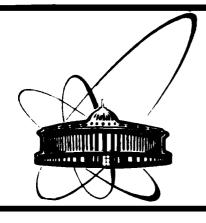
89-555



Объединенный институт ядерных исследований дубна

K 76

E2-89-555

A.L.Koshkarov*, V.V.Nesterenko

OPEN BOSONIC STRINGS
IN A BACKGROUND ISOTROPIC
ELECTROMAGNETIC FIELD

Submitted to "International Journal of Modern Physics A"

^{*} Petrozavodsk State University, Petrozavodsk, SU-185018 USSR

I. Introduction

Considerable progress has recently achieved in investigation of the string dynamics in background fields corresponding to the first levels of the string spectrum 1-6. In this approach one gets a new insight into the properties of the string itself, and some information on the local fields generated by the interacting strings in the low energy limit can be obtained.

The background fields corresponding to massless states in the string spectrum are of special interest in the string approach to the particle physics $^{1-5}$. It is these states that should be compared with the elementary particles observed in the energy range available in the experiment now 7,8 . The Abelian gauge vector field (electromagnetic field) corresponds to the massless state in the spectrum of open strings x). This field does not act on the fermionic variables of the string 1 , and as a consequence we can confine ourselves to the consideration of a bosonic part of the string spectrum.

A constant homogeneous electromagnetic field in the D-dimensional space-time can be of two types, non-isotropic and isotropic. This corresponds to two canonical forms of the strength tensor $\int_{\mu\nu}$. In the case of the four-dimensional space-time the non-isotropic electromagnetic field can be cast by a appropriate Lorentz transformation into the parallel electric and magnetic fields. An isotropic electromagnetic field in an arbitrary reference frame describes electric and magnetic fields which are equal and perpendicular to each other.

The dynamics of open strings has been investigated in a non-isotropic electromagnetic field $^{1-3}$. The string dynamics

x) To introduce into the string theory the non-Abelian gauge symmetry, one should to use the Chan- Paton mechanism 9.

turns out to be stable only if a constraint on the invariants of the tensor is satisfied. In a special reference frame this requirement reduces to the constraint on the absolute value of the electric field. This result is a direct indication that the vector Abelian gauge field generated by interacting strings in the low energy limit should be described by a nonlinear Lagrangian instead of the Maxwell Lagrangian (for example, by the Born - Infeld Lagrangian 2, 10).

In Ref. 1 it has been shown that for the isotropic configuration of an external electromagnetic field such a restriction is absent. This conclusion was based on the analysis of the boundary conditions without using the solution of the equations of motion.

In the present paper, the investigation of the string dynamics in a background non-isotropic electromagnetic field carried out in Ref. 1 is extended to the case of an isotropic external field. Two types of open strings are considered, neutral strings and strings with a net charge. The generalized light-like gauge is introduced, the general solution of the equations of motion are constructed and the string mass spectrum is analysed. On the basis of the obtained results one can conclude that the string dynamics in a background isotropic electromagnetic field is stable at an arbitrary value of the external field strength.

The rest of this paper is arranged in the following way.

In the second section, the action of an open string in an external electromagnetic field is introduced and the equations of motion and boundary conditions are obtained. Here the canonical form of the strenth tensor \(\sum_{\mu \nu} \) for the isotropic electromagnetic field is given too.

In the third section, the dynamics of the neutral strings in the isotropic field is investigated. We introduce the light-like gauge conditions that can be used both for neutral and charged strings moving in an isotropic external field. The independent dynamical variables are separated and the general solutions for them are constructed. On this basis the string spectrum is investigated. In the forth section, the same is done for a charged open string in an isotropic background electromagnetic field. In conclusion the obtained results are shortly discussed.

2. Action functional and equations of motion

The dynamics of an open bosonic string with charges Q_1 and Q_2 at its ends placed in an external electromagnetic field $A_1(x)$ is defined by the action C_2 F_3

$$S = -7 \int_{\tau_{i}}^{\tau_{2}} d\tau \int_{0}^{\pi} d\delta \sqrt{-g} - \sum_{\alpha=i}^{2} q_{\alpha} \int_{\tau_{i}}^{\tau_{2}} d\tau \dot{x}_{\mu}(\tau, \xi_{\alpha}) A^{\mu}(x), \quad (2.1)$$

where T is the string tension, x''(7,6), $\mu = 0, 1, ..., D-1$ are the string coordinates, T = u', G = u'; $g = \det \| \partial_x x'' \partial_x \|$, i, j = 0, 1; G = 0, 0; $x = \partial_x x$, $x = \partial_x x$. In the enveloping space-time the metric with signature (+, -, -, -, ...) is used.

In the orthonormal gauge

$$\left(\dot{\mathcal{X}} \pm \mathcal{X}'\right)^2 = 0 \tag{2.2}$$

the string is described by the equations of motion

$$\ddot{x}_{\mu} - x_{\mu}'' = 0 \tag{2.3}$$

and boundary conditions

$$Tx'_{\mu} + q F_{\mu\nu} \dot{x}' = 0, \quad 6 = 0,$$

$$Tx'_{\mu} - q F_{\mu\nu} \dot{x}' = 0, \quad 6 = \pi,$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}.$$
(2.4)

Further we suppose that $\int_{\mu\nu}$ does not depend on $\mathcal X$ and consider the isotropic background field when matrix $\int_{\mu\nu}$ has the following block-diagonal form

$$F = \begin{vmatrix} A & O \\ O & B \end{vmatrix}, \qquad (2.5)$$

where A is a (4x4) matrix of the form

$$A = \begin{vmatrix} 0 & F & 0 & 0 \\ -F & 0 & 0 & F \\ 0 & 0 & 0 & 0 \\ 0 & -F & 0 & 0 \end{vmatrix}$$
 (2.6)

and D is the block-diagonal (D-4)x(D-4) matrix

$$B = \text{diag}(B_2, B_3, ..., B_{(D/2)-1}).$$
 (2.7)

Here \mathcal{B}_{α} , $\alpha = 2, 3, ..., (D/2) - 1$ are (2x2) matrices

$$B_{\alpha} = \begin{vmatrix} 0 & -H_{\alpha} \\ H_{\alpha} & 0 \end{vmatrix}. \tag{2.8}$$

We assume for definiteness that D is an even number.

The matrix A defined in (2.6) maintains its form in an arbitrary reference frame. The rest part of the tensor $f_{\mu\nu}$ can be transformed to the form (2.7), (2.8) by an appropriate rotation of the last (D-4) space-like coordinates.

3. Neutral string

In this section we investigate the dynamics of the neutral string $(q = -q_2 = q)$ in an isotropic background electromagnetic field. In the case under consideration the total canonical momentum of the string

$$P^{\mu} = \int_{0}^{\pi} P^{\mu}(7,6) d6, \qquad (3.1)$$

$$p^{\mu}(\hat{r},6) = -\frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu}} = T\dot{x}^{\mu} + q F^{\mu} x, \qquad (3.2)$$

is conserved and the light-like gauge can be introduced in the folliwing way

$$T_{\mu}x'' + q_{\mu}F_{\mu\nu}\dot{x}' = 0,$$
 $T_{\mu}\dot{x}'' + q_{\mu}F_{\mu\nu}\dot{x}'' = \pi'n_{\mu}P''.$ (3.3)

where n^{μ} is a constant isotropic vector $n^2 = 0$. It is convenient to choose the components of this vector so that the gauge conditions (3.3) do not depend on the external field. Putting

$$M^{H}=(1,0,0,1,0,...)$$
 (3.4)

we obtain from (2.5)-(2.8) and (3.3)

$$x' = 0, \quad \dot{x} = \frac{\beta}{7\pi}, \tag{3.5}$$

where $\sqrt{2} \alpha^{\pm} = \alpha^{0} \pm \alpha^{3}$.

The boundary conditions (2.4) with allowance for (3.5) take the form $(6=0, \mathcal{H})$

$$x' = 0, \quad x' + \sqrt{2} f \dot{x}^{1} = 0,$$
 (3.6)

$$x''^{1} + \sqrt{2} f \frac{\rho}{7\pi} = 0, \quad x'^{2} = 0, \quad f = (q/7)F;$$
 (3.7)

$$\xi^{\alpha} - i h_{\alpha} \xi^{\alpha} = 0, \quad \xi^{\alpha} = x^{2\alpha} + i x^{2\alpha+1},$$

$$h = (q/T)H_{\alpha}, \quad \alpha = 2, 3, ..., (D/2) - 1.$$
(3.8)

The light-like gauge (3.3) enables one to split the string coordinates $\mathcal{X}''(7,6)$ into dependent and independent ones. As usual, we shall treat as independent variables the transverse string coordinates $\overrightarrow{\mathcal{X}}_1 = \left\{ x^1, x^2, x^4, x^5, \dots, x^{D-1} \right\}$, and light-cone variables $\mathcal{X}^{\pm} = (x^{\pm}x^{3})/\sqrt{2}$ will be considered as dependent ones.

From the orthonormal gauge conditions (2.2) using (3.5) we obtain

$$\dot{x}^{+} = \frac{T\pi}{2P} (\dot{\vec{x}}_{\perp}^{2} + \vec{x}_{\perp}^{2}), \quad x'^{+} = \frac{T\pi}{P} \dot{\vec{x}}_{\perp} \dot{\vec{x}}_{\perp}^{2}. \quad (3.9)$$

Thus, equations (3.5) and (3.9) express the dependent variables in terms of the independent ones. It is easy to be convinced that eqs. (3.5) and (3.9) are consistent with the equations of motion (2.3) and boundary conditions (3.6)-(3.8) in the following sense: if the transverse variables $\mathcal{X}_{1}(\mathcal{T}, \mathcal{E})$ obey equations of motion (2.3) and boundary conditions (3.7),(3.8), then dependent string coordinates \mathcal{X}^{\pm} and \mathcal{X}^{\pm} expressed in terms of \mathcal{X}_{1} and \mathcal{X}_{1} according to (3.5) and (3.9) satisfy Eqs. (2.3) and their boundary conditions (3.6).

The independent string coordinates are represented in the form

$$x'(\tau,6) = Q^{1} + \frac{\rho^{1}}{\pi \tau} \tau - \sqrt{2} f \frac{\rho}{\pi \tau} 6 + \frac{t}{\sqrt{\pi \tau}} \sum_{n \neq 0} e^{-in\tau} \frac{1}{n} \cos n6,$$

$$x^{2}(\tau,6) = Q^{2} + \frac{\rho^{2}}{\pi \tau} \tau + \frac{t}{\sqrt{\pi \tau}} \sum_{n \neq 0} e^{-in\tau} \frac{\alpha_{n}^{2}}{n} \cos n6,$$

$$\xi'(\tau,6) = Q^{2} + \frac{\rho^{2}}{\pi \tau} \tau + \frac{t}{\sqrt{\pi \tau}} \sum_{n \neq 0} e^{-in\tau} \frac{\alpha_{n}^{2}}{n} \cos n6,$$

$$(3.10)$$

$$\xi'(\tau,6) = Q^{2} + \frac{\rho^{2} + i \rho^{2}}{\pi \tau} (\tau + ih_{\alpha} 6) + \frac{t}{\pi \tau} \sum_{n \neq 0} e^{-in\tau} \xi^{\alpha} \cos n6 + h \sin n6, \quad \alpha = 2, 3, ..., (D/2) - 1.$$

The momentum variables (3.2) can also be divided into the dependent

$$\rho^{+} = T\dot{x}^{+} + \sqrt{2}q Fx'^{1}, \qquad \rho^{-} = T\dot{x}^{-}$$
 (3.11)

and independent ones

$$\rho^{A} = T \dot{x}^{A} = \frac{\rho}{\pi} + \frac{1}{\sqrt{\pi}T} \sum_{n \neq 0} e^{-inT} \left(\frac{A}{n} \cos n6, A = 1, 2, \frac{1}{\pi} \right)$$

$$h^{\alpha} = \rho^{2\alpha} + i \rho^{2\alpha + 1} = T \left(\frac{A}{5} - ih_{\alpha} \right) = (3.12)$$

$$= \frac{1}{\pi} \left(\rho^{2\alpha} + i \rho^{2\alpha + 1} \right) + \left(1 + h_{\alpha}^{2} \right) \sqrt{\frac{T}{51}} \sum_{n \neq 0} e^{-inT} \cos n6,$$

$$\alpha = 2, 3, \dots, (D/2) - 1.$$

Now we can investigate the string mass squared

$$M^{2} = P P^{\mu} = 2 P^{\dagger} P^{-} - \vec{p}_{1}^{2}. \tag{3.13}$$

By virtue of eqs. (3.9) and (3.11) we obtain for β^{+} $\beta^{+} = \frac{7^{2}\pi}{2\beta^{-}} \int_{0}^{\pi} d6 \left(\dot{\vec{x}}_{\perp}^{2} + \dot{\vec{x}}_{\perp}^{12} \right) + \sqrt{2} q \int_{0}^{\pi} d6 x'^{1}. \quad (3.14)$

Using expansion (3.10) we can calculate M^2 straightforwardly

$$M^{2} = -2 \left(\frac{QF}{T}\right)^{2} \left(\beta\right)^{2} - \frac{1}{2} \frac{h_{\alpha} + \frac{1}{2}}{1 + h_{\alpha}^{2}} + \frac{1}{2} \left(\beta\right)^{2} +$$

$$[x'(7,6), p'(7,6')] = i \delta_{ij} \delta(6-6'),$$

$$i, j = 1, 2, 4, 5, ..., D-1$$
(3.16)

Then eq. (3.15) becomes

$$M^{2} = -M^{2} + 2\pi T \sum_{i=1}^{\infty} \sum_{j=1}^{D-2} \alpha_{j}^{+i} \alpha_{j}^{i}, \qquad (3.17)$$

where M_{tr}^2 is a tachyonic contribution due to the motion of the string as a whole in transverse directions

$$M_{tr}^{2} = 2\left(\frac{qF}{T}\right)^{2} \left(\beta^{-1}\right)^{2} - \sum_{\alpha=2}^{2} \frac{h_{\alpha} \vec{D}_{\alpha}^{2}}{1 + h_{\alpha}^{2}}.$$
 (3.18)

The operators \mathcal{Q}^{i} obey the usual commutation relations

$$\left[\alpha_{n}^{i},\alpha_{m}^{i}\right]=\delta_{ij}\delta_{nm}. \tag{3.19}$$

It is interesting to consider the string energy in a special reference frame where the component ρ^3 of the total momentum of the string vanishes. The transition to this reference frame maintains the isotropic structure of the electromagnetic strength tensor (2.5) and (2.6).

By wirtue of (3.14) we obtain

$$E^{2} = \left[1 + \left(\frac{q}{T}F\right)^{2}\right] \left\{\sum_{\alpha=2}^{\frac{n}{2}-1} \frac{\overrightarrow{p}_{\alpha\perp}^{2}}{1 + h_{\alpha}^{2}} + 2\pi \overrightarrow{T} \sum_{n=1}^{\infty} \sum_{i=1}^{n} \alpha_{n}^{+i} \alpha_{n}^{i}\right\}$$
(3.20)

In contrast to M^2 in (3.17), E^2 is positive definite at the classical level. In quantum theory the zero point oscillations of the string give a techyonic contribution into Eq.

$$\mathcal{M}_{o}^{2} = -5.7 \frac{D-2}{12} \tag{3.21}$$

and into Eq. (3.20)

$$E_0^2 = -57 \frac{D-2}{12} \left[1 + \left(\frac{9}{7} F \right)^2 \right]^{-1}. \tag{3.22}$$

Thus the analysis of the mass and energy spectrum of the open neutral string does not give any restriction on the external isotropic electromagnetic field. The same result has been obtained in Ref. 1 by investigating only the boundary conditions (2.4). Probably the string dynamics under consideration is stable at arbitrary values of the field stringth f in Eq. (2.6), while in the case of a non-isotropic background electromagnetic field we have the constraint 1,3

$$\left(\frac{q}{T}E\right)^2 < 1. \tag{3.23}$$

Nevertheless, it should be noted that if is absolutely unclear what is the physical implication of the tachyonic contribution $-\frac{2}{2}$ to the squared mass of the string (3,17) caused by the translation motion of the string as a whole in transverse directions.

4. Charged string in a background isotropic electromagnetic field

As it was shown in Ref. 1, the light-like gauge cannot be introduced in the theory of the open charged strings when the external electric field does not vanish. (See also Ref. 11). However, in the case of an isotropic background field the light-like gauge (3.5) can be introduced for the open strings with a net charge. Indeed in Eqs. (3.5) there is no dependence on the external field and as a consequence these equations can be in agreement with the boundary conditions. In addition, the projection of the total momentum of the string onto the constant vector \mathcal{N}^{μ} given by (3.4) is conserved. It is easy to show if one takes the electromagnetic potential $\mathcal{A}_{\mu}(\mathcal{X})$ in the form

$$A_{\mu}(x) = -\frac{1}{2} F_{\mu\nu} x, \quad F_{\mu\nu} = const. \quad (4.1)$$

Taking into account Eqs. (2.5) and (2.6) we obtain

(4.2)

$$A'(x) = -\frac{F}{2}x', \quad A'(x) = -\frac{F}{\sqrt{2}}x^{-}, \quad A'(x) = 0, \quad A(x) = -\frac{F}{2}x', \dots$$

The density of the momentum variables is given by

$$P^{\mu}(\tilde{x},6) = -\frac{\partial \mathcal{L}}{\partial \dot{x}_{\mu}} = T \dot{x}^{\mu} + \sum_{\alpha=1}^{2} q_{\alpha} A^{\mu}(\alpha) \delta(6-6_{\alpha}),$$

$$G_{1} = 0, G_{2} = \overline{x}.$$
(4.3)

From Eqs. (4.2) and (4.3) it follows that

$$p^{-}(7,6) = T \dot{x}^{-}(7,6) \tag{4.4}$$

6= 56

because of A(x) = 0.

6 = 0

The boundary conditions in the case under consideration are

$$Tx'^{+} + \sqrt{2}q_{1}F\dot{x}' = 0$$
, $Tx'^{+} - \sqrt{2}q_{2}F\dot{x}' = 0$, $x'^{-} = 0$, (4.5)

 $Tx'^1+\sqrt{2}q_1Fx'=0$; $Tx'^1-\sqrt{2}q_2Fx'=0$. The boundary conditions for the remaining components of x''(7,6) are the same as in the case of the non-isotropic background field (see Eq. (5.2) in Ref. 1). With allowance for the equations of motion (2.3) and boundary conditions (4.5) it may be verified that

the projection of the total string momentum onto the vector \mathcal{N}^{μ}

$$\beta^{-} = \int_{0}^{\pi} \rho^{-}(7,6) d6 = T \int_{0}^{\pi} \dot{x}^{-}(7,6) d6 \qquad (4.6)$$

is conserved.

The boundary conditions for \mathcal{X}^{1} in (4.5) are simplified by virtue of the light-like gauge conditions (3.5)

$$Tx'^{1}+\sqrt{2}q, \frac{F}{T\pi}\beta=0, 6=0; Tx'^{1}-\sqrt{2}q, \frac{F}{T\pi}\beta=0, 6=\pi.$$
 (4.7)

As in the preceding section we treat the transverse string coordinates $\widehat{\mathcal{X}} = \left\{ \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \dots \right\}$ as the independent dynamical variables and the light-cone coordinates $\widehat{\mathcal{X}} = (\mathcal{X} \stackrel{\circ}{=} \mathcal{X}) / V \widehat{Z}$ as the dependent ones. One can easily verify again that Eqs. (3.9) representing the dependent variables in terms of the independent ones are in agreement with the equations of motion. (2.3) and boundary conditions (4.5) and (4.7).

Let us go now to the construction of the general solution for the independent string coordinates. For $\mathcal{X}^{2}(\mathcal{I},\mathcal{C})$ we have obviously the same expansion as in (3.10). For the components $\mathcal{X}^{n}(\mathcal{I},\mathcal{C})$, μ =4,5, ... D-1 one can use the solution obtained under consideration of the non-isotropic background field (see Eqs. (5.13) and (5.16) in Ref. 1). A new solution is for $\mathcal{X}^{2}(\mathcal{I},\mathcal{C})$. It contains now additional terms proportional to \mathcal{I}^{2} and \mathcal{C}^{2}

$$x'(\bar{z},6) = \sqrt{2}(q_1 + q_2) \frac{F}{T} \frac{\rho}{7\pi} \frac{\bar{z}^2 + 6^2}{2\pi} + \alpha \bar{z} - \frac{\rho}{\sqrt{\pi}} \frac{\bar{z}^2 + 6^2}{7\pi} + \alpha \bar{z} - \frac{\rho}{\sqrt{\pi}} \frac{\rho}{7\pi} \delta + Q + \frac{i}{\sqrt{\pi}} \sum_{n \neq 0} e^{-in\tau} \frac{d_n^{1}}{n} \cos n \delta.$$
(4.8)

By virtue of (4.1)-(4.3) the density of the corresponding canonical momentum is

$$p'(7,6) = Tx'(7,6) + \sum_{\alpha=1}^{2} q_{\alpha} A(x) \delta(6-\delta_{\alpha}) =$$

$$= \sqrt{2} (q_{1}+q_{2}) \frac{F}{7_{2}} \frac{D}{\pi} \frac{T}{\pi} + T\alpha + \sqrt{T} \sum_{\alpha=1}^{2} C A_{\alpha} \cos n\delta -$$

$$-\frac{F}{\sqrt{2}} x (7,6) \sum_{\alpha=1}^{2} q_{\alpha} \delta(6-\delta_{\alpha}), \quad 6, = 0, \quad 6_{2} = \pi.$$
The integration constant α in the expansion (4.8) and

(4.9) can be written in terms of P'(o), where

$$\beta'(\tau) = \int_{0}^{\pi} \rho'(\tau, 6) d6.$$
 (4.10)

From (4.9) and (4.10) it follows that

$$Q = \frac{\rho'(o)}{T_{\mathcal{F}}}. (4.11)$$

The general solution of the equations of motion enables to obtain for the string mass squared the same formula as in (3.17) with a new expression for M^2 . Only the first term in (3.18) should be modified. Now it can depend on the evolution parameter \mathcal{T} exactly because the total energy-momentum vector of the string $\mathcal{D}^{\mathcal{M}}$ is not conserved in the case under consideration.

Hence the structure of the mass spectrum of the charged string in an isotropic background electromagnetic field remains the same as in the case of neutral string. Any constraints on the external field do not appear.

5. Conclusion

The absence of the constraints on the external isotropic electromagnetic field in the open string theory can be interpreted at first sight as a contradiction with the conclusion that the electromagnetic field generated by an interacting strings should be described by the nonlinear Born - Infeld Lagrangian^{2, IO};

$$\mathcal{L}_{B-I} = \left(\det \| \delta_{\mu}^{\nu} + \frac{q}{\tau} F_{\mu}^{\nu} \| \right)^{1/2}$$
 (5.1)

As a matter of fact, it is not so. The constraint (3.23) for the non-isotropic electromagnetic field in the framework of the Born - Infeld theory is a consequence of positivity of the radicand in (5.1). For the isotropic tensor this expression equals 1, and as a consequence, there are no constraints on in this case.

References

- 1. V. V. Nesterenko, Int. J. Mod. Phys. A4 (1989) 2627.
- A.A. Aboulsacod, C.G. Callan, C.R. Nappi and S.A. Yost, Nucl. Phys. B280 [FS18] (1987), 599.
- 3. C.P. Burgess, Nucl. Phys. B294 (1987) 427.
- 4. C.G.Callan, E.Martinec, D.Friedan and M.Perry, Nucl. Phys. B262 (1985) 593.
- 5. C.G. Callan and J. Gan, Nucl. Phys. B272 (1986) 647.
- 6. J.M.F. Labastida and M.A.H. Vozmediano, <u>Nucl. Phys.</u> B312 (1989) 308.

- 7. M.B. Green, J.H. Schwarz and E. Witten. Superstring Theory, v's 1,2 (Cambridge University Press, Cambridge, 1987).
- 8. B.M. Barbashov, V.V. Nesterenko, <u>Usp. Fiz. Nauk</u> (in Russian), 150 (1986) 489, Sov. Phys. Usp. 29, No 12 (1986) 1077.
- 9. H.M. Chan and J. Paton, Nucl. Phys. B40 (1969) 519.
- IO.E.S. Fradkin and A.A. Tseytlin, Phys. Lett. 158B (1985) 516.
- 11. H. Kanasugi and H. Okada, Progr. Theor. Phys. 79 (1988) 1197.

Received by Publishing Department on Jule 24, 1989.