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MESONS IN THE LOW-ENERGY LIMIT OF QCD_h

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Мезоны в низкоэнергетическом пределе
$КХД_{\alpha}$

Для класса кварковых моделей с точечным 4-кварковым взаимодействием описаны кварк-антикварковые состояния в рамках подхода Бете-Солпитера. Используется разложение вершинных и волновых функций Бете-Солпитера по лоренцовым структурам и по квантовым числам частиц. Получено условие нормировки для функции основного состояния. В качестве приложения общей схемы определен спектр масс низколежащих мезонов без разложения по энергии. Эти вычисления выполнены в специальной модели Намбу-Иона-Лазинио, следующей из КХД, для адронов с помощью процедуры локализации потенциала. Определены константы распадов π и k-мезонов. Получено хорошее согласие с экспериментальными данными.

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Mesons in the Low-Energy Limit of QCD_h

Quark-antiquark bound states are described within the Bethe-Salpeter approach for a class of quark models with instantaneous 4-quark interaction. Thereby decompositions of the Bethe-Salpeter vertex and wave functions according to their Lorentz structures and the particle content are used. Normalization conditions for the bound state functions are given. As an application of the general scheme, we determine the mass spectrum of low-lying mesons without expanding in energy. This calculation is performed for a special Nambu-Jona-Lasinio model which follows from the so-called QCD for hadrons, QCD_h, by a localization of the potential. Furthermore the pion and kaon decay constants are determined. We receive good agreement with the experimental data.

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1. Introduction

In investigating QCD as a theory of hadrons it is useful to utilize analogies with QED. In this sense QCD should be at least at the theoretical level as the atomic theory in QED.

As was shown in [1], the S-matrix theory with nonlocal atoms in QED differs from the one with free asymptotical states because elementary particles in atoms are off mass-shell. It turns out that for the description of the atomic spectra and the interaction of atoms it is necessary to add to the theoretical symmetry principles two empirical ones. These are the minimal quantization [2] and the choice of the time axis of quantization [1, 3].

The minimal quantization consists in the projection of the Belinfante energy-momentum tensor on the explicit solutions of the equations for the time gauge-field component. It leads to the Coulomb gauge and to a Lorentz transformation changing the gauge. Concerning the choice of the time axis of quantization, we suggest to take it parallel to the eigenvector of the bound state total momentum operator [3]. In this way, for a separate atom the latter corresponds to the Coulomb field moving together with the particles whose bound state it forms. The two empirical principles explained just now for QED are the prize to be paid for the application of a local S-matrix theory to nonlocal objects.

Now let us turn to QCD. A minimal construction of the QCD-Hamiltonian leads to the appearance of a new type (in comparison with QED) of static infrared divergences localized in the region of small spatial momenta (whereas in QED one deals with infrared divergences localized on the light cone). Therefore, an infrared redefinition of the QCD-Hamiltonian is required. Now, from the phenomenology of a lattice and heavy quarkonia it follows naturally that this can be achieved by adding in the Hamiltonian, to the colour current interaction, a rising potential as a background for a modified perturbation theory. This theory called the QCD for hadrons and denoted by QCD_h was investigated in [1]. In particular, there was shown the absence of infrared divergences, the existence of the parton limit, and the smallness of the effective coupling constant for all transfer momenta.

Furthermore, in [1] it was stated that the low-energy limit of the bound-state interaction corresponds to a localization of the bound-state wave functions with respect to the relative coordinate, and that such a localization is equivalent (for the properties of the solutions of the Bethe-Salpeter equation) to a localization of the rising potential. Therefore, in the low-energy limit the latter is replaced by a 4-quark Nambu-Jona-Lasinio potential [4] with a definite dependence on the quantization axis η_ν :

$$K^{\eta}_{NJL}(x-y) = \frac{N_c}{\mu^2} \not\eta\, \delta^4(x-y) \not\eta \tag{1.1}$$

with $\not\eta = \eta_\nu \gamma_\nu$ and $\eta^2 = 1$ (in the rest frame $\not\eta \equiv \gamma_0$). The parameter μ is fixed by the masses of low-lying resonances.

This paper is devoted to the investigation of the Bethe-Salpeter equation in the ladder approximation for vertex as well as wave functions of quark-antiquark bound states. In this context we consider quark models with an instantaneous 4-quark interaction like the quark sector of QCD_h and Nambu-Jona-Lasinio models.

The Bethe-Salpeter equations are transformed by means of decompositions for the vertex and wave functions according to their Lorentz structures into equations for new (lower-component) functions. Furthermore, we consider the normalization conditions for the Bethe-Salpeter functions.

We apply the general scheme to the investigation of the Nambu-Jona-Lasinio model of type (1.1) for the case of $SU(3)_f$. Thereby the difference from other modern treatments of the Nambu-Jona-Lasinio model [5, 6] lies not only in the matrix structure (1.1) but also in an exact calculation of the energy dependence in order to investigate the reasons for the appearance of tachyons in the QCD low-energy expansion [7]. Furthermore, the P-A, V-T, and S-V mixings have been exactly taken into account. They occur automatically by solving the Bethe-Salpeter equation with the help of projection operators on the particle and antiparticle states.

This paper is organized as follows. In Sect.2 we define the class of quark models under consideration throughout this paper. The corresponding Bethe-Salpeter equations in the ladder approximation for different vertex and wave functions of quark-antiquark bound states are given. Sect.3 is devoted to the normalization conditions for the vertex and wave functions. In Sect.4 we solve the Bethe-Salpeter equation for the Nambu-Jona-Lasinio model of type (1.1) and discuss the mass spectrum for low-lying mesons. Furthermore, the pion and kaon decay constants are calculated. Sect.5 contains the conclusion.

2. Bethe-Salpeter equation

2.1. Definition of the model

In this section we will investigate the Bethe-Salpeter equation for quark-antiquark bound states in the ladder approximation. Decompositions for the Bethe-Salpeter vertex and wave functions will be given. We derive the corresponding equations for the lower-component Bethe-Salpeter functions.

Throughout this paper we will consider quark models with the following effective action [1]:

$$\begin{aligned} S_{eff} = \int d^4x \Big\{ \bar{q}_{\alpha_1}(x) \left[G^{-1}_{\hat{m}^o}(x)\right]_{\alpha_1\beta_1} q_{\beta_1}(x) - \\ - \frac{1}{2N_c} \int d^4y\, q_{\beta_2}(y)\, \bar{q}_{\alpha_1}(x) \left[K^{\eta}(x-y)\right]_{\alpha_1\beta_1;\,\alpha_2\beta_2} q_{\beta_1}(x)\, \bar{q}_{\alpha_2}(y) \Big\}. \end{aligned} \tag{2.1}$$

Here $G^{-1}_{\hat{m}^o}(x) = i\not\partial - \hat{m}^o$ means the Green function for free quarks with the bare mass matrix $\hat{m}^o = \mathrm{diag}\,(m^o_1, m^o_2, \ldots, m^o_{N_f})$; α_i and β_i, $i = 1, 2$, are a compact notation for Dirac as well as flavour indices. Flavour and colour numbers are denoted by N_f and N_c , respectively. $K^{\eta}(x-y)$ is the instantaneous interaction kernel with the definite time axis η_μ ($\eta^2 = 1$) , which we choose for simplicity in the rest frame, $\eta_\mu = (1, 0, 0, 0)$:

$$K^{\eta}(x) = \not\eta\, V(x^{\perp})\, \delta(x\eta)\, \not\eta = \gamma_o\, V(\vec{x})\, \delta(x_o)\, \gamma_o \;,$$

$$x^{\perp}_{\mu} = x_{\mu} - x^{\parallel}_{\mu} \;, \quad x^{\parallel}_{\mu} = \eta_{\mu}\, (x\eta) \;.$$

In the short-hand notation action (2.1) can be written as [8]

$$S_{eff} = (q\bar{q}, -G^{-1}) - \frac{1}{2N_c}\, (q\bar{q},\, K^{\eta} q\bar{q}) \;.$$

After quantization over the quark fields it takes the form

$$S_{eff}[M] = N_c \left\{ \frac{1}{2}\, (M,\, (K^{\eta})^{-1} M) - i\, \mathrm{Tr}\, \ln\, (-G^{-1}_{\hat{m}^o} + M) \right\} \tag{2.2}$$

with $M = M(x,y)$ being the bilocal field; Tr means both the integration over continuous variables and the trace over discrete indices.

The extremum condition for action (2.2) coincides with the Schwinger-Dyson equation for the quark mass operator Σ:

$$\Sigma(x-y) = \hat{m}^o\, \delta^4(x-y) + i\, (K^{\eta}(x-y),\, G_{\Sigma}(x-y)) \;, \tag{2.3}$$

where

$$G^{-1}_{\Sigma}(x-y) = i\not\partial\, \delta^4(x-y) - \Sigma(x-y) \;.$$

This equation defines the quark mass spectrum.

2.2. Bound-state vertex functions

Now let us turn to the homogeneous Bethe-Salpeter equation in the ladder approximation for the vertex function of the bound state. For that we need the plane wave expansion of the bound state field:

$$M(x) = \sum_{H} \int \frac{d^3\mathcal{P}}{(2\pi)^{3/2}}\, \frac{1}{\sqrt{2\omega_H}} \left\{ e^{i\mathcal{P}x}\, a^{+}_{H}(\mathcal{P})\, \Gamma^{H}(\mathcal{P}) + e^{-i\mathcal{P}x}\, a^{-}_{H}(\mathcal{P})\, \bar{\Gamma}^{H}(\mathcal{P}) \right\} \;,$$

where

$$\omega_H = \sqrt{\vec{\mathcal{P}}^2 + M^2_H} \tag{2.4}$$

is the bound state energy ($\mathcal{P} = (\omega_H, \vec{\mathcal{P}})$). Furthermore, $a^{+}_{H}(\mathcal{P})$ ($a^{-}_{H}(\mathcal{P})$) are creation (annihilation) operators of states with the momentum $\mathcal{P}$ and the quantum numbers are denoted all together by H. $\Gamma^{H}(\mathcal{P})$ and $\bar{\Gamma}^{H}(\mathcal{P})$ mean the vertex functions of the bound state.

Then the Bethe-Salpeter equation in terms of Γ^{H} in an arbitrary reference frame is of the form [1]

$$\Gamma^{H}(q|\mathcal{P}) = -i \int \frac{d^4p}{(2\pi)^4}\, V(q^{\perp}-p^{\perp})\, \not\eta \left[G_a(p+\mathcal{P}/2)\, \Gamma^{H}(p|\mathcal{P})\, G_b(p-\mathcal{P}/2) \right] \not\eta \,. \tag{2.5}$$

Here the quark propagator

$$G_a(p) \equiv G_{m_a}(p) = \frac{1}{\not p - m_a + i\varepsilon}$$

was introduced. Equation (2.5) follows after the Fourier transformation from variation of the free part of the effective action (2.2) over fluctuations $(M - \Sigma)$. In the following we will consider this equation only in the rest frame:

$$\Gamma^{H}(q) = -i \int \frac{d^4p}{(2\pi)^4}\, V(\vec{q}-\vec{p})\, \gamma_o \left[G_a(p+M_H/2)\, \Gamma^{H}(p)\, G_b(p-M_H/2) \right] \gamma_o \,. \tag{2.6}$$

To obtain equations for the lower-component Bethe-Salpeter vertex functions, it is necessary to introduce projection operators for the two particles [9]. In the rest frame they are given by (for a = 1, 2)

$$\Lambda^{(a)}_{\pm}(p) = S^{-1}_a(p)\, \Lambda^{o}_{\pm}\, S_a(p) = \frac{1}{2}\, (1 \pm S^{-2}_a(p)\, \gamma_o) = \frac{1}{2}\, (1 \pm \gamma_o\, S^{2}_a(p)) \;,$$

$$\bar{\Lambda}^{(a)}_{\pm}(p) = S_a(p)\, \bar{\Lambda}^{o}_{\pm}\, S^{-1}_a(p) = \frac{1}{2}\, (1 \pm S^{2}_a(p)\, \gamma_o) = \frac{1}{2}\, (1 \pm \gamma_o\, S^{-2}_a(p)) \;, \tag{2.7}$$

where

$$S^{\pm 2}_a(p) = \sin \phi_a(p) \pm \hat{p} \cos \phi_a(p) = \exp\left[\pm 2\hat{p}\, \upsilon_a(p) \right] \;,$$

$$\hat{p} = \hat{p}_i \gamma_i \;, \quad \hat{p}_i = \frac{p_i}{|\vec{p}|} \;, \quad \hat{p}^2 = -1 \;,$$

$$\sin \phi_a(p) = \frac{m_a}{E_a(p)}, \qquad \cos \phi_a(p) = \frac{|\vec{p}|}{E_a(p)},$$

$$\upsilon_a(p) = \frac{1}{2}\left[-\phi_a(p) + \frac{\pi}{2} \right],$$

$$E_a(p) = \sqrt{\vec{p}^{\,2} + m_a^2}$$

and

$$\Lambda^o_\pm = \bar{\Lambda}^o_\pm = \frac{1}{2}(1 \pm \gamma_o).$$

In the following we will use the representation

$$S_a^{\pm 1} = c_a \pm \hat{p}\, s_a \tag{2.8}$$

with

$$s_a \equiv \sin \upsilon_a(p) \quad \text{and} \quad c_a \equiv \cos \upsilon_a(p). \tag{2.9}$$

Now, by means of the projection operators the propagator $G_a(p)$ can be represented as

$$\begin{aligned} G_a(p) &= \frac{1}{p_o\gamma_o - p_i\gamma_i - m_a + i\varepsilon} = \\ &= \left[\frac{\Lambda_+^{(a)}(p)}{p_o - E_a(p) + i\varepsilon} + \frac{\Lambda_-^{(a)}(p)}{p_o + E_a(p) - i\varepsilon} \right] \gamma_o = \\ &= \gamma_o \left[\frac{\bar{\Lambda}_+^{(a)}(p)}{p_o - E_a(p) + i\varepsilon} + \frac{\bar{\Lambda}_-^{(a)}(p)}{p_o + E_a(p) - i\varepsilon} \right]. \end{aligned} \tag{2.10}$$

After inserting (2.10) into (2.6) and integrating over p_o one obtains

$$\Gamma^H(q) = -\hat{I}_p(q)\, \gamma_o \left[\frac{\Pi_+(p)}{E(p) - M_H - i\varepsilon} + \frac{\Pi_-(p)}{E(p) + M_H - i\varepsilon} \right] \gamma_o \tag{2.11}$$

with

$$\begin{aligned} \Pi_+(p) &= \Lambda_+^{(a)}(p)\, \gamma_o\, \Gamma^H(p)\, \gamma_o\, \bar{\Lambda}_-^{(b)}(p), \\ \Pi_-(p) &= \Lambda_-^{(a)}(p)\, \gamma_o\, \Gamma^H(p)\, \gamma_o\, \bar{\Lambda}_+^{(b)}(p) \end{aligned} \tag{2.12}$$

and the total energy $E = E_a + E_b$, the bound state mass M_H as well as the integral operator

$$\hat{I}_p(q) = \int \frac{d^3p}{(2\pi)^3} V(p-q).$$

Now, using standard methods equation (2.11) for Γ^H decouples into equations for new wave functions Γ_1^H and Γ_2^H. In an arbitrary reference frame these are defined by means of the decomposition

$$\Gamma^H(p|\mathcal{P}) = \Gamma_1^H(p|\mathcal{P}) + \frac{\not{\mathcal{P}}}{M_H} \Gamma_2^H(p|\mathcal{P}),$$

$$\Gamma_l^H = 1S_l + \gamma_5 P_l + \left(\gamma_\mu - \mathcal{P}_\mu \frac{\not{\mathcal{P}}}{M_H^2}\right) V_l^\mu + \gamma_5 \left(\gamma_\mu - \mathcal{P}_\mu \frac{\not{\mathcal{P}}}{M_H^2}\right) A_l^\mu, \quad l = 1, 2.$$

However, to study the meson mass spectrum, it is enough to consider the bound states at rest, $\vec{\mathcal{P}} = 0$. In this case one has

$$\left.\begin{aligned} \Gamma^H &= \Gamma_1^H + \gamma_o \Gamma_2^H, \\ \Gamma_l^H &= \gamma^S L_l^S + \gamma^P L_l^P + \gamma_i^V L_{li}^V + \gamma_i^A L_{li}^A, \quad l = 1, 2, \quad i = 1, 2, 3, \end{aligned}\right\} \tag{2.13}$$

where $\gamma^S = 1$, $\gamma^P = \gamma_5$, $\gamma_i^V = \gamma_i$, $\gamma_i^A = \gamma_i\gamma_5$.

Furthermore, we make the decompositions ($a = 1, 2$)

$$L_{li}^I(p) = p_i L_l^I + e_i^a(p)\, L_l^{Ia}, \qquad I = V, A$$

with

$$\hat{e}^a(p) = \hat{e}_i^a(p)\, \gamma_i, \quad \hat{e}_i^a(p)\, \hat{e}_i^b(p) = \delta^{ab}, \quad \hat{p}_i\, \hat{e}_i^a(p) = 0.$$

Therefore, (2.13) takes the form

$$\Gamma^{H} = \frac{1}{2}\sum_{I}\left(\Gamma_1^{HI} + \gamma_0\,\Gamma_2^{HI}\right)\tilde{\gamma}^{I}\,, \tag{2.14}$$

where the matrices $\tilde{\gamma}^{I}$, I = 1, 2, ..., 6, are given by

$$\tilde{\gamma}^{1} = \gamma_5\,,\quad \tilde{\gamma}^{2} = \hat{e}^{a}\,,\quad \tilde{\gamma}^{3} = \hat{p}\,,\quad \tilde{\gamma}^{4} = 1\,,\quad \tilde{\gamma}^{5} = \gamma_5\hat{e}^{a}\,,\quad \tilde{\gamma}^{6} = \gamma_5\hat{p}\,. \tag{2.15}$$

Inserting decomposition (2.14) into (2.11) yields

$$\Pi^{\pm} = \sum_{I}\Pi_I^{\pm}\,,\qquad I = 1,\,2,\,\ldots,\,6\,,$$

$$\Pi_I^{\pm} = -\frac{1}{4}\left[\left(c^{-} - s^{-}\hat{p}\right) \pm \left(c^{+} + s^{+}\hat{p}\right)\gamma_0\right]\tilde{\gamma}^{I}\Gamma_{c\pm}\,,\qquad \text{for } I = 1,\,2,$$

$$\Pi_I^{\pm} = -\frac{1}{4}\left[\left(c^{+} - s^{+}\hat{p}\right) \pm \left(c^{-} + s^{-}\hat{p}\right)\gamma_0\right]\tilde{\gamma}^{I}\Gamma_{c\pm}\,,\qquad \text{for } I = 3,$$

$$\Pi_I^{\pm} = \frac{1}{4}\left[\left(s^{+} + c^{+}\hat{p}\right) \mp \left(s^{-} - c^{-}\hat{p}\right)\gamma_0\right]\tilde{\gamma}^{I}\Gamma_{s\mp}\,,\qquad \text{for } I = 4,\,5,$$

$$\Pi_I^{\pm} = \frac{1}{4}\left[\left(s^{-} + c^{-}\hat{p}\right) \mp \left(s^{+} - c^{+}\hat{p}\right)\gamma_0\right]\tilde{\gamma}^{I}\Gamma_{s\mp}\,,\qquad \text{for } I = 6.$$

Here the following notation has been introduced: $s^{\pm}$ and $c^{\pm}$ stand for the combinations

$$\left.\begin{aligned} s^{\pm} &= s_p^{\pm} \equiv s_1(p)c_2(p) \pm s_2(p)c_1(p)\,,\\ s^{\pm} &= c_p^{\pm} \equiv c_1(p)c_2(p) \mp s_1(p)s_2(p) \end{aligned}\right\} \tag{2.16}$$

of the trigonometrical functions (2.9), respectively. $\Gamma^{I}_{s\pm}$ and $\Gamma^{I}_{c\pm}$ are the modified vertex functions:

$$\Gamma^{I}_{c\pm} = c^{-}\,\Gamma^{I}_{1} \pm c^{+}\,\Gamma^{I}_{2}\qquad \text{for } I = 1,\,2,$$

$$\Gamma^{I}_{c\pm} = c^{+}\,\Gamma^{I}_{1} \pm c^{-}\,\Gamma^{I}_{2}\qquad \text{for } I = 3,$$

$$\Gamma^{I}_{s\pm} = s^{+}\,\Gamma^{I}_{1} \pm s^{-}\,\Gamma^{I}_{2}\qquad \text{for } I = 4,\,5,$$

$$\Gamma^{I}_{s\pm} = s^{-}\,\Gamma^{I}_{1} \pm s^{+}\,\Gamma^{I}_{2}\qquad \text{for } I = 6.$$

Now, we are able to write down the Bethe-Salpeter equations for the lower-component vertex functions in the rest frame. The result can be given in the following manner:

$$\Gamma^{[1-6]}_{\binom{1}{2}}(q) = \frac{\hat{I}_p(q)}{8}\left[\frac{1}{E-M}\left(\mathcal{C}_{\pm} - \xi\mathcal{S}_{\mp}\right)\Gamma_1^{[1-6]}(p) + \frac{1}{E+M}\left(\mathcal{C}_{\mp} + \xi\mathcal{S}_{\pm}\right)\Gamma_2^{[1-6]}(p)\right],$$

$$\Gamma^{[3-4]}_{\binom{1}{2}}(q) = \frac{\hat{I}_p(q)}{8}\left[\frac{1}{E-M}\left(\mathcal{S}'_{\pm} \mp \xi\mathcal{C}_{\pm}\right)\Gamma_1^{[3-4]}(p) + \frac{1}{E+M}\left(\mathcal{S}'_{\mp} \pm \xi\mathcal{C}_{\mp}\right)\Gamma_2^{[3-4]}(p)\right],$$

$$\Gamma^{2a}_{c\pm}(q) = -\frac{\hat{I}_p(q)}{8}\,\underline{\delta}^{ab}\left[\frac{1}{E-M}\,\mathcal{C}_{\pm}\,\Gamma^{2b}_{c+}(p) + \frac{1}{E+M}\,\mathcal{C}_{\mp}\,\Gamma^{2b}_{c-}(p)\right],$$

$$\Gamma^{5a}_{s\pm}(q) = -\frac{\hat{I}_p(q)}{8}\,\underline{\delta}^{ab}\left[\frac{1}{E-M}\,\mathcal{S}'_{\mp}\,\Gamma^{5b}_{s-}(p) + \frac{1}{E+M}\,\mathcal{S}'_{\pm}\,\Gamma^{5b}_{s+}(p)\right]$$

$$\tag{2.17}$$

with

$$\Gamma^{[1-6]}_{\binom{1}{2}} = \Gamma^{1}_{c\pm} - \Gamma^{6}_{s\mp}\,,\qquad \Gamma^{[3-4]}_{\binom{1}{2}} = \Gamma^{3}_{c\pm} - \Gamma^{4}_{s\mp}\,,$$

$$\mathcal{S}_{\pm} \equiv s_q^{-}\,s_p^{-} \pm s_q^{+}\,s_p^{+}\,,\qquad \mathcal{S}'_{\pm} \equiv s_q^{-}\,s_p^{+} \pm s_q^{+}\,s_p^{-}\,,$$

$$\mathcal{C}_{\pm} \equiv c_q^{-}\,c_p^{-} \pm c_q^{+}\,c_p^{+}\,,$$

$$\xi \equiv \hat{p}_i\,\hat{q}_i\,, \tag{2.18}$$

$$\underline{\delta}^{ab} \equiv e_i^{a}(q)\,e_i^{b}(p)\,, \tag{2.19}$$

and $E \equiv E(p)$.

2.3. Bound-state wave functions

Sometimes it is favourable to work not with the vertex function Γ ($\equiv \Gamma^{H}$) but with the Bethe-Salpeter wave function Ψ. In the rest frame both these quantities are connected with each other by the relation

$$\Gamma(q) = -\,\hat{I}_p(q)\,\gamma_0\,\Psi(p)\,\gamma_0\,, \tag{2.20}$$

so that Ψ is defined as

$$\Psi(p) = \left\{ \frac{\Pi_+(p)}{E(p) - M} + \frac{\Pi_-(p)}{E(p) + M} \right\} .$$

By using (2.11), (2.12), and (2.7) $\dot{\Psi}(p)$ and $\Pi_{\pm}(p)$ can be represented in the form

$$\Psi(p) = S_1^{-1}(p)\, \overset{o}{\Psi}(p)\, S_2^{-1}(p) \tag{2.21}$$

$$\Pi_{\pm}(p) = S_1^{-1}(p)\, \overset{o}{\Pi}_{\pm}(p)\, S_2^{-1}(p) \tag{2.22}$$

with the definitions

$$\overset{o}{\Psi}(p) \equiv - \left\{ \frac{\overset{o}{\Pi}_+(p)}{E(p) - M} + \frac{\overset{o}{\Pi}_-(p)}{E(p) + M} \right\} , \tag{2.23}$$

$$\overset{o}{\Pi}_{\pm}(p) = \Lambda^{o}_{\pm}\, \overset{o}{\Gamma}(p)\, \Lambda^{o}_{\mp} \tag{2.24}$$

and

$$\overset{o}{\Gamma}(p) = S_1^{-1}(p)\, \Gamma(p)\, S_2^{-1}(p) . \tag{2.25}$$

The function $\overset{o}{\Psi}(p)$ fulfills the condition

$$\Lambda^{o}_{\pm}\, \overset{o}{\Psi}(p)\, \Lambda^{o}_{\pm} = 0 .$$

Therefore, it exhibits an unambiguous expansion with respect to the Lorentz structures:

$$\overset{o}{\Psi}(q) = \gamma_5 L^{o} + \hat{e}^{\alpha}(q)\, N^{o\alpha} + \hat{q}\, \Sigma^{o} \tag{2.26}$$

with the decomposition

$$L^{o} = L^{o}_1 + \gamma_0 L^{o}_2 \tag{2.27}$$

and analogously for $N^{o\alpha}$ and σ^{o}.

Inserting (2.25) into (2.24) and the resulting expression in (2.23) one obtains with the help of (2.20)

$$\overset{o}{\Psi}(p) = \left\{ \frac{1}{E_T(p) - M} \Lambda^{o}_{+}\, \hat{I}_q(p)\, S'_1(p,q)\, \Psi(q)\, S'_2(q,p)\, \Lambda^{o}_{-} + \right.$$

$$\left. + \frac{1}{E_T(p) + M} \Lambda^{o}_{-}\, \hat{I}_q(p)\, S'_1(p,q)\, \Psi(q)\, S'_2(q,p)\, \Lambda^{o}_{+} \right\} ,$$

where $S'_i(p,q)$, $i = 1, 2$, is given by

$$S'_i(p,q) = S_i^{-1}(p)\, \gamma_0\, S_i^{-1}(q) . \tag{2.28}$$

The action of the projectors $\Lambda^{o}_{\pm}$ on the wave function $\overset{o}{\Psi}$ is given by

$$\Lambda^{o}_{\pm}\, \overset{o}{\Psi}(p) = \frac{1}{E(p) \mp M}\, \overset{o}{\Pi}_{\pm}(p) , \tag{2.29}$$

with

$$\overset{o}{\Pi}_{\pm}(p) = \Lambda^{o}_{\pm}\, \hat{I}_q(p)\, S_1(p,q)\, \Psi(q)\, S_2(q,p)\, \Lambda^{o}_{\mp} ,$$

From (2.29) one can derive the following two Schrödinger-type equations:

$$\left. \begin{aligned} M \gamma_0 \overset{o}{\Psi}(p) &= E(p)\, \overset{o}{\Psi}(p) - \overset{o}{\Pi}_+(p) - \overset{o}{\Pi}_-(p) , \\ M\, \overset{o}{\Psi}(p) &= E(p)\, \gamma_0 \overset{o}{\Psi}(p) - \overset{o}{\Pi}_+(p) + \overset{o}{\Pi}_-(p) \end{aligned} \right\} . \tag{2.30}$$

By means of the decomposition

$$\overset{o}{\Psi}(p) = \sum_{J=1}^{3} \left[\overset{o}{\Psi}{}^J_1(p) + \gamma_0 \overset{o}{\Psi}{}^J_2(p) \right] \bar{\gamma}^J_p ,$$

where the matrices $\bar{\gamma}^J$ are given in accordance with (2.26) and (2.27) by

$$\bar{\gamma}^1_p = \gamma_5 , \quad \bar{\gamma}^2_p = \hat{e}^{\alpha}(p) , \quad \bar{\gamma}^3_p = \hat{p} \tag{2.31}$$

and

$$\overset{o}{\Psi}{}^1_i = L^{o}_i , \quad \overset{o}{\Psi}{}^2_i = N^{o\alpha}_i , \quad \overset{o}{\Psi}{}^3_i = \Sigma^{o} , \quad i = 1, 2,$$

equations (2.30) can be rewritten for $\overset{o}{\Psi}{}^J_1$ and $\overset{o}{\Psi}{}^J_2$. In a common form they can be transformed to

$$M\, \overset{o}{\Psi}{}^J_{\binom{2}{1}}(p)\, \mathrm{tr}(\bar{\gamma}^K_q \bar{\gamma}^J_p) = E\, \overset{o}{\Psi}{}^J_{\binom{1}{2}}(p)\, \mathrm{tr}\,(\bar{\gamma}^K_q \bar{\gamma}^J_p) - \mathrm{tr}\,[\bar{\gamma}^K_q(\overset{o}{\Pi}_+(p) \pm \overset{o}{\Pi}_-(p))],\ I = 1,\ 2,\ 3. \tag{2.32}$$

Evaluating the traces in (2.32) for each case (J = 1, 2, 3) we obtain the following equations for L^o_ι, N^{oa}_ι, and Σ^o_ι :

$$M\, L^o_{\binom{2}{1}}(p) = E(p)\, L^o_{\binom{1}{2}}(p) + \hat{I}_q(p) \left[(c^\mp_p c^\mp_q - \xi s^\mp_p s^\mp_q)\, L^o_{\binom{1}{2}}(q)\right],$$

$$M\, \Sigma^o_{\binom{2}{1}}(p) = E(p)\, \Sigma^o_{\binom{1}{2}}(p) - \hat{I}_q(p) \left[(\xi c^\pm_p c^\pm_q - s^\pm_p s^\pm_q)\, \Sigma^o_{\binom{1}{2}}(q) + \underline{\eta}^b c^\pm_p c^\pm_q\, N^{ob}_{\binom{1}{2}}(q)\right]$$

$$M\, N^{oa}_{\binom{2}{1}}(p) = E(p)\, N^{oa}_{\binom{1}{2}}(p) - \hat{I}_q(p) \Big\{[c^\mp_p c^\mp_q \underline{\delta}^{ab} + s^\mp_p s^\mp_q(\xi \underline{\delta}^{ab} - \eta^a \underline{\eta}^b)]\, N^{ob}_{\binom{1}{2}}(q) +$$

$$+ (\eta^a c^\mp_p c^\pm_q)\, \Sigma^o_{\binom{1}{2}}(q)\Big\}. \tag{2.33}$$

Here $s^\pm_p$ and $c^\pm_p$ are defined according to (2.16). The quantities η^a and $\underline{\eta}^a$ are given by

$$\eta^a = q_\iota e^a_\iota(p), \qquad \underline{\eta}^a = p_\iota e^a_\iota(q), \tag{2.34}$$

and ξ and $\underline{\delta}^{ab}$ have been introduced earlier by (2.18) and (2.19), respectively.

From (2.21) one obtains the relation between Ψ^J and $\overset{o}{\Psi}{}^J$:

$$\Psi^J_{\binom{1}{2}}\, \mathrm{tr}\,(\bar{\gamma}^K \bar{\gamma}^J) = (\pm)\, \mathrm{tr}\left(\bar{\gamma}^K \bar{\gamma}^J\, S_1^{(\pm\alpha_J)} S_2\right) \overset{o}{\Psi}{}^J_{\binom{1}{2}}$$

with

$$\alpha_J = \begin{cases} 1, & J = 3, \\ -1, & J = 1, 2. \end{cases} \tag{2.35}$$

By means of (2.8) we get

$$S_1^{(\mp\alpha_J)}\, S_2^{-1} = c^{(\pm\alpha_J)} \mp \alpha_J\, s^{(\pm\alpha_J)}\, \hat{q} \tag{2.36}$$

so that

$$\Psi^J_{\binom{1}{2}} = \pm\, c^{(\pm\alpha_J)}\, \overset{o}{\Psi}{}^J_{\binom{1}{2}}, \qquad J = 1,\ 2,\ 3,$$

i.e.

$$L_{\binom{1}{2}} = \pm\, c^\mp\, L^o_{\binom{1}{2}}, \qquad \Sigma_{\binom{1}{2}} = \pm\, c^\pm\, \Sigma^o_{\binom{1}{2}}, \qquad N^a_{\binom{1}{2}} = \pm\, c^\mp\, N^{oa}_{\binom{1}{2}}.$$

Next let us consider the Bethe-Salpeter equation in terms of $\overset{o}{\Gamma}$. From (2.25), (2.20) and (2.21) there follows the representation

$$\overset{o}{\Gamma}(p) = -\hat{I}_q(p)\, S'_1(p,q)\, \overset{o}{\Psi}(q)\, S'_2(q,p), \tag{2.37}$$

where S' is given by (2.28). Decomposing $\overset{o}{\Gamma}$ as

$$\overset{o}{\Gamma} = \sum_I \left(\overset{o}{\Gamma}{}^I_1 + \gamma_o \overset{o}{\Gamma}{}^I_2\right) \tilde{\gamma}^I,$$

and $\overset{o}{\Psi}$ according to (2.26) one obtains from (2.37)

$$\overset{o}{\Gamma}{}^I_{\binom{1}{2}}(p)\, \mathrm{tr}\,(\tilde{\gamma}^K_p \tilde{\gamma}^I_p) = -\hat{I}_q(p)\, \overset{o}{\Psi}{}^J_{\binom{1}{2}}(q)\, \mathrm{tr}\left[\tilde{\gamma}^K_p S_1'^{(\pm 1)}(p,q) \bar{\gamma}^J_q\, S'_2(q,p)\right] =$$

$$= -\hat{I}_q(p)\, \overset{o}{\Psi}{}^J_{\binom{1}{2}}(q)\, \mathrm{tr}\left\{\tilde{\gamma}^K_p \gamma^o \bar{\gamma}^J_q \left[S_1^{(\mp\alpha_J)}(q)\, S_2^{-1}(q)\right] \gamma^o \left[S_2^{-1}(p)\, S_1^{(\mp\beta_K)}(p)\right]\right\}$$

with I, K = 1, 2, ..., 6 and J = 1, 2, 3 , α_J defined by (2.35) and

$$\beta_K = \begin{cases} 1, & K = 3, 4, 5 \\ -1, & K = 1, 2, 6. \end{cases} \tag{2.38}$$

The matrices $\tilde{\gamma}^K_p$ and $\bar{\gamma}^J_q$ are defined due to (2.15) and (2.31), respectively. Their lower indices denote the corresponding momenta.

Now, using relations of the type (2.36) the desired Bethe-Salpeter equations linking the vertex functions $\overset{o}{\Gamma}{}^I$ and the wave functions $\overset{o}{\Psi}{}^J$ take the form

$$\overset{o}{\Gamma}{}^{1}_{\binom{1}{2}}(p) = \hat{I}_q(p)\left[\left(c_q^{\pm}c_p^{\mp} - \xi s_q^{\mp}s_p^{\mp}\right) L^{o}_{\binom{1}{2}}(q)\right] ,$$

$$\overset{o}{\Gamma}{}^{2a}_{\binom{1}{2}}(p) = -\hat{I}_q(p)\left[\left(c_q^{\pm}c_p^{\mp}\underline{\delta}^{ab} - s_q^{\pm}s_p^{\mp}(\xi\underline{\delta}^{ab} - \eta^{a}\underline{\eta}^{b})\right) N^{ob}_{\binom{1}{2}}(q)\right] ,$$

$$\overset{o}{\Gamma}{}^{3}_{\binom{1}{2}}(p) = -\hat{I}_q(p)\left[\left(\xi c_q^{\mp}c_p^{\pm} - s_q^{\pm}s_p^{\pm}\right) \Sigma^{o}_{\binom{1}{2}}(q)\right] ,$$

$$\overset{o}{\Gamma}{}^{4}_{\binom{1}{2}}(p) = (\pm)\hat{I}_q(p)\left[\left(s_q^{\pm}c_p^{\pm} - \xi c_q^{\mp}s_p^{\pm}\right) \Sigma^{o}_{\binom{1}{2}}(q) - \underline{\eta}^{a}c_q^{\pm}s_p^{\pm} N^{oa}_{\binom{1}{2}}(q)\right] ,$$

$$\overset{o}{\Gamma}{}^{5a}_{\binom{1}{2}}(p) = (\pm)\hat{I}_q(p)\left[s_q^{\mp}c_p^{\pm}\,\eta^{a}\, L^{o}_{\binom{1}{2}}(q)\right] ,$$

$$\overset{o}{\Gamma}{}^{6a}_{\binom{1}{2}}(p) = (\pm)\hat{I}_q(p)\left[\left(\xi s_q^{\mp}c_p^{\mp} + s_q^{\mp}c_p^{\mp}\right) L^{o}_{\binom{1}{2}}(q)\right] .$$

On the other hand, one obtains from the Schrödinger-like equation (2.30) after replacing $\Pi_{\pm}(p)$ according to (2.22) and decomposing Γ^{o} like in (2.38) the following relations:

$$\overset{o}{\Gamma}{}^{J}_{\binom{1}{2}} = M\,\overset{o}{\Psi}{}^{J}_{\binom{2}{1}} - E\,\overset{o}{\Psi}{}^{J}_{\binom{1}{2}} , \qquad J = 1, 2, 3$$

or

$$\overset{o}{\Gamma}{}^{1}_{\binom{1}{2}} = M\, L^{o}_{\binom{2}{1}} - E\, L^{o}_{\binom{1}{2}}$$

and analogously for N^{oa} and Σ^{o} .

For completeness we also give the relations connecting $\overset{o}{\Gamma}{}^{I}$ and Γ^{K}. These can be derived from (2.25):

$$\Gamma^{1}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{1}_{\binom{1}{2}} \mp s^{\pm}\,\overset{o}{\Gamma}{}^{6}_{\binom{1}{2}} , \qquad \Gamma^{4}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{4}_{\binom{1}{2}} \mp s^{\pm}\,\overset{o}{\Gamma}{}^{3}_{\binom{1}{2}} ,$$

$$\Gamma^{2a}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{2a}_{\binom{1}{2}} , \qquad \Gamma^{5a}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{5a}_{\binom{1}{2}} ,$$

$$\Gamma^{3}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{3}_{\binom{1}{2}} \pm s^{\pm}\,\overset{o}{\Gamma}{}^{4}_{\binom{1}{2}} , \qquad \Gamma^{6a}_{\binom{1}{2}} = c^{\pm}\,\overset{o}{\Gamma}{}^{6a}_{\binom{1}{2}} \pm s^{\pm}\,\overset{o}{\Gamma}{}^{1}_{\binom{1}{2}} .$$

From these equations follows that, as already stated in (2.17), there takes place a mixing of vertex functions for $I = 1, 6$ and $I = 3, 4$. No mixing occurs for $I = 2, 5$.

3. Normalization of the Bethe-Salpeter function

The Bethe-Salpeter vertex function is normalized as follows:

$$\sum_{H} \int \frac{d^4 p}{(2\pi)^4} (\Gamma^{H}(\mathcal{P}))^{+}\, \Gamma^{H}(\mathcal{P})\, (\mathcal{P}_o - \omega_H)\, M_H = 1 . \tag{3.1}$$

Here $\mathcal{P}_o$ denotes the bound state energy and ω_H is given by (2.4). A normalization is possible only for $\vec{\mathcal{P}} \neq 0$. To derive a normalization condition for the lower-component Bethe-Salpeter vertex functions we consider the action (2.2) and make an expansion in powers of small values $(\mathcal{P}_o - \omega_H)$. Let us concentrate upon the term

$$S_{free2} = -i\,\frac{N_c}{2}\,\mathrm{tr} \int \frac{d^4\mathcal{P}}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4}\, \gamma_o\, G_1(\tfrac{\mathcal{P}}{2}+k)\, M(\mathcal{P})\, G_2(\tfrac{\mathcal{P}}{2}-k)\, \gamma_o\, M(\mathcal{P}) ,$$

which is obtained from the second part of the action (2.2) by means of a Fourier transformation. By using (2.10) it can be rewritten analogously to (2.11) in the following manner:

$$S_{free2} = -\frac{N_c}{2}\,\mathrm{tr} \int \frac{d^4\mathcal{P}}{(2\pi)^4} \frac{d^3 k}{(2\pi)^3} \left\{ \frac{\Lambda_{+}(k)\gamma_o\Gamma^{H}(\mathcal{P})\gamma_o\bar{\Lambda}_{-}(k)}{E(k) - \sqrt{\mathcal{P}^2} - i\varepsilon} + \frac{\Lambda_{-}(k)\gamma_o\Gamma^{H}(\mathcal{P})\gamma_o\bar{\Lambda}_{+}(k)}{E(k) + \sqrt{\mathcal{P}^2} - i\varepsilon} \right\} \Gamma(\mathcal{P}) . \tag{3.2}$$

Now we expand the dominators in (3.2) in powers of $(\mathcal{P}_o - \omega_H)$. Then, by taking into account

$$\sqrt{\mathcal{P}_o^2 - \vec{\mathcal{P}}_H^2} - M_H = \sqrt{\mathcal{P}_o^2 - \omega_H^2 + M_H^2} - M_H = (\mathcal{P}_o - \omega_H)\frac{\omega_H}{M_H} + \ldots$$

and introducing the notation $\mathcal{P}_H \equiv \sqrt{\mathcal{P}_H^2}$ the equation (3.2) turns into

$$S_{free2} \approx -\frac{N_c}{2}\,\mathrm{tr}\int\frac{d^4\mathcal{P}}{(2\pi)^4}\left[1-\frac{\omega_H}{M_H}(\mathcal{P}_o-\omega_H)\frac{\partial}{\partial\mathcal{P}_H}\right]\cdot$$

$$\cdot\int\frac{d^3k}{(2\pi)^3}\left(\frac{\Lambda_+(k)\gamma_o\Gamma^H(\mathcal{P})\gamma_o\bar{\Lambda}_-(k)}{E(k)-\mathcal{P}_H-i\varepsilon}+\frac{\Lambda_-(k)\gamma_o\Gamma^H(\mathcal{P})\gamma_o\bar{\Lambda}_+(k)}{E(k)+\mathcal{P}_H-i\varepsilon}\right)\Bigg|_{\mathcal{P}_H=M_H}\cdot\Gamma\ .$$

Comparing this expression with (3.1) one receives as normalization condition for the lower-component Bethe-Salpeter wave functions

$$\frac{N_c}{2M_H}\frac{\partial}{\partial\mathcal{P}_H}\,\mathrm{tr}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\Lambda_+(k)\gamma_o\Gamma^H\gamma_o\bar{\Lambda}_-(k)}{E(k)-\mathcal{P}_H-i\varepsilon}+\frac{\Lambda_-(k)\gamma_o\Gamma^H\gamma_o\bar{\Lambda}_+(k)}{E(k)+\mathcal{P}_H-i\varepsilon}\right)\Bigg|_{\mathcal{P}_H=M_H}\cdot\Gamma=1. \tag{3.3}$$

4. Meson Masses and decay constants from a special Nambu-Jona-Lasinio model

4.1 Mass spectrum for low-lying mesons

Now let us turn to the special Nambu-Jona-Lasinio model defined by the effective action (2.2) with K^{η} being the instantaneous interaction kernel (1.1). The corresponding Schwinger-Dyson equation (2.3) for the quark mass operator which is now given by $\Sigma(x-y) = \delta^4(x-y)\,\mathrm{diag}\,(m_1, m_2, ..., m_{N_f})$ reduces to the following equations:

$$m_i = m_i^o - i\frac{N_c}{\mu_i^2}\int\frac{d^4q}{(2\pi)^4}\frac{1}{\not{q}-m_i+i\varepsilon} =$$

$$= m_i^o + \frac{N_c}{2\mu_i^2}\int_0^L\frac{d\vec{q}}{(2\pi)^3}\frac{1}{(\vec{q}^{\,2}+m_i^2)^{1/2}}\ ,\qquad i=1,2,...,N_f\ . \tag{4.1}$$

Here we introduced, for each constituent quark mass m_i, a corresponding parameter μ_i so that the parameter μ in (1.1) is replaced by

$$\mu^2=\mu_a\,\mu_b\ ,\quad a,b=u,d,s\ .$$

Integration in (4.1) leads to

$$8\pi^2\mu_i^2 = N_c\frac{m_i}{m_i-m_i^o}\left[L^2-m_i^2\ln\frac{2L}{m_i}\right]\ . \tag{4.2}$$

This relation linking the parameters μ_i and L with the masses m_i^o and m_i will be of importance in our subsequent calculations.

Our aim is to calculate, from our Nambu-Jona-Lasinio model within the Bethe-Salpeter approach, the mass spectrum for low-lying mesons in the three-flavour case (the calculation for two flavours has been performed in [9]). To do this, it is enough to consider the bound states at rest, $\vec{\mathcal{P}}=0$. In our case the Bethe-Salpeter equation (2.6) takes the form

$$\Gamma^H = -i\frac{N_c}{\mu^2}\int\frac{d^4p}{(2\pi)^4}\,\gamma_o\,G_a(p+\mathcal{P}_H^o/2)\,\Gamma^H\,G_b(p-\mathcal{P}_H^o/2)\,\gamma_o\ .$$

After inserting decomposition (2.13) for Γ^H it decouples into four sets of two algebraic equations for the quantities L_1^I and L_2^I:

$$\left.\begin{aligned}-\frac{\mu^2}{N_c}L_1^I &= C^I L_1^I + B^I L_2^I\ ,\\ &\qquad\qquad I=S,P,V,A\\ -\frac{\mu^2}{N_c}L_2^I &= D^I L_2^I + B^I L_1^I\ ,\end{aligned}\right\} \tag{4.3}$$

($L_i^I \equiv L_{ii}^I$ for $I=V,A$, $i=1,2,3$).

The expressions for the coefficients B^I, C^I, and D^I are given in table 1. There is employed the following short-hand notation:

$$A(M_H)=\frac{1}{2}\left[m_1\,\alpha_1(M_H)-m_2\,\alpha_2(M_H)\right]\ ,$$

$$F(M_H)=\frac{1}{2}\left[m_1\,\alpha_1(M_H)+m_2\,\alpha_2(M_H)\right]\ ,$$

where α_1, α_2, and β denote the integrals

$$\alpha_1(M_H)=\int^L\frac{d^3k}{(2\pi)^3}\frac{1}{E_1}\frac{1}{(E_1+E_2)^2-M_H^2+i\varepsilon} \tag{4.4}$$

(α_2 is obtained from α_1 by interchanging indices 1 and 2),

Table 1.: Coefficients B^I , C^I , and D^I in the system of equations (4.3) for I = S, P, V, A.

I	B^I	C^I	D^I
S	$M_H A(M_H)$	$(m_1 - m_2)A(M_H) + \beta$	$(m_1 - m_2)A(M_H)$
P	$M_H F(M_H)$	$(m_1 + m_2)F(M_H) + \beta$	$(m_1 + m_2)F(M_H)$
V	$M_H F(M_H)$	$(m_1 + m_2)F(M_H) + 2\beta/3$	$(m_1 + m_2)F(M_H) + \beta/3$
A	$M_H A(M_H)$	$(m_1 - m_2)A(M_H) + 2\beta/3$	$(m_1 + m_2)A(M_H) + \beta/3$

$$\beta(M_H) = \int^{L} \frac{d^3k}{(2\pi)^3}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)\frac{k^2}{(E_1 + E_2)^2 - M_H^2 + i\varepsilon} .$$

β can be expressed by means of α_1 and α_2 :

$$\beta(M_H) = \frac{\mu^2}{2N_c}\left(2 - \frac{m_{01}}{m_1} - \frac{m_{02}}{m_2}\right) + \frac{1}{2}\left[\left(M_H^2 - 3m_1^2 - m_2^2\right)\alpha_1(M_H) + \left(M_H^2 - 3m_2^2 - m_1^2\right)\alpha_2(M_H)\right] .$$

The coefficients B^I , C^I , and D^I in system (4.3) of homogeneous algebraic equations are in general complex quantities. In this way system (4.3) has an unambiguous solution if

$$\left.\begin{aligned}&\left(\mathrm{Re}\, C^I - \frac{\mu^2}{N_c}\right)\cdot\left(\mathrm{Re}\, D^I - \frac{\mu^2}{N_c}\right) - (\mathrm{Re}\, B^I)^2 = 0\\ &(\mathrm{Im}\, C^I)\cdot(\mathrm{Im}\, D^I) - (\mathrm{Im}\, B^I)^2 = 0\end{aligned}\right\} . \tag{4.5}$$

We will use just these equations to determine the mass spectrum.

First of all one has to calculate the integrals α_1 and α_2 .For α_1 one gets:

$$\mathrm{Re}\,\alpha_1(M_H) = \frac{N_c}{8\pi^2\mu^2}\cdot$$

$$\begin{cases}\left[d_1(M_H) - \sqrt{z_2(M_H)}\arctan\left(r_2\sqrt{z_2(M_H)}\right)^{-1}\right], & z_2 > 0 ,\\ \left[d_1(M_H) - \sqrt{|z_2(M_H)|}\ \ln\dfrac{1 + r_2\sqrt{|z_2(M_H)|}}{1 - r_2\sqrt{|z_2(M_H)|}}\right], & z_2 < 0 ,\end{cases} \tag{4.6}$$

$$\mathrm{Im}\,\alpha_1(M_H) = \begin{cases}0 , & z_2 > 0 ,\\ Z(M_H), & z_2 < 0 ,\end{cases} \tag{4.7}$$

$$Z(M_H) = \frac{N_c}{8\pi^2\mu^2}\,\frac{\pi}{4}\sqrt{|z_2(M_H)|}\left[\left(1 - \frac{\Delta_2^2}{M_H^4}\right)\sqrt{\frac{z_2(M_H)}{z_1(M_H)}} + 1 + \Delta_2\left(m_2^2 - m_1^2 + M_H^2\right)\right] , \quad z_2 < 0$$

with

$$d_1(M_H) = \frac{L^2}{2}(r_2 - r_1) + \frac{M_H^2}{2}\ln\left[\frac{(1 + r_1)(1 + r_2)}{m_1 m_2}L^2\right] + \left(m_1^2 - \frac{\Delta_2^2}{2M_H^2}\right)\ln\left(\frac{1 + r_1}{1 + r_2}\,\frac{m_2}{m_1}\right) ,$$

$$\Delta_2 = m_2^2 - m_1^2 ,$$

$$r_i = \sqrt{1 + \frac{m_i^2}{L^2}} , \quad i = 1, 2 ,$$

$$z_2(M_H) = \frac{4m_2^2M_H^2}{(M_H^2 - \Delta_2)^2} - 1 .$$

Furthermore, in solving the mass relation for the a_1-meson we modify integrals (4.4) for α_1 and α_2 , taking into account the decay width $\Gamma(a_1 \to \bar{q}q)$ and substituting $M_H \equiv M_{a_1}$ by $M_{a_1} + i\Gamma$. The new integrals are denoted by $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$. For $\tilde{\alpha}_1$ one has:

$$\mathrm{Re}\,\tilde{\alpha}_1(M_{a_1}) = \frac{M^2_{a_1}}{M^2_{a_1}+\Gamma^2_{a_1}}\,\mathrm{Re}\,\alpha_1(M_{a_1}) + \frac{M_{a_1}\Gamma_{a_1}}{M^2_{a_1}+\Gamma^2_{a_1}}\,\mathrm{Im}\,\alpha_1(M_a)\,, \tag{4.8}$$

$$\mathrm{Im}\,\tilde{\alpha}_1(M_{a_1}) = -\frac{M_{a_1}\Gamma_{a_1}}{M^2_{a_1}+\Gamma^2_{a_1}}\,\mathrm{Re}\,\alpha_1(M_{a_1}) + \frac{M^2_{a_1}}{M^2_{a_1}+\Gamma^2_{a_1}}\,\mathrm{Im}\,\alpha_1(M_{a_1})\,. \tag{4.9}$$

The expressions for α_2 and $\tilde{\alpha}_2$ are obtained from (4.6) - (4.9) interchanging the indices 1 and 2. In the same way one gets the formulae for $d_2(M_H)$, Δ_1 and $z_1(M_H)$.

4.2 Determination of the pion and kaon decay constants

According to (2.13) the Bethe-Salpeter vertex function for the pseudoscalar sector with the bound state at rest is given by

$$\Gamma^P = \gamma_5 L^P_1 + \gamma_0\gamma_5 L^P_2\,.$$

The corresponding Bethe-Salpeter equation has the form

$$\left.\begin{aligned} 4\mu^2 L^P_1 &= C^P(M_P)\,L^P_1 + B^P(M_P)\,L^P_2\,,\\ 4\mu^2 L^P_2 &= D^P(M_P)\,L^P_2 + B^P(M_P)\,L^P_1 \end{aligned}\right\},$$

where

$$B^P(M_P) = 8N_c M_P F(M_P)\,,$$

$$C^P(M_P) = 8N_c(m_1 + m_2)F(M_P) + 4N_c\beta(M_P)\,,$$

$$D^P(M_P) = 8N_c(m_1 + m_2)F(M_P)\,.$$

Here in distinction to (4.3), tr_γ has been calculated.

The pseudoscalar decay constants are defined by the axial-vector coupling in the second term of action (2.1) which we denote by S_{free2}. The corresponding pseudoscalar part reads

$$S^P_{free2}(M_P) = \frac{1}{2}\,tr_{fl}\Big[\,C^P(M_P)(L^P_1)^2 + D^P(M_P)(L^P_2)^2 + 2B^P(M_P)\,L^P_1L^P_2\,\Big]\,. \tag{4.10}$$

This representation follows immediately from equation (3.2). The last term of (4.10) is of interest here. It yields

$$F_{P_i} = 2N_c F(M_{P_i})\,L^{P_i}_1$$

for the pion ($i = u$) and kaon ($i = s$) decay constants.

To calculate F_π and F_K, the normalization of the pseudoscalar vertex functions $\gamma_5\,L^{P_i}_1$ is required. It follows from the normalization condition (3.3) for the lower-component Bethe-Salpeter vertex functions which for the pseudoscalar sector is given by

$$\frac{1}{\mathcal{P}_P}\frac{\partial}{\partial\mathcal{P}_P}\,S^P_{free2}(\mathcal{P}_P)\Big|_{\mathcal{P}_P=M_P} = 1$$

with $\mathcal{P}_P = \sqrt{\mathcal{P}^2_P}$. Then, with the help of (4.10) we obtain the normalization condition for pion and kaon vertex functions:

$$\mathcal{N}_i^{-2}\left(L^{P_i}_1\right)^2 = 1$$

with

$$\mathcal{N}_i^{-2} = \frac{1}{2\mathcal{P}_P}\frac{\partial}{\partial\mathcal{P}_P}\,C^P(\mathcal{P}_P)\Big|_{\mathcal{P}_P=M_P}\,,$$

so that

$$L^{P_i}_1 = \mathcal{N}_i$$

and finally

$$F_{P_i} = 2N_c F(M_{P_i})\,\mathcal{N}_i\,. \tag{4.11}$$

4.3. Numerical results

In this paper we calculate the masses for low-lying mesons in the iso-spin symmetric case $m_{ou} = m_{od}$, $m_u = m_d$. Then the pion mass relation takes approximately the form (for $m_u^2/L^2 \ll 1$ and $M_\pi^2 \ll 4m_u^2$):

$$M_\pi^2 = 2\,\frac{m_{ou}}{m_u}\left[\,L^2 + m_u^2\left(2 - 3\ln\frac{2L}{m_u}\right)\right]\cdot\left(\ln\frac{2L}{m_u} - 1\right)^{-1}. \tag{4.12}$$

From expression (4.12) it is clearly seen that the pion appears as a Goldstone particle ($M_\pi = 0$ for $m_o = 0$).

Now let us present the numerical results. Our theory contains, as parameters, the bare quark masses $m_{ou} = m_{od}$, m_{os}, the parameters

μ_i^2 , i = u, d, s with $\mu_u^2 = \mu_d^2$ and the cut-off parameter L . From the Schwinger-Dyson equation it follows that μ_i^2 and L are connected by relation (4.2).

To fix the parameters, we take, as input data $m_u = m_d = 330$ MeV, $m_s = 400$ MeV, $M_\pi = 140$ MeV, $M_K = 494$ MeV and $M_\rho = 770$ MeV. Then one obtains for $N_c = 3$ with $L = 1590$ MeV , $\mu_u^2 = \mu_d^2 = 8.7 \cdot 10^4$ MeV^2 , $\mu_s^2 = 9.7 \cdot 10^4$ MeV^2 from (4.5) - (4.9):

$$m_{ou} = m_{od} = 2.1 \text{ MeV}, \quad m_{os} = 56 \text{ MeV},$$

$$M_\sigma = 660 \text{ MeV},$$

$$M_{a_1} = (1132 - i654) \text{ MeV},$$

$$M_{K^*} = 896 \text{ MeV},$$

$$M_\Phi = 970 \text{ MeV}.$$

From (4.11) one gets for the decay constants $F_\pi = 108$ MeV and $F_K = 149$ MeV, so that

$$\frac{F_K}{F_\pi} = 1.38 .$$

The results obtained in our earlier work [10] for the two-flavour case are in agreement with the corresponding values reported here.

5. Conclusion

We have derived general equations for different Bethe-Salpeter vertex and wave functions (including the lower-component ones) for quark models with instantaneous 4-quark interaction like, for instance, the quark sector of QCD_h or Nambu-Jona-Lasinio-type models. Thereby, we used the projection operators (2.9) on particle and antiparticle states and decompositions (2.14), (2.15) of Γ^H , and (2.26) for $\Psi(q)$. The main results of sect.2 are equations (2.17) and (2.33).

The general scheme is applied to determine the masses of low-lying mesons from the Nambu-Jona-Lasinio model of type (1.1) in the case of three flavours. Thereby, in distinction to the usual procedure we did not expand in energy. In this connection let us refer to some recent papers [5, 11] on different Nambu-Jona-Lasinio models. We want also to mention that in [12] there was considered a Nambu-Jona-Lasinio model for the two-flavour case within the Bethe-Salpeter approach.

In our approach no tachyons appear, whereas they do appear in the QCD low-energy expansion as claimed in [7]. Furthermore , within our approach the P-A , V-T , and S-V mixings have been taken into account from the very beginning by using an adequate decomposition of the Bethe-Salpeter vertex functions. With the help of the general normalization condition (3.3) we were able to determine the pion and kaon decay constants. The numerical results reported in sect.4.3. are in good agreement with experiment.

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