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ON CONTRIBUTION OF INSTANTONS TO NUCLEON SUM RULES

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1 Introduction

QCD sum rule (SR)[1,2] is a well-known method to describe hadron properties[3]. It is based on the first principles of QCD and allows one to relate phenomenological information on the nontrivial QCD vacuum structure expressed through nonzero averages of vacuum fields with physical characteristics of hadron ground states.

The success of QCD sum rules is related with the fact that the operator expansion of the correlator of currents[4] may usually restricted only to several operators of lowest dimensions such as quark $< 0 \mid \bar{q}q \mid 0 >$ and gluon $< 0 \mid G^a_{\mu\nu} G^a_{\mu\nu} \mid 0 >$ condensates.

However, there are channels which require a more detailed information on the QCD vacuum structure. The correlator of scalar currents is a typical example where direct instantons[5], high-frequency vacuum fluctuations, are important. Within the model of QCD vacuum as an instanton liquid[6,17] it has been shown [7] that the contributions of instantons to the sum rule for pseudoscalar mesons allows the explanation of the features of their spectrum, in particular, determination of an almost massless $\pi-$meson, and qualitative solution of the $U_A(1)$ problem related with the anomalously large mass of the $\eta'-$ meson.

These results were also confirmed in the quark model[8,9] that takes the QCD-vacuum properties into account explicitly and shows that quark interaction through instantons defines the hadron ground state spectrum. There the problem of describtion of isotopic differences of hadron masses[10] was solved and a deeply bound $H-$ dihyperon[11] whose stability is provided by instanton interaction was predicted.

It has also been shown in[9] that for the nucleon the instanton-induced interaction is of primary importance because the nucleon wave function contains the part where two quarks are in a spin-zero state (a scalar diquark) in the same manner as a quark-antiquark pair in the $\pi-$ meson.

However, in the standard SR for a nucleon[12] this fact has not been taken into account. Note that the nucleon SR[12] does not possess a sufficient stability. To cure this difficulty, several ways were proposed to improve the SR stability, in particular, by adding operators of higher dimensions, taking account of anomalous dimensions, the differences of continuum thresholds[13] or next-to-leading corrections in α_s of the perturbation QCD theory[14]. Although these corrections slightly influence absolute values of mass and residue of nucleon, they could not provide an essential improvement of the stability.

Here we shall show that, like in the pseudoscalar SR, the direct instanton contribution to the nucleon SR allows us not only to stabilize the corresponding SR but also to reproduce experimental value of nucleon mass.

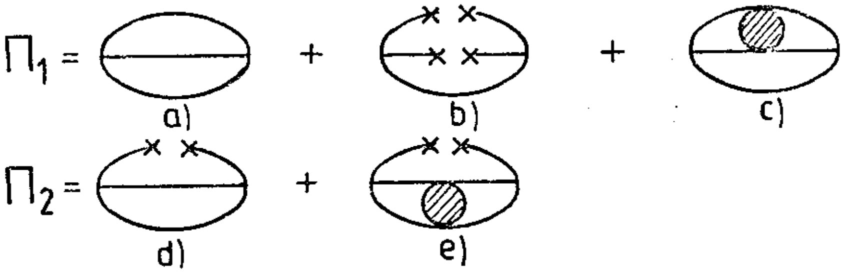

Figure 1: The contributions of a) quark loop, b) four-quark condensate $< 0 \mid \bar{q}q \mid 0 >^2$, c,e) direct instantons, d) quark condensate $< 0 \mid \bar{q}q \mid 0 >$ to the nucleon sum rule.

2 Nucleon sum rule

The most general expression of the nucleon current is[12,13,15]

$$O(x) = aO_1(x) + bO_2(x), \tag{1}$$

where

$$O_1(x) = \epsilon^{abc}(u^a C d^b)\gamma_5 u^c,$$
$$O_2(x) = \epsilon^{abc}(u^a C \gamma_5 d^b) u^c,$$

and a, b are any real parameters.

The current correlator (1) has two Dirac structures for which the dispersion relations are written ($Q^2 = -q^2$):

$$i \int dx\ e^{iqx} < 0 \mid T(O(x)\bar{O}(0)) \mid 0 >= \hat{q}\Pi_1(Q^2) + \Pi_2(Q^2), \tag{2}$$

where apparently Π_2 is connected with the spontaneous chiral symmetry breaking and is propotional to the nucleon mass.

A standard analysis of SR for baryons[12,13,15] is limited by the contributions of six-dimensional operators in Π_1, Π_2 (others give negligible contributions[13]). Here we also shall take account of the contribution of direct instantons (Fig. 1).

There are two types of direct instanton contributions: diagram 1c) manifestly breaking the hypothesis of factorization of the four-quark operator $< 0 \mid \bar{q}\Gamma q\ \bar{q}\Gamma q \mid 0 >$[2] usually accepted in SR contributes to Π_1, where diagram 1e) gives an exponential contribution of direct instantons. We do not take into account the gluon condensate $< 0 \mid G^a_{\mu\nu} G^a_{\mu\nu} \mid 0 >$ because the analysis shows that it practically does not affect final results for values of the nucleon mass and residue.

The contribution of direct instantons was evaluated by substituting into the expression for polarization operator (2), the Green function of the quark in the zero mode in the instanton field[16]:

$$S_\pm(x,y) = \frac{\psi_\pm(x)\psi^\dagger_\pm(y)}{m^*}, \tag{3}$$

where the zero mode:

$$\psi_\pm(x) = \frac{1}{\sqrt{2}\,\pi} \frac{\rho_c(1 \pm \gamma_5)\gamma_\mu(x-z)_\mu}{[\rho_c^2 + (x-z)^2]^{3/2} \mid x - z \mid} U,$$

$\rho_c \approx 1.6\ Gev^{-1}$ is the average size of the instanton in QCD vacuum, z is its displacement, U is a color-spin matrix, plus (minus) sign refers to the instanton (anti-instanton),

$$m^* = -\frac{2}{3}\pi^2 < 0 \mid \bar{q}q \mid 0 > \rho_c^2$$

is the effective mass of a quark in the instanton liquid.

Substituting (3) into (2), averaging over instantons displacement and borelizing analogously to[2] we obtain

$$\begin{aligned}
\Pi_1(\tau) &= F\exp(-M_N^2\tau^2) = 2\tau^{-6}\{E_2(\tau)\chi + 64z^{-6}[f\eta \qquad (4)\\
&+ 0.9(1 - \frac{24}{7}\,z^{-2} + \frac{5\sqrt{\pi}}{32}z^3\exp(-z^2))\phi]\},\\
\Pi_2(\tau) &= FM\exp(-M_N^2\tau^2) = k\tau^{-4}\{E_1(\tau)\eta \qquad (5)\\
&+ 2\sqrt{\pi}z\exp(-z^2)\varphi\},
\end{aligned}$$

where τ is a Borel parameter, F is a residue defined as $F = (4\pi)^4\lambda_N^2$, (λ_N is defined as $< 0 \mid j_N \mid N_{k,\lambda} >= \lambda_N u_N(k,\lambda)$),

$$z = \rho_c/\tau;\ k = -(4\pi)^2 < 0 \mid \bar{q}q \mid 0 >;$$

$$E_n(\tau) = 1 - \exp(-s_0\tau^2)\sum_{k=0}^{n}(s_0\tau^2)^k,\ f = 2n_c\frac{\pi^2\rho_c^4}{2} \propto 1/20$$

is a packing fraction in the model of instanton liquid (n_c is the instanton density),

$$\chi = \frac{5(a^2+b^2)+2ab}{8}, \ \varphi = \frac{13(a^2+b^2)+10ab}{16},$$

$$\eta = \frac{7b^2-5a^2-2ab}{4}, \ \varphi = b^2 - a^2.$$

Standard procedure of treating the SR[2,12] consists in searching for plateau of stability that is the region of τ where right and left parts of SR are in agreement within a certain accuracy. Then in this region both the corrections from higher-dimensional operators and continuum threshold have to be small.

Note that direct instantons contribute to SR (4,5) at any choice of nucleon current (1). Moreover without direct instantons the SR for Π_1 and Π_2 do not posses the plateau in the region $\tau \propto 1/M_N$. It is important that the contribution of direct instantons is different for Π_1 and Π_2. Π_1 contains both power and exponential terms but Π_2 contains only exponential terms. Then it appears that the SR for Π_1 structure is very sensitive to the choice of current (1). Indeed if at $\mid a \mid = \mid b \mid$[12] there are no instanton contributions, then at a small deviation from this relation their contributions becomes one order larger than contributions of diagrams 1a and 1b in the region $\tau \propto 1\ Gev^{-1}$. So at any $(a \neq b)$ the SR for Π_1 does not simultaneously satisfy the criteria of smallness of the continuum and nonperturbative corrections, becomes unstable and we eliminate it from our consideration. Note that the α_s corrections to Π_1[14] are also significant and narrow the scale of convergence of that Π_1 although α_s corrections to Π_2 are small.

At the same time exponential instanton contributons to Π_2 stabilize the SR and practically do not depend on the choice of nucleon current (1).

The graphs of the right-hand side of expressions

$$M_N^2(\tau) = -\frac{\partial_{\tau^2}\Pi_2(\tau)}{\Pi_2(\tau)}; \tag{6}$$

$$F(\tau) = \exp(M_N^2\tau^2)\Pi_2(\tau) \tag{7}$$

are given in Fig. 2,3 at different choices of current (1). From Fig. 2,3 it is seen that without direct instantons the bound nucleon state does not arise. Further, the choice of the nucleon current in the form with the

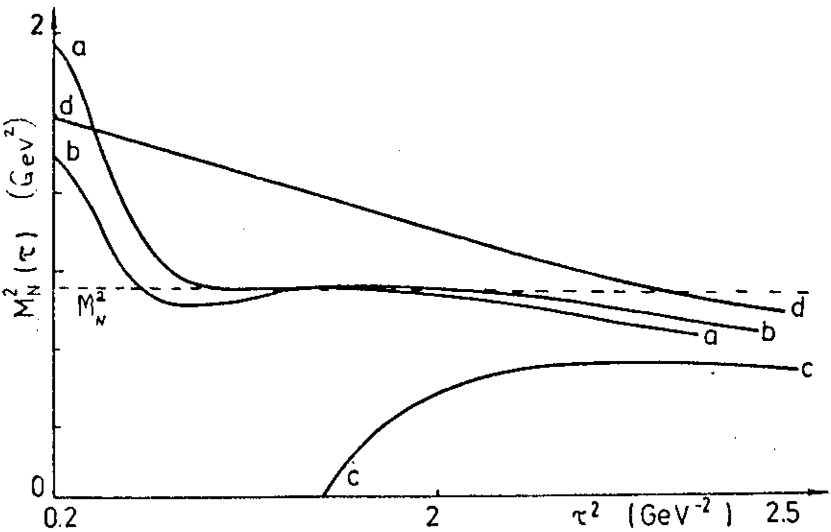

Figure 2: Right-hand side of sum rules (6) at different choice of nucleon current a) Ioffe's current a=-b=1, b) scalar "diquark" current b=0, c) the case without continuum b=5, a=-7, d) the case without instantons $\varphi = 0$

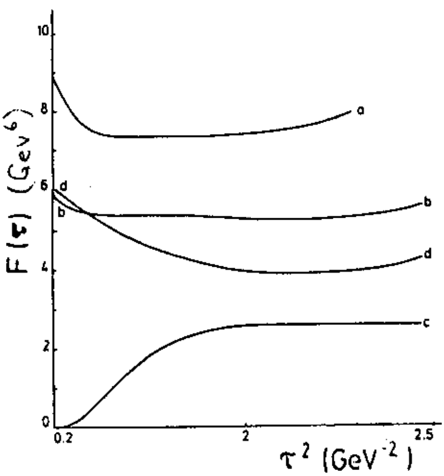

Figure 3: Right-hand side of sum rule (7).

vector diquark ($a = -b = 1$) and scalar diquark ($b = 0$) stabilizes the SR with practically experimental nucleon mass.

$$a)\ M_N = 0.94\ Gev,\ F = 7.5\ Gev^6,\ s_0^2 = 3.1\ Gev^2,$$

$$b)\ M_N = 0.96\ Gev,\ F = 5.5\ Gev^6,\ s_0^2 = 2.5\ Gev^2,$$

For the current ($b = 5,\ a = -7$) where the contribution of continuum (diagram 1d) is eliminated, the bound state arises only due to direct instantons. And finally, at any (a,b) the nucleon residue is approximately twice as larger as the value obtained in[13]. Note that the given results in fact insensitive to the parameters of instanton liquid because the coefficient of the stabilizating term in Π_2 (5) is a number. So taking account of direct instantons for the correlator of the nucleon current allows us to obtain stable result for the mass and residue of the nucleon.

3 Conclusion

So we have shown that the contributions of direct instantons to the nucleon sum rules stabilizes one. Note that in fact the nucleon mass is proportional to the inverse size of the instanton $1/\rho_c$ (plateau region, Fig. 2). The agreement between the obtained and experimental mass testifies that the effective size of instantons in the QCD vacuum corresponds to the instanton liquid model with $\rho_c \approx 1.6\ Gev^{-1}$ which corresponds to phenomenological estimation[7,17]. From our results it also follows that the nucleon is formed as a bound state in the instanton field, where two quarks are in zero modes (3) and the third acquires some effective mass due to the condensate in the instanton field.

So, the QCD sum rules confirm the conclusion on the dominant role of instanton-induced interaction in the hadron spectroscopy earlier obtained within the quark model[9].

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