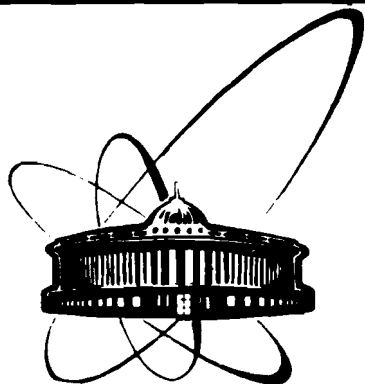


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AXIAL ANOMALY AND SUM RULE FOR PHOTON
SPIN STRUCTURE FUNCTION

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Investigations of the axial Adler - Bell - Jackiw anomaly contribution to the proton structure function g_1^p /1-3/, initiated by the so-called "spin crisis" (see e.g. /4/) suggested by the EMC-data /5/, draw attention to the fact /6/ well-known during about ten years that the classical light-by-light scattering diagram ("the box") contains a contribution due to the anomaly /7/. Therefore it is interesting to investigate this contribution in other processes where this diagram does work. A high P_T dijet production /3,8/ is one of them. Another information can be obtained from deep inelastic scattering of a polarized electron on a polarized photon where the longitudinal spin asymmetry is determined by the structure function g_1^γ of the photon.

As it is known the photon structure functions are investigated at e^+e^- - colliders in the reaction $e^+e^- \rightarrow e^+e^- X$ (Fig.1), where the momentum transfer squared of one lepton is large, $-q_1^2 = Q^2 \gg \gg M_{\text{hadron}}^2$ and that of the second lepton q_2^2 is about zero.

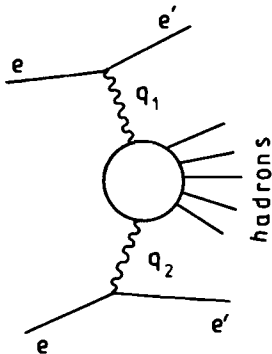


Fig.1

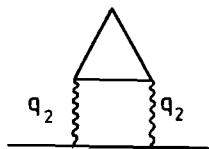
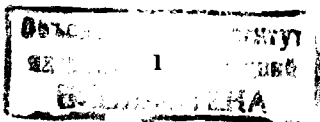


Fig.2

However, the registration of lepton with a small momentum transfer is practically a difficult problem. A more simple way is to detect only the lepton with a large momentum transfer (the so-called "Single Tag experiment"). Due to a rapid decrease of the cross-section with growing Q_2^2 , its dominating part is in a region $|Q_2^2| \approx 0$. So, in practice, it is more suitable to deal with



asymmetry of deep inelastic longitudinally polarized electron-positron scattering. The amplitude and cross-section of the process have the same form as the electron-proton ones with the natural change of the proton mass by the electron one and with structure functions g_1^e and g_2^e . For the structure function g_1^e one can write down the sum rule similar to that for the proton ^{/1-3/}. However, the hadron part of it in the lowest order of α is determined by the diagram, Fig.2, with the triangle anomaly. It has been calculated by Adler ^{/7/} and results in

$$\Gamma_1^e = \int_0^1 dx g_1^e(x) = - \frac{\langle e^4 \rangle}{2} N_f \frac{3}{8} \left(\frac{\alpha}{\pi}\right)^2 \ln \frac{Q^2}{4m^2}. \quad (1)$$

The right-hand-side of (1) can be represented as the first moment of the photon structure function, call it $g_1^\gamma = \alpha W_4$

$$\Gamma_1^\gamma = \int_0^1 g_1^\gamma(x, Q^2) dx = - \frac{\langle e^4 \rangle}{2} N_f \frac{\alpha}{2\pi} \quad (2)$$

times the difference of the photon numbers with opposite helicities in the lepton. The latter can be obtained from the spin-dependent Weizsacker - Williams splitting function

$$\Delta\gamma = n_\gamma^{(+)} - n_\gamma^{(-)} = \frac{3}{4} \frac{\alpha}{\pi} \ln \frac{Q^2}{4m^2}. \quad (3)$$

This contribution was not considered in the paper ^{/10/} due to it have no logarithmically growing term. However, the value of Γ_1^γ in this case is determined by the triangle anomaly contribution in matrix elements of the axial quark current ^{/3/} in hadron part of the operator product expansion. In the other approach ^{/1,2/} Γ_1^γ is determined by a matrix elements of the axial photon current which coefficient function is the same anomaly.

Turn now to the QCD corrections. If the axial anomaly were not renormalized, the sum rules (1), (2) would be exact. It has become clear recently that the nonrenormalizability is broken in three-loop diagrams ^{/11,12/}. Notice in passing that this fact discovered first in ^{/11/} was misintrepreted as a breaking of one-loop character of the anomaly. As a matter of fact, it is a necessary condition of the one-loop character of the Adler-Bardeen relation ^{/12/}.

With taking into account the multiplicative renormalization of the anomaly ^{/12/} the sum rule (2) acquires the form

$$\int_0^1 g_1^\gamma(x, Q^2) dx = - \frac{\langle e^4 \rangle}{2} \frac{\alpha}{2\pi} N_f Q(Q^2), \quad (4)$$

where $Q(Q^2)$ in the leading two-loop approximation for the anomalous dimension ^{/6,7/} obeys the equation

$$\frac{dQ(Q^2)}{d \ln Q^2} = - \frac{1}{2} \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 N_f Q(Q^2) \quad (5)$$

$$Q(\infty) = 1$$

with the evident solution

$$Q(Q^2) = \exp \left\{ \frac{\alpha_s(Q^2)}{\pi} \frac{12N_f}{33-2N_f} \right\}. \quad (6)$$

It is not difficult to estimate that in the region of a few $(\text{GeV}/c)^2$, where $\alpha_s \approx 0.2-0.3$ the deviation of the asymptotical value is about 10%.

Naturally, the same factor has to appear in the sum rule (1) for the structure function of the electron.

Experimental check of the sum rules at the e^+e^- -colliders is of interest, in our opinion.

In conclusion we consider as a pleasure to thank S.B.Gerasimov and A.V.Radyshkin for fruitful discussions.

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Ефремов А.В., Теряев О.В. E2-89-528
 Аксиальная аномалия и правило сумм
 для структурной функции фотона

Показано, что первый момент структурной функции поляризованного фотона g_1^Y определяется треугольной аномалией. Ренормировка аномалии обуславливает Q^2 -зависимость соответствующего правила сумм.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

Efremov A.V., Teryaev O.V. E2-89-528
 Axial Anomaly and Sum Rule for Photon
 Spin Structure Function

It is shown that the first moment of the spin structure function g_1^Y of a polarized photon is determined by the axial anomaly. Renormalization of the anomaly determines the Q^2 -dependence of the corresponding sum rule.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1989

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