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Ефимов Г.В., Неделько С.Н. E2-89-478
О регуляризации обрезанием интегралов по собственному времени в SU(N)-модели Янга-Миллса

В рамках метода фонового поля рассматривается SU(N)-модель Янга-Миллса. Предлагается регуляризация производящего функционала для функций Грина, которая сохраняет инвариантность относительно калибровочных преобразований фонового поля. Перенормировка может быть осуществлена калибровочно инвариантным образом. На уровне диаграмм Фейнмана предлагаемая регуляризация сводится к известному методу обрезания интегралов по собственному времени.

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Efimov G.V., Nedel'ko S.N. E2-89-478
On a Regularization of the SU(N)-Yang-Mills Model by Cutoff of the Proper-Time Integrals

SU(N)-Yang-Mills model is considered within the background field method. Ultraviolet regularization of the generating functional for Green functions, maintaining invariance under gauge transformations of the background field, is proposed. Gauge-invariant renormalization can be realized. In terms of Feynman diagrams the regularization reduces to the known method of cutoff of the proper-time integrals.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

In this paper we shall propose a new formulation of the known method of ultraviolet regularization. We shall investigate the pure SU(N)-Yang-Mills (Y-M) model within the background-field method [1-3] following the standard Euclidean path-integral approach.

The proper-time integrals appear naturally within the background-field calculations (see, e.g. ref. [4-6]). Cutoff of the proper-time integrals (CPTI) at the lower limit is frequently used for regularization of Feynman diagrams [5,6] (mainly in the one-loop calculations).

The gauge invariance to hold is the principal demand upon the regularization of a non-Abelian theory.

Our problem can be formulated as follows: does the CPTI-regularization break the gauge invariance of the initial nonregularized theory? We shall investigate the invariance properties of CPTI within the background-field method. For this purpose we shall introduce the regularization which is realized in terms of the path-integral representation of the generating functional for Green functions. At the level of Feynman diagrams this regularization is equivalent to CPTI. That is the reason for considering it as a generalized CPTI-regularization (GCPT).

The idea of background-field method can be formulated in the following way. The field in the classical Lagrangian is represented as a sum of quantum (Q_μ^a) and classical (B_μ^a) fields. Then the gauge-fixing condition invariant under gauge transformations of B and isotopic rotations of Q (B - invariance) is chosen. At the same time the Faddeev - Popov quantization ensures the invariance of the integrand in the generating functional $Z[J,B]$ under the gauge transformation of Q, until $J = 0$ (Q-invariance).

Thus, the gauge invariance of $Z[J,B]$ has a double meaning within the background-field method.

Q-invariance of $Z[J,B]$ is broken when $J \neq 0$. That is the reason why the problem of construction of the gauge-invariant effective action $\Gamma[B]$ is not yet solved completely. $\Gamma[B]$ is invariant under the gauge transformations of B but it depends on the functional form of the gauge-fixing condition (because $J \neq 0$) [1-3].

GCPT maintains the B-invariance but breaks the Q-invariance. It is sufficient for ensuring the gauge-invariant tensor structure of the regularized effective action $\Gamma_\lambda[B]$. $\Gamma_\lambda[B]$ depends on the form of gauge condition because $J \neq 0$ (as usual) and through the regularization.

The GCPT is realized in the following way . The part quadratic over quantum gluonic and ghost fields of the exponent in the integrand of $Z[J,B]$ is modified by introducing form-factors. As a consequence, gluonic and ghost propagators turn out to be regularized. It makes the theory superrenormalizable. An additional regularization of the one-loop section should be done. The theory becomes finite when the regularization parameter is finite.

GCPT may be useful for the calculation of the effective potential of the Y-M theory with the background constant field [5,6] specifically, within the nonperturbative methods (e.g., variational evaluation of functional integrals [7]).

We will not consider the problem of infrared divergences because it is a separate issue.

1. FORMULATION OF THE PROBLEM

The Euclidean generating functional of the Y-M theory with a background field B_μ^a has the form [2]

$$Z[J,B] = N^{-1}\int \delta Q \, det\left[\frac{\delta G^a}{\delta \omega^b}\right] exp\left\{ \int d^4x \left(\mathcal{L}_{Q+B}(x) - \right.\right.$$

$$\left.\left. - \frac{1}{2\xi}(G^a)^2 + Q_\mu^a(x) J_\mu^a(x) \right)\right\} , \tag{1.1}$$

where $\mathcal{L}_{Q+B}$ is the Y-M Lagrangian,

$$G^a = (\delta^{ab}\partial_\mu + ig B_\mu^{ab}) Q_\mu^b$$

is the background gauge-fixing term. $B_\mu = B_\mu^a T^a$, T^a are the generators of SU(N) in the adjoint representation: $(T^a)^{bc} = i f^{bac}$.

The determinant in (1.1) is defined by the transformation

$$B_\mu \to B_\mu$$

$$Q_\mu \to U(Q_\mu + B_\mu) U^+ - \frac{i}{g} U \partial_\mu U^+ - B_\mu \quad , \tag{1.2}$$

$$U = exp\{-i\omega^a(x) T^a\} .$$

If $J=0$, then the integrand in (1.1) is invariant under the transformation

$$B_\mu + Q_\mu \to U(B_\mu + Q_\mu) U^+ - \frac{i}{g} U \partial_\mu U^+ \tag{1.3}$$

which can be considered as (1.2), so as

$$B_\mu \to U B_\mu U^+ - \frac{i}{g} U \partial_\mu U^+ \tag{1.4}$$

$$Q_\mu \to U Q_\mu U^+ . \tag{1.5}$$

The effective action is related to $Z[J,B]$ via the Legendre transform

$$\Gamma[\tilde{Q},B] = W[J,B] - \int d^4x \, \tilde{Q}_\mu^a J_\mu^a ,$$

$$W[J,B] = \ln Z[J,B] , \quad \tilde{Q}_\mu^a = \frac{\delta W[J,B]}{\delta J_\mu^a} .$$

The functional $W[J,B]$ is invariant under (1.4) and

$$J_\mu \to U J_\mu U^+ . \tag{1.6}$$

This fact leads to invariance of $\Gamma[\tilde{Q},B]$ under (1.4) and

$$\tilde{Q}_\mu \to U \tilde{Q}_\mu U^+ . \tag{1.7}$$

It is shown [2] that the background-field effective action $\Gamma[\tilde{Q},B]$ is equivalent to the conventional one $\bar{\Gamma}[\tilde{Q}]$ calculated in a special gauge

$$\Gamma[\tilde{Q},B] = \bar{\Gamma}[\tilde{Q}+B] , \tag{1.8}$$

$$\Gamma[0,B] = \bar{\Gamma}[B] .$$

Relationship (1.8) means that $\Gamma[0,B]$ contains the same information, as $\bar{\Gamma}[Q]$. Explicit gauge invariance of $\Gamma[0,B]$ leads to the gauge-invariant renormalization [2,3] .

The above-mentioned scheme of construction of the gauge-invariant effective action is a formal one. It is necessary to regularize the theory.

We shall consider the regularization, maintaining the invariance under (1.3) considered only as (1.4),(1.5), but not as (1.2). We shall give arguments for the possibility of the gauge-invariant renormalization.

It is convenient for our purpose to rewrite (1.1) in the following form

$$Z[J,B] = N^{-1}\int \delta Q\,\delta C^{+}\delta C\, \exp\{\Gamma_{ce}[Q,C^{+},C,B] + \int d^4x\, J^a_\mu Q^a_\mu\}.$$

The action Γ_{ce} has the structure

$$\Gamma_{ce} = \Gamma_Q[Q,B] + \Gamma_{ce}[B] + \Gamma_{gh}[C^{+},C,B] + \Gamma_{int}[Q,C^{+},C,B]$$

in conformity with the Lagrangian

$$\mathcal{L}(x) = \mathcal{L}_Q(x) - \tfrac{1}{4}B^a_{\mu\nu}(x)B^a_{\mu\nu}(x) + \mathcal{L}_{gh}(x) + \mathcal{L}_{int}(x),$$

$$\mathcal{L}_Q = -\tfrac{1}{2}Q^a_\mu(x)\,K^{ab}_{\mu\nu}(B,\xi/x)\,Q^b_\nu(x),$$

$$K^{ab}_{\mu\nu}(B,\xi/x) = \left[-\nabla^2\delta_{\mu\nu} - 2igT^cB^c_{\mu\nu} + (1-\tfrac{1}{\xi})\nabla_\mu\nabla_\nu\right]^{ab}, \quad (1.9)$$

$$\mathcal{L}_{gh} = C^{+a}(x)M^{ab}(B/x)C^b(x),\quad M^{ab}(B/x) = (-\nabla^2)^{ab}, \quad (1.10)$$

$\mathcal{L}_{int}(x)$ is the interaction Lagrangian,

$$\nabla_\mu = \partial_\mu + igB_\mu(x),\quad B_\mu(x) = T^aB^a_\mu(x),$$

$$T^aB^a_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) + ig[B_\mu(x), B_\nu(x)],$$

T^a are the generators in the adjoint representation, ξ is the gauge parameter, $C(x)$ is the ghost field. The field B^a_μ obeys the classical equation of motion.

In a standard way, the functional $Z[J,B]$ is represented as

$$Z[J,B] = \exp\{\Gamma_{int}[\tfrac{\delta}{\delta J},\tfrac{\delta}{\delta \eta},\tfrac{\delta}{\delta \eta^{+}},B]\}\, Z^{\circ}[J,\eta^{+},\eta,B]\Big|_{\eta^{+}=\eta=0};$$

$$Z^{\circ}[J,\eta^{+},\eta,B] = N^{-1}\exp\{\Gamma_{ce}[B]\}\int \delta Q\,\delta C^{+}\delta C\,\times$$

$$\times\exp\{\Gamma_Q[Q,B] + \Gamma_{gh}[C^{+},C,B] + \int d^4x\,(Q^a_\mu J^a_\mu + C^{+a}\eta^a + \eta^{+a}C^a)\}.$$

Let us proceed to the construction of the regularized generating functional.

2. REGULARIZATION OF THE PROPAGATORS

Let us modify the quadratic part of the Lagrangian substituting

$$\mathcal{L}^\lambda_Q = -\tfrac{1}{2}Q^a_\mu(x)\,K^{ab}_{\mu\nu}(B,\xi,\lambda/x)\,Q^b_\nu(x),$$

$$\mathcal{L}^\lambda_{gh} = C^{+a}(x)M^{ab}(B,\lambda/x)C^b(x)$$

for $\mathcal{L}_Q$, $\mathcal{L}_{gh}$ (1.9), (1.16), where

$$K^{ab}_{\mu\nu}(B,\xi,\lambda/x) = \left[\Phi\left(\frac{K(B,1/x)}{2\lambda^2}\right)K(B,\xi/x)\,\Phi\left(\frac{K(B,1/x)}{2\lambda^2}\right)\right]^{ab}_{\mu\nu}, \quad (2.1)$$

$$M^{ab}(B,\lambda/x) = \left[\Phi\left(\frac{M(B/x)}{\lambda^2}\right)M(B/x)\right]^{ab}, \quad (2.2)$$

$$K^{ab}_{\mu\nu}(B,1/x) = -(\nabla^2\delta_{\mu\nu} + 2igT^cB^c_{\mu\nu})^{ab},$$

$$\Phi^{ab}_{(\mu\nu)}(\cdot) = [\exp\{\cdot\}]^{ab}_{(\mu\nu)}. \quad (2.3)$$

The substitution does not break the invariance of $\mathcal{L}(x)$ under the transformations (1.4),(1.5), since $K(B,1/x)$, $M(B/x)$ and, consequently, $\Phi(\frac{1}{2\lambda^2}K(B,1/x))$, $\Phi(\frac{1}{\lambda^2}M(B/x))$ are transformed covariantly.

The expression for $\tilde{Z}^{\circ}_\lambda[J,\eta^{+},\eta,B]$ corresponding to $\mathcal{L}^\lambda_Q$, $\mathcal{L}^\lambda_{gh}$, takes the following form after the standard integration over Q,C,C^{+}

$$\tilde{Z}^{o}_{\lambda}[J,\zeta^{+},\zeta,B] = N^{-1} \det[M(B,\lambda|\cdot)] \det^{-\frac{1}{2}}[K(B,\xi,\lambda|\cdot)] \times$$
$$\times \exp\{\Gamma_{cl}[B] - \iint d^4x\, d^4y\, (\tfrac{1}{2} J^a_\mu(x) G^{ab}_{\mu\nu}(B,\xi,\lambda/x,y) J^b_\nu(y) + \tag{2.4}$$
$$+ \zeta^{+a}(x) D^{ab}(B,\lambda/x,y) \zeta^b(y))\} .$$

The functions $G^{ab}_{\mu\nu}(B,\xi,\lambda/x,y)$ and $D^{ab}(B,\lambda/x,y)$ are the gluonic and ghost propagators in the external field B^a_μ, obeying the equations

$$[K(B,\xi,\lambda/x)\, G(B,\xi,\lambda/x,y)]^{ab}_{\mu\nu} = \delta^{ab}\delta_{\mu\nu}\,\delta(x-y) , \tag{2.5}$$

$$[M(B,\lambda/x) D(B,\lambda/x,y)]^{ab} = \delta^{ab}\delta(x-y) . \tag{2.6}$$

The function $G^{ab}_{\mu\nu}(B,\xi,\lambda/x,y)$ can be represented as (see Appendix A)

$$G^{ab}_{\mu\nu}(B,\xi,\lambda/x,y) = G^{ab}_{\mu\nu}(B,1,\lambda/x,y) + (1-\xi)\int d^4z \times \tag{2.7}$$
$$\times [G_{\mu\alpha}(B,1,\lambda\sqrt{2}/x,z) \nabla^z_\alpha \nabla^z_\beta G_{\beta\nu}(B,1,\lambda\sqrt{2}/z,y)]^{ab} ,$$

where $G^{ab}_{\mu\nu}(B,1,\lambda/x,y)$ satisfies

$$[K(B,1,\lambda/x) G(B,1,\lambda/x,y)]^{ab}_{\mu\nu} = \delta^{ab}\delta_{\mu\nu}\,\delta(x-y) . \tag{2.8}$$

Thus, it is enough to solve (2.6),(2.8) in order to find the propagators.

Equation (2.8) can be rewriten as

$$[K(B,1/x) \exp\{\frac{K(B,1/x)}{\lambda^2}\} G(B,1,\lambda/x,y)]^{ab}_{\mu\nu} = \delta^{ab}\delta_{\mu\nu}\delta(x-y). \tag{2.9}$$

Formally, the solutions of (2.6), (2.9) has the integral representation [6]

$$D^{ab}(B,\lambda/x,y) = \int_{\lambda^{-2}}^{\infty} ds [e^{-SM(B/x)}\delta(x-y)]^{ab} , \tag{2.10}$$

$$G^{ab}_{\mu\nu}(B,1,\lambda/x,y) = \int_{\lambda^{-2}}^{\infty} ds [e^{-SK(B,1/x)}\delta(x-y)]^{ab}_{\mu\nu} , \tag{2.11}$$

where S is the so-called proper time. It is just the exponential form-factor Φ that leads to cut-off of the integrals at the lower limit.

The singularity of propagators when $x \to y$, showing itself in the proper-time representation as a pole of the integrand in (2.10), (2.11) when $S \to 0$ is the source of ultraviolet divergences. Since the integrals are cut, the propagators turn out to be regularized.

Thus, we arrive at the superrenormalizable theory, as all multi-loop diagrams are finite, because only regularized propagators correspond to their internal lines.

Of course, formulars (2.10) and (2.11) will be useful practically if it is possible to calculate integrands explicitly. A special choice of B^a_μ makes this possible (e.g. $B^a_{\mu\nu}$ = const).

3. REGULARIZATION OF THE ONE-LOOP SECTION AND COMPLETELY REGULARIZED GENERATING FUNCTIONAL

Let us consider the one-loop section. It is defined by determinants in (2.4). Both gluonic and ghost determinants contain ultraviolet divergences, that require an additional regularization.

According to (2.1), (2.2) we find that

$$\det[K(B,\xi,\lambda|\cdot)] = \det[\Phi(\frac{K(B,1|\cdot)}{\lambda^2})] \det[K(B,\xi|\cdot)] ,$$
$$\det[M(B,\lambda|\cdot)] = \det[\Phi(\frac{M(B|\cdot)}{\lambda^2})] \det[M(B|\cdot)] .$$

As it is shown in [5], $\det[K(B,\xi|\cdot)]$ is ξ-independent up to the constant term.

Thus, we can introduce the regularization as

$$\det_{reg}\left[\frac{M(B|\cdot)}{M(0|\cdot)}\right] = \det\left[\frac{M(B|\cdot)}{M(0|\cdot)}\;\frac{M(0|\cdot)+\lambda^2}{M(B|\cdot)+\lambda^2}\right] , \tag{3.1}$$

$$\det_{reg}\left[\frac{K(B,\xi|\cdot)}{K(0,\xi|\cdot)}\right] = \det\left[\frac{K(B,1|\cdot)}{K(0,1|\cdot)}\;\frac{K(0,1|\cdot)+\lambda^2}{K(B,1|\cdot)+\lambda^2}\right] . \tag{3.2}$$

The parameter λ is chosen the same as in formulars (2.1),(2.2). Formulars (3.1) and (3.2) correspond to the Pauli-Villars regularization.

Substitution of (3.1), (3.2) into (2.4) gives the completely regularized Z°_{λ}.

It is simple now to write down the expression for the regularized functional $Z_{\lambda}[J,B]$

$$Z_{\lambda}[J,B] = N^{-1}\det{}^{-1}\left[\Phi\left(\frac{M(B|\cdot)}{\lambda^2}\right)\right]\det{}^{\frac{1}{2}}\left[\Phi\left(\frac{K(B,1|\cdot)}{\lambda^2}\right)\right]\times$$
$$\times\det\left[\frac{M(0|\cdot)+\lambda^2}{M(B|\cdot)+\lambda^2}\right]\det{}^{\frac{1}{2}}\left[\frac{K(B,1|\cdot)+\lambda^2}{K(0,1|\cdot)+\lambda^2}\right]\times \tag{3.3}$$
$$\times\exp\{\Gamma_{ce}[B]\}\int\delta Q\,\delta C^{+}\delta C\,\exp\Big\{\int d^4x\big(\mathcal{L}^{\lambda}_{Q}(x)+\mathcal{L}^{\lambda}_{gh}(x)+$$
$$+\mathcal{L}_{int}(x)+Q^a_{\mu}J^a_{\mu}\big)\Big\}.$$

The regularized effective action is defined as

$$\Gamma_{\lambda}[\hat{Q},B] = W_{\lambda}[J,B]-\int d^4x\,\tilde{Q}^a_{\mu}J^a_{\mu}\ , \tag{3.4}$$

$$W_{\lambda}[J,B] = \ln Z_{\lambda}[J,B]\ ,\quad \tilde{Q}^a_{\mu} = \frac{\delta W_{\lambda}[J,B]}{\delta J^a_{\mu}}\ . \tag{3.5}$$

Since $Z_{\lambda}[J,B]$ ((3.3)) is invariant under transformations (1.4) and (1.6), $\Gamma_{\lambda}[\tilde{Q},B]$ is invariant under (1.4) and (1.7) (see Appendix B). Consequently, $\Gamma_{\lambda}[0,B]$ is invariant under the transformation (1.4).

$\Gamma_{\lambda}[0,B]$ is the regularized invariant effective action, derived within the Abbott [2] formulation of the background-field method.

We conclude that the divergences (as $\lambda\to\infty$) of $\Gamma_{\lambda}[0,B]$ must have the gauge-invariant tensor structure and can be only of the logarithmic type (as the dimensional analysis shows). So, the renormalization can be realized by means of gauge-invariant counterterms.

In conclusion we mention the two important items remaining out of our consideration: a detailed formulation of the renormalization within the regularization, proposed in this paper, and the question about ξ -dependence of renormalization constants. (One can find the investigation of these questions in [3] (C.F.Hart)).

APPENDIX A

Let us show that $G^{ab}_{\mu\nu}(B,\xi,\lambda|x,y)$ represented by (2.7) obeys equation (2.5).

Substitution of (2.7) into (2.5) gives (see (2.1),(1.9))

$$K_{\mu\rho}(B,\xi,\lambda|x)\,G_{\rho\nu}(B,\xi,\lambda|x,y) = K_{\mu\rho}(B,1,\lambda|x)\,G_{\rho\nu}(B,1,\lambda|x,y)+$$
$$+(1-\xi)K_{\mu\rho}(B,1,\lambda|x)\int d^4z\,G_{\rho\alpha}(B,1,\lambda\sqrt{2}|x,z)\nabla^z_{\alpha}\nabla^z_{\beta}G_{\beta\nu}(B,1,\lambda\sqrt{2}|z,y)+$$
$$+\left(1-\frac{1}{\xi}\right)\Phi_{\mu\alpha}\left(\frac{K(B,1|x)}{2\lambda^2}\right)\nabla_{\alpha}\nabla_{\beta}\Phi_{\beta\rho}\left(\frac{K(B,1|x)}{2\lambda^2}\right)G_{\rho\nu}(B,1,\lambda|x,y)+ \tag{A.1}$$
$$+\left(1-\frac{1}{\xi}\right)(1-\xi)\,\Phi_{\mu\alpha}\left(\frac{K(B,1|x)}{2\lambda^2}\right)\nabla_{\alpha}\nabla_{\beta}\Phi_{\beta\rho}\left(\frac{K(B,1|x)}{2\lambda^2}\right)\times$$
$$\times\int d^4z\,G_{\rho\sigma}(B,1,\lambda\sqrt{2}|x,z)\nabla^z_{\sigma}\nabla^z_{\alpha}G_{\alpha\nu}(B,1,\lambda\sqrt{2}|z,y)\ .$$

We omit the color indices.

Obviously (see (2.3))

$$\Phi_{\mu\rho}\left(\frac{K(B,1|x)}{2\lambda^2}\right)\Phi_{\rho\nu}\left(\frac{K(B,1|x)}{2\lambda^2}\right) = \Phi_{\mu\nu}\left(\frac{K(B,1|x)}{\lambda^2}\right)\ . \tag{A.2}$$

One gets for the first item in (A.1) taking into account equation (2.8)

$$K_{\mu\rho}(B,1,\lambda|x)\,G_{\rho\nu}(B,1,\lambda|x,y) = \delta_{\mu\nu}\,\delta(x-y)\ . \tag{A.3}$$

Using (A.2) and (2.9) we get for the second item

$$(1-\xi)K_{\mu\rho}(B,1,\lambda|x)\int d^4z\,G_{\rho\alpha}(B,1,\lambda\sqrt{2}|x,z)\,\nabla^z_{\alpha}\nabla^z_{\beta}G_{\beta\nu}(B,1,\lambda\sqrt{2}|z,y)=$$
$$=(1-\xi)\,\Phi_{\mu\rho}\left(\frac{K(B,1|x)}{2\lambda^2}\right)\nabla_{\rho}\nabla_{\beta}G_{\beta\nu}(B,1,\lambda\sqrt{2}|x,y)\ . \tag{A.4}$$

Let us take into account

$$\Phi_{\beta\rho}\left(\frac{a}{\lambda^2}K(B,1|x)\right)G_{\rho\nu}(B,1,\lambda|x,y) = G_{\beta\nu}\left(B,1,\frac{\lambda}{\sqrt{1-a}}|x,y\right) \tag{A.5}$$

to rewrite the third item in the form

$$(1-\frac{1}{\xi})\phi_{\mu\alpha}\left(\frac{K(B,1/X)}{2\lambda^2}\right)\nabla_\alpha\nabla_\beta G_{\beta\nu}(B,1,\lambda\sqrt{2}/x,y) \,. \tag{A.6}$$

The fourth item takes the form (see (A.5))

$$(1-\frac{1}{\xi})(1-\xi)\phi_{\mu\alpha}\left(\frac{1}{2\lambda^2}K(B,1/X)\right)\nabla_\alpha^x\int d^4z\, \nabla_\rho^x G_{\rho\sigma}(B,1/x,z)\nabla_\sigma^{z\,x} \tag{A.7}$$

$$\times\nabla_\beta^z G_{\beta\nu}(B,1,\lambda\sqrt{2}/z,y) \,,$$

where $G_{\rho\sigma}(B,1/X,Z)=\lim_{\lambda\to\infty} G_{\rho\sigma}(B,1,\lambda/X,Z)$ is the nonregularized propagator.

From the identity

$$\nabla_\mu K_{\mu\rho}(B,1/X) = -\nabla^2\nabla_\rho$$

it is possible to obtain /8/ that

$$\nabla_\rho^x G_{\rho\sigma}(B,1/X,Z)\nabla_\sigma^z = -\delta(X-Z)\,. \tag{A.8}$$

To apply (A.8), one has to assume that $\nabla_\mu^z G_{\mu\nu}(B,1/Z,X)$ vanishes sufficiently fast for $Z\to\infty$ as to allow partial integration.

Substituting (A.8) into (A.7) we arrive at the expression

$$-(1-\frac{1}{\xi})(1-\xi)\phi_{\mu\alpha}\left(\frac{K(B,1/X)}{2\lambda^2}\right)\nabla_\alpha\nabla_\beta G_{\beta\nu}(B,1,\lambda\sqrt{2}/x,y)\,. \tag{A.9}$$

If we take into account (A.9), (A.6), (A.4) and (A.3), then we shall find that (A.1) will take the form of equation (2.5).

APPENDIX B

Let us show that $\Gamma_\lambda[\tilde{Q},B]$ is invariant under transformations (1.4), (1.7). For simplicity we omit Lorentz and color indices and integrations, if they are unimportant for understanding.

From (3.4), (3.5) one obtains

$$\frac{\delta\Gamma_\lambda[\tilde{Q},B]}{\delta\tilde{Q}} = -J \,. \tag{B.1}$$

The invariance of $W_\lambda[J,B]$ under (1.5), (1.8) means that

$$\frac{\delta W_\lambda}{\delta J}\delta J + \frac{\delta W_\lambda}{\delta B}\delta B = 0, \tag{B.2}$$

where δB and δJ correspond to infinitesimal transformations of (1.4), (1.6) type

$$\delta J_\mu^a = -i(\omega^c T^c)^{ab} J_\mu^b \,. \tag{B.3}$$

Let us take the derivative of (3.4) with respect to B (J is fixed)

$$\frac{\delta\Gamma_\lambda}{\delta B} + \frac{\delta\Gamma_\lambda}{\delta\tilde{Q}}\frac{\delta\tilde{Q}}{\delta B} = \frac{\delta W_\lambda}{\delta B} - J\frac{\delta\tilde{Q}}{\delta B}\,.$$

Taking into account (B.1) we get

$$\frac{\delta\Gamma_\lambda}{\delta B} = \frac{\delta W_\lambda}{\delta B} \tag{B.4}$$

In accordance with (B.3), (B.1), (3.5)

$$\frac{\delta W_\lambda}{\delta J^a}\delta J^a = -i\tilde{Q}^a(\omega^c T^c)^{ab}J_\mu^b = i\tilde{Q}^a(\omega^c T^c)^{ab}\frac{\delta\Gamma_\lambda}{\delta\tilde{Q}^b}\,.$$

Thus

$$\delta J^a\frac{\delta W_\lambda[J,B]}{\delta J^a} = \frac{\delta\Gamma_\lambda[\tilde{Q},B]}{\delta\tilde{Q}^a}\delta\tilde{Q}^a \,, \tag{B.5}$$

where $\delta\tilde{Q}^a = -i(\omega^c T^c)^{ab}\tilde{Q}^b$.

So we can rewrite relationship (B.2) in the form (see (B.4), (B.5))

$$\frac{\delta\Gamma_\lambda[\tilde{Q},B]}{\delta B}\delta B + \frac{\delta\Gamma_\lambda[\tilde{Q},B]}{\delta\tilde{Q}}\delta\tilde{Q} = 0\,. \tag{B.6}$$

Relationship (B.6) means that $\Gamma_\lambda[\tilde{Q},B]$ is invariant under transformations (1.4),(1.7).

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