \neq 0$. This is in accordance with the result A i.e. with the case without a condensate $\varkappa < 1.6/\sqrt{\rho/m^4}$ if $N_c = 0$. So, when the value of $\varkappa$ is in the range $1.6\sqrt{\rho/m^4} \lesssim \varkappa < 0.2$ and $T > m$ there exists a condenate.

B.2. At last we will consider the case $\varkappa > 0.2$ corresponding to nonrelativistic $\mathcal{X}$ particles $m > T$ and to the presence of a condensate. In this case the set of equations (IO) has the form:

$$\rho = N_c m + g_x \frac{\Gamma(\frac{5}{2})}{2\pi^2}\left(\frac{2mT}{4m}\right)^{5/2}\left(\xi(\tfrac{5}{2}) + \sum_{n=1}^{\infty}\frac{\exp(-\frac{2nm}{T})}{n^{5/2}}\right) + g_x \frac{\Gamma(\frac{3}{2})}{2\pi^2}\left(\frac{2mT}{2}\right)^{3/2} m\left(\xi(\tfrac{3}{2}) + \sum_{n=1}^{\infty}\frac{\exp(-\frac{2nm}{T})}{n^{3/2}}\right) +$$
$$+ \frac{g_f}{2\pi^2}\left(\frac{m^4}{4^5} + \frac{\pi^2 m^2 T^2}{2.4^2} + \frac{7}{60}\pi^4 T^4\right) + g_0 \frac{\pi^2 T^4}{30}$$

$$B = N_c \varkappa + g_X \frac{\Gamma(\frac{3}{2})}{2\pi^2}\left(\frac{2mT}{2}\right)^{3/2}\left[\xi(\tfrac{3}{2}) + \sum_{n=1}^{\infty}(-1)^n \frac{\exp(-\frac{2nm}{T})}{n^{3/2}}\right] + \frac{g_q B_q}{2\pi^2}\,\frac{1}{3}\left(\frac{m^3}{4^3} + \frac{\pi^2 m T^2}{4}\right)$$

Neglecting exponentially small terms we consider different contributions into the total energy density. It is easy to see that the right hand side of the first equation, excluding the term $\rho_c = N_c m$ is small. The condensate contains the main part of the energy density $\rho \simeq \rho_c$. Consequently, one can expect a low temperature $T \sim \rho - \rho_c$.

Excluding N_c from the set of equations one can calculate the ratio $x = m/T$ as a function of $\varkappa$

$$4.6x^{5/2} + 5.97x^{3/2}\left(\frac{\varkappa - 1}{\varkappa}\right) + 0.0008\left(\frac{3\varkappa - 4}{\varkappa}\right) + 0.5x^2\left(\frac{1.5\varkappa - 1}{\varkappa}\right) + 36.2x^4 = 0 \qquad (11)$$

The dependence $T/m = f_1(\varkappa)$ is presented in fig.3. The case $m > T$ is realized if $\varkappa \gtrsim 0.2$.

$$\frac{\rho}{m^4} = N_c/m^3 + 4.6x^{5/2} + 5.97x^{3/2} + 0.0024 + 0.75x^2 + 36.2x^4$$

$$\frac{B}{m^3} = N_c\varkappa/m + 0.5x^2 + 5.97x^{3/2} + 0.0032$$

$$N_c = \rho/m - m^3(5.97x^{3/2} + 0.0032 + 0.5x^2)/\varkappa$$

It is interesting, that if $\varkappa \sim 1$ so that the following condition is fulfilled:

$$\left(\frac{\rho}{m^4} - \frac{N_c}{m^3}\right)(1-\varkappa) < \frac{B_q g_q}{2\pi^2}\frac{1}{3}\left(\frac{M_q^3}{m^3} - \frac{18}{8}\frac{g_f}{g_q}\frac{M_q^4}{m^4}\right),$$

till the moment of the condensate evapotation, then in this range the temperature of the plasma is almost constant i.e. it neither depends on the energy density, nor on the baryon charge. It is approximately given by the expression

$$\rho\frac{(1-\varkappa)}{m^4} - \frac{N_c}{m^3}(1-\varkappa) + \frac{g_q}{18.4^3\pi^2}\left(1 - \frac{18}{8}\frac{g_f}{g_q}\frac{M_q}{M_\chi}\right) =$$

$$= \left[\frac{g_f}{4}\left(\frac{M_q}{M_\chi}\right)^2 - \frac{g_q}{18}\left(\frac{M_q}{M_\chi}\right) + \frac{3}{2}\frac{g_\chi}{(2\pi)^{3/2}}\xi\left(\tfrac{3}{2}\right)\sqrt{\frac{T}{m}}\right]\left(\frac{T}{m}\right)^2$$

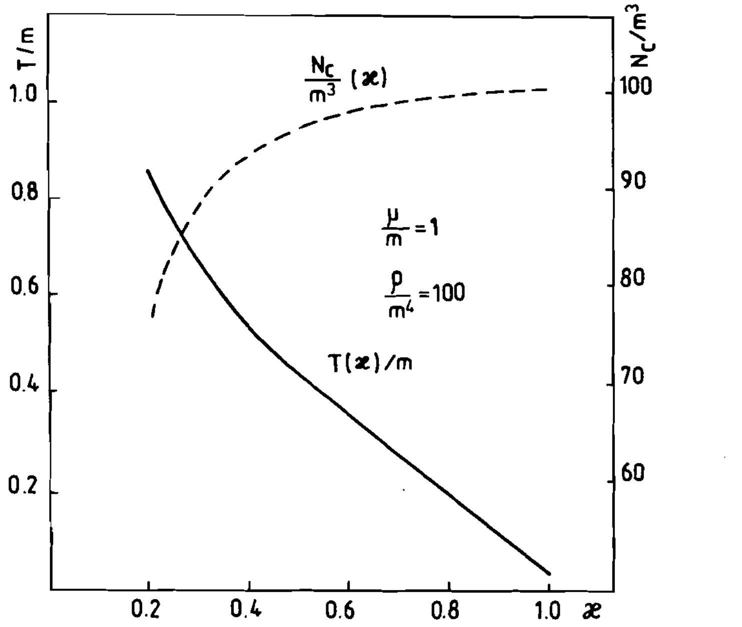

Fig.3. The solid line gives the dependence of the plasma temperature on the initial baryon charge of the system: $T(\varkappa)$. The dashed line gives the dependence of the condensate particle number density N_c on $\varkappa$: $N_c(\varkappa)$. χ particles are non relativistic (B.2).

. Then at $(1-\varkappa)/\chi \sim O(1)$ and $g_q = 36$, $g_f = 48$ we will obtain $T/m \sim 0.0279$.

The numerical solution of the equation $T/m = f_1(x)$ gives the result $T/m = 0.0277$, at $\varkappa \sim 1$.

It is note worthy, that in the case $m > T$ the plasma temperature, in the presence of the condensate, also does not depend on the energy density, it depends only on $\varkappa$. The decrease of the energy density influences only the density of the condensate. The condition of the condensate evaporation can be written in the form:

$$\frac{\rho_{ev.}}{m^4} = \frac{5.97x^{3/2} + 0.0032 + 0.5x^2}{\varkappa}$$

When $\varkappa = 1$, $(\rho/m^4)_{ev.} \sim 3.10^{-2}$ while when $\varkappa = 0.2, \frac{\rho_{ev}}{m^4} \sim 5$

After the evaporation of the condensate, the temperature can be determined from the equations (8). In case of small $\varkappa$ the evaporation is possible at big energy densities, as well. If $\rho/m^4 > O(1)$ the evaporation is possible only at $\varkappa$ as small as $\varkappa < 0.2$ which is outside the case considered, i.e. $m > T$ (for these $\varkappa$ values see B.1).

The baryon asymmetry in this case is determined by the expression:

$$\beta = \frac{B}{N_{tot}} = \frac{1}{a}\left[\frac{N_c \varkappa}{T^3} + 5.97\left(\frac{m}{T}\right)^{3/2} + 0.0032\left(\frac{m}{T}\right)^3 + 0.5\left(\frac{m}{T}\right)\right]$$

$$N_{tot} = N_\ell + N_{\bar\ell} + N_q + N_{\bar q} + N_{\tilde\ell} + N_\gamma = aT^3 = f_2\left(g_i, \frac{m}{T}\right)T^3$$

where $i = \gamma, \ell, q, \tilde\ell$

The condition $\rho/m^4 \gg 1$ leads to $B/T^3 > 1$. The part of the baryon asymmetry contained in quarks, at $m > T$ is less than unity $\beta_f \sim [0.0032 (\frac{m}{T})^3 + 0.5(\frac{m}{T})]\cdot a^{-1} << 1$.

Evidently, the main part of the baryon charge is stored in the condensate. Using the obtained expression $T/m = f_1(\varkappa)$ (see eq. (11)) we can determine the baryon asymmetry of the quarks.

So, for $\varkappa = 1$ $m/T \sim 36$ $\beta_f \sim 0.004$, and for $\varkappa = 0.17$ $m/T \sim 1.1$ $\beta_f \sim 0.04$.

In the course of the Universe expansion, the energy density decreases at t^{-2}. In case of nonrelativistic particles the condensate density $\rho_c = N_c m$ has the same behaviour. μ_χ stays constant $\mu_\chi = m$ till the moment, when N_c disappears. An evaporation of the condensate with the decrease of the energy density proceeds. Till the moment of the evaporation the temperature of the system also remains constant, $T = const > m$ determined by $\varkappa$. After the condensate evaporation the chemical potential decreases as the temperature goes down with the expansion.

Let us estimate the condensate evaporation time in the process of the Universe expansion. For simplicity we take the case $\varkappa \sim 1$ then as it has already been calculated $\rho_{ev}/m^4 \sim 3.10^{-2}$.

Using $\rho = \rho_0 (t_0/t)^2$

we get $t_{ev.} \sim 6 (\rho_0/m^4)^{1/2} t_0$.

After the evaporation the temperature time dependence is the same as in the standard Fridman cosmology $(T/T_{ev.}) \sim (t_{ev.}/t)^{2/3}$

for the case of nonrelativistic matter dominance. In our case this can be realized, if except χ field and its decay products there exists an additional nonrelativistic matter in the Universe, giving the dominant contribution into the energy density. It could be for example the coherently oscillating inflanton field ψ.

As it was shown above only the energy density ρ_c decreases with the Universe expansion, while the temperature stays constant till the condensate evaporation. So, the results for β_f obtained above are applicable till the moment of evaporation.

To get an agreement with the observations a reheating sources are necessary, like, for example, the inflanton decay if is takes place after the condensate evaporation $\Gamma_\psi < \Gamma_\chi$.

If the inflanton decays earlier and after some moment ρ_χ dominates the energy density, some B and L nonconserving processes are necessary. They could wash the excess of B and L charges and could lead to the necessary reheating.

As we show in a separate publication the process of particle production by the classical oscillating field χ leads to a very small value of $\varkappa$ and so to a reasonably small value of β.

Thus no reheating is necessary.

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