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дубна

E2-89-439

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NEW RELATIONS BETWEEN ℓN-SCATTERING CROSS SECTIONS AND NEUTRAL CURRENT PARAMETERS

Submitted to "Zeitschrift für Physik C"

1989

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Новые соотношения между сечениями ℓN-рассеяния и параметрами нейтральных токов

Получены новые соотношения в глубоконеупругом и квазиупругом $\overset{(-)}{\nu}$ N-, $e^{\pm}(\mu)^{\pm}$N-рассеянии, устанавливающие связь сечений с параметрами нейтральных токов и независящие от структурных функций и формфакторов нуклона. Известный пример такого рода - соотношение Пашоса-Вольфенштейна в $\overset{(-)}{\nu}$ N-рассеянии. Соотношения получены с учетом вклада дополнительного Z'-бозона, что позволяет использовать их как для извлечения параметров Стандартной модели (ρ, $\sin^2\theta_W$), так и поиска некоторых проявлений новой физики.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

Bednyakov V.A., Kovalenko S.G. E2-89-439

New Relations Between ℓN-Scattering Cross Sections and Neutral Current Parameters

New relations which connect cross sections with neutral current parameters have been obtained in deep inelastic and (quasi-) elastic $\overset{(-)}{\nu}$ N-, $e^{\pm}(\mu)^{\pm}$N-scattering; the relations are independent of the structure functions and formfactors of the nucleon. A known example is the Paschos-Wolfenstein relation in $\overset{(-)}{\nu}$ N-scattering. The relations have been obtained with allowance for the contribution of the extra Z'-boson which makes it possible to use them both for extractions of the Standard Model parameters (ρ, $\sin^2\theta_W$) and for the search for some manifestations of new physics.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1989

Precision measurement of the Standard Model (SM) parameters and the search for new-physics effects require special effort for elimination of the factors that are difficult to be theoretically checked. In lepton-nucleon scattering this is first of all the nucleon structure characterized by the structure functions (SF) and formfactors of the nucleon (FFN). Neither of them was found on the basic of the first principles of the theory as yet. So the values of the neutral current parameters extracted from experimental data may have large systematic errors due to the uncertainty in the theoretical description of the nucleon structure. It is well known that in νN-scattering there are relations for special combinations of the cross sections which allow the uncertainty to be eliminated. An example is the Paschos-Wolfenstein relation [1]:

$$R_1(\nu) = \frac{\sigma_{NC}(\nu N^{I=0}) - \sigma_{NC}(\bar{\nu} N^{I=0})}{\sigma_{CC}(\nu N^{I=0}) - \sigma_{CC}(\bar{\nu} N^{I=0})} = \rho^2 (\tfrac{1}{2} - \sin^2\theta_W). \qquad (1)$$

Despite the fact that the formula involves the cross sections of the deep inelastic νN-scattering ($N^{I=0}$ is the isoscalar target), nevertheless the right-hand side of the relation contains only the SM parameters ρ, $\sin^2\theta_W$ and is independent of SF.

In this paper new relations of this kind are found for scattering of (anti-)neutrino and longitudinally polarized electrons and positrons or $\mu^\pm$ - mesons on non-polarized nucleons and nuclei. The relations are obtained with allowance for the contribution of the extra Z′-boson which is considered in ℓN-scattering as the most probable manifestation of new physics at the tree level. We discuss the Z′-boson occuring in the superstring inspired E_6- grand unified model [2], which is an object of thorough investigations now [3]-[17]. The relations obtained can be used for analysis of the results of precision measurements which will probably be carried out in the near future. The use of them will allow a decrease in the influence of the uncertainties related to nucleon structure, which is important for achieving a high accuracy of the extracted values of the SM parameters $(\rho, \sin^2\theta_W)$, and for the search for Z′-boson manifestations.

§1. Effective Lagrangian and ℓN-scattering cross sections

As a basis, we shall take the SM Lagrangian extended to include the Z′-boson:

$$L_{NC} = eA^{\mu}J^{em}_{\mu} + g_Z Z^{\mu}_1 J^{(1)}_{\mu} + g_{Z'} Z^{\mu}_2 J^{(2)}_{\mu}. \tag{2}$$

The fields Z_1, Z_2 are the eigenstates of the mass matrix whose non-diagonal elements determine the mixing of current Z^o and Z′ states. Mixing angle θ is:

$$\tan^2\theta = (M^2_{zo(sm)} - M^2_{z1})/(M^2_{z2} - M^2_{zo(sm)}),$$

where $M_{zo(sm)} = M_W/\cos\theta_W$, M_{z1} and M_{z2} are the masses of the physical fields Z_1, Z_2. The neutral currents entering into Lagrangian (2) will be written down as

$$J^{(k)}_{\mu} = \Sigma\{\varepsilon^{(k)}_L \bar{f}_L \gamma_{\mu} f_L + \varepsilon^{(k)}_R \bar{f}_R \gamma_{\mu} f_R\},\ k = 1,2. \tag{3}$$

The chiral constants are of the form

$$\varepsilon^{(1)}_i(f) = \varepsilon^{zo}(f)\cos\theta + g_{Z'}/g_Z\, \varepsilon^{Z'}(f)\sin\theta, \quad i = L,R \tag{4}$$

$$\varepsilon^{(2)}_i(f) = \varepsilon^{Z'}(f)\cos\theta - g_Z / g_{Z'} \varepsilon^{zo}(f)\sin\theta. \tag{5}$$

$$\varepsilon^{zo}(f) = T_{3L}(f) - X_W\, Q^{em}(f), \tag{6}$$

Here $X_W = \sin^2\theta_W$; $T_{3L}(f)$ and $Q^{em}(f)$ are the third component of the weak isospin and the electric charge of the fermion f. The chiral constants $\varepsilon^{Z'}$ are given in Table. Angle θ_{E6}, involved in their definition, characterizes the scheme of the E_6 gauge symmetry breaking and is a free parameter of the theory. The relation between the coupling constants g_Z and $g_{Z'}$ also depends upon the symmetry breaking. The following result of the renormalization group analysis is known in the general case [7]:

$$(g_{Z'}/g_Z)^2 \le \tfrac{5}{3} X_W.$$

When studying ℓN-scattering in the Born approximation, it is convenient to follow the effective Lagrangian [18]

$$L^{eff}_{NC} = -\frac{4G}{\sqrt{2}}[\bar{\ell}_L \gamma_{\mu} \ell_L J^{\ell\mu}_{(L)} + \bar{\ell}_R \gamma_{\mu} \ell_R J^{\ell\mu}_{(R)}], \tag{7}$$

which is obtained from the initial Lagrangian (2), ($\ell_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\ell$, $\ell = \nu, e, \mu$). Effective hadron current (i = L,R) are introduced:

$$J^{\ell\mu}_{(i)} = \sum_q \{E^{\ell q}_{(i)L} \bar{q}_L \gamma^{\mu} q_L + E^{\ell q}_{(i)R} \bar{q}_R \gamma^{\mu} q_R\} = \tag{8}$$

$$= \tfrac{1}{2}[\alpha^{\ell}_{(i)} V^{3\mu} + \beta^{\ell}_{(i)} A^{3\mu} + \gamma^{\ell}_{(i)} V^{o\mu} + \delta^{\ell}_{(i)} A^{o\mu}]. \tag{9}$$

Here V^3, V^o are the isovector and isoscalar vector currents; A^3, A^o are the isovector and isoscalar axial-vector currents. The effective current parameters depend upon Q^2 and have the form:

$$E^{\ell q}_{(i)j} = \frac{\chi}{2}\frac{m^2_p}{Q^2}\Big[Q^{em}(\ell)\, Q^{em}(q) + $$

$$+ \frac{Q^2}{X_W(1-X_W)}\Big(\frac{\varepsilon^{(1)}_i(\ell)\varepsilon^{(1)}_j(q)}{M^2_1 + Q^2} + \Big(\frac{g_{Z'}}{g_Z}\Big)^2 \frac{\varepsilon^{(2)}_i(\ell)\varepsilon^{(2)}_j(q)}{M^2_2 + Q^2}\Big)\Big], \tag{10}$$

$$\alpha^{\ell}_{(i)}(Q^2)= \chi\frac{m_p^2}{Q^2}\, Q^{em}(\ell) + a^{\ell}_{(i)}(Q^2)\alpha^{Z} + b^{\ell}_{(i)}(Q^2)\alpha^{Z'},$$

$$\beta^{\ell}_{(i)}(Q^2)= a^{\ell}_{(i)}(Q^2)\beta^{Z} + b^{\ell}_{(i)}(Q^2)\beta^{Z'}, \tag{11}$$

$$\gamma^{\ell}_{(i)}(Q^2)= \frac{\chi}{3}\,\frac{m_p^2}{Q^2}\, Q^{em}(\ell) + a^{\ell}_{(i)}(Q^2)\gamma^{Z} + b^{\ell}_{(i)}(Q^2)\gamma^{Z'},$$

$$\delta^{\ell}_{(i)}(Q^2)= a^{\ell}_{(i)}(Q^2)\delta^{Z} + b^{\ell}_{(i)}(Q^2)\delta^{Z'},$$

$$a^{\ell}_{i}(Q^2)= \frac{2}{(1-X_W)}\left[\varepsilon^{(1)}_{i}(\ell)\cos\theta\,\frac{M_W^2}{M_1^2+Q^2} - \left(\frac{g_{Z'}}{g_Z}\right)\varepsilon^{(2)}_{i}(\ell)\sin\theta\,\frac{M_W^2}{M_2^2+Q^2}\right], \tag{12}$$

$$b^{\ell}_{i}(Q^2)= \frac{2}{(1-X_W)}\left(\frac{g_{Z'}}{g_Z}\right)^2\left[\varepsilon^{(1)}_{i}(\ell)\sin\theta\left(\frac{g_Z}{g_{Z'}}\right)\frac{M_W^2}{M_1^2+Q^2} + \varepsilon^{(2)}_{i}(\ell)\cos\theta\,\frac{M_W^2}{M_2^2+Q^2}\right],$$

where $\chi = 2\pi\alpha\sqrt{2}/Gm_p^2 \approx 0.6\ 10^4$, m_p is the proton mass.

In the tree approximation $\alpha^Z= 1-2X_W$, $\beta^Z= 1$, $\gamma^Z= -\frac{2}{3}X_W$, $\delta^Z= 0$, $\alpha^{Z'} = -\beta^{Z'} = -\gamma^{Z'} = 2\,\frac{\sin\theta_{E6}}{\sqrt{10}}$, $\delta^{Z'} = 2\,\frac{\cos\theta_{E6}}{\sqrt{6}}$.

Lagrangian (7) is convenient because it is similar to the Lagrangian of the νN-scattering. It allows the formulae for deep inelastic and (quasi-)elastic scattering cross sections of polarized leptons on nucleons with allowance for the Z′-boson contribution to be written down without calculations. It is enough to make the following substitutions in corresponding formulae for the νN-scattering cross sections: $\nu \to e_L$, $\bar\nu \to \bar e_R$, $\varepsilon_{L,R} \to E^{\ell}_{(L)L,R}$ or $\bar\nu \to \bar e_L$, $\nu \to e_R$, $\varepsilon_{L,R} \to E^{\ell}_{(R)L,R}$.

Lets us write down the final formulae for deep inelastic ℓN-scattering

$$\frac{d^2\sigma^{NC}}{dxdy}(\ell_L N)= \varphi_N(E^{\ell}_{(L)L},E^{\ell}_{(L)R}); \qquad \frac{d^2\sigma^{NC}}{dxdy}(\bar\ell_R N)= \varphi_N(E^{\ell}_{(L)R},E^{\ell}_{(L)L}), \tag{13}$$

$$\frac{d^2\sigma^{NC}}{dxdy}(e^+_L N)= \varphi_N(E^{e}_{(R)L},E^{e}_{(R)R}); \qquad \frac{d^2\sigma^{NC}}{dxdy}(e^-_R N)= \varphi_N(E^{e}_{(R)R},E^{e}_{(R)L}), \tag{14}$$

$$\frac{d^2\sigma^{CC}}{dxdy}(\nu(\bar\nu)N)= \sigma_o x\{f_{s(\bar s)} + f_{d(\bar d)} + (f_{\bar u(u)} + f_{\bar c(c)})(1-y)^2\}, \tag{15}$$

and (quasi-)elastic ℓN-scattering

$$\frac{d\sigma}{dQ^2}(\ell_L(\bar\ell_R)N\to\ell_L(\bar\ell_R)N)= \varphi_{\pm}(F^{\ell}_{VL(N)},F^{\ell}_{ML(N)},F^{\ell}_{AL(N)}) \tag{16}$$

$$\frac{d\sigma}{dQ^2}(\bar\ell_L(\ell_R)N\to\bar\ell_L(\ell_R)N)= \varphi_{\pm}(F^{\ell}_{VR(N)},F^{\ell}_{MR(N)},F^{\ell}_{AR(N)}) \tag{17}$$

$$\frac{d\sigma}{dQ^2}(\nu n(\bar\nu p)\to e^-_L p(e^+_R n))= \frac{d\sigma}{dQ^2}(e^-_L p(e^+_R n)\to\nu n(\bar\nu p))= \cos^2\theta_c\varphi_{\pm}(F^{CC}_V,F^{CC}_M,F^{CC}_A). \tag{18}$$

To shorten the writing, the following notation is introduced:

$$\varphi_N(\varepsilon_L,\varepsilon_R)= \sigma_o x\ \Sigma\{f^N_q(x,Q^2)[\,|\varepsilon_L(q)|^2 + (1-y)^2|\varepsilon_R(q)|^2] + f^N_{\bar q}(x,Q^2)[\,|\varepsilon_R(q)|^2 + (1-y)^2|\varepsilon_L(q)|^2]\}, \tag{19}$$

$$\varphi_{\pm}(F_V,F_M,F_A)= \frac{G^2}{\pi}\{\left(\frac{F_V\pm F_A}{2}\right)^2+(1-y)^2\left(\frac{F_V\mp F_A}{2}\right)^2+ \frac{My}{4E}(F_A^2-F_V^2)+ + \frac{y}{2}E\ F_M[(1-y)\frac{E}{2M}F_M + y(F_V + \tfrac{1}{4}F_M \mp F_A) \pm 2F_A]\}; \tag{20}$$

where $\sigma_o= \frac{2G^2ME}{\pi} \approx 1.72\cdot10^{-41}(E/1GeV)cm^2$, $f^N_q(x,Q^2),f^N_{\bar q}(x,Q^2)$- are the distribution functions (DF) of quarks and antiquarks in the nucleon; N= p,n; $F_{V,M,A}(Q^2)$ are the FFN. E is the initial lepton energy in the lab system, $Q^2= 2MExy = Sxy$.

FFN $F^{\ell}_{k(N)}(Q^2)$, $F^{CC}_{k}(Q^2)$ are determined from the matrix elements:

$$\langle p,n|J^{\ell\mu}_{(i)}|p,n\rangle=\bar u(p_2)\{F^{\ell}_{Vi(p,n)}\gamma^{\mu}-\frac{\sigma^{\mu\nu}q_\nu}{2M}F^{\ell}_{Mi(p,n)}-\gamma^{\mu}\gamma^{5}F^{\ell}_{Ai(p,n)}\}u(p_1),$$

$$\langle p|J^{CC}_{\mu}|n\rangle= \bar u(p_2)\{F^{CC}_{V}\gamma^{\mu}- \frac{\sigma^{\mu\nu}q_\nu}{2M}F^{CC}_{M} - \gamma^{\mu}\gamma^{5}F^{CC}_{A}\}u(p_1). \tag{21}$$

Following the isotopic symmetry of strong interactions and the CVC hypothesis, one can put down:

$$F^{CC}_{V,M} = F^{p}_{1,2} - F^{n}_{1,2}. \tag{22}$$

$$F^{\ell}_{Vi(p,n)}(Q^2)= r^{\ell}_{+i}\ F^{p}_{1}(Q^2) - r^{\ell}_{-i}\ F^{n}_{1}(Q^2), \qquad i= L,R$$

$$F^{\ell}_{Mi(p,n)}(Q^2)= r^{\ell}_{+i}\ F^{p}_{2}(Q^2)\ -\ r^{\ell}_{-i}\ F^{n}_{2}(Q^2), \tag{23}$$

$$F^{\ell}_{Ai(p,n)}(Q^2)= \pm\ \beta^{\ell}_{i}\ F^{CC}_{A}(Q^2)\ +\ \delta^{\ell}_{i}\ F^{o}_{A}(Q^2),$$

where $r^{\ell}_{\pm i}= \frac{1}{2}(\alpha^{\ell}_{i}\ \pm\ 3\gamma^{\ell}_{i})$ and $F^{p,n}_{1,2}$ are the electromagnetic FFN. According to the definition, $G^{p,n}_{M}= F^{p,n}_{1}\ +\ F^{p,n}_{2}$ is the magnetic FFN; the isoscalar axial-vector FFN F^{o}_{A} is in the matrix element

$$\langle N|A^{o}_{\mu}|N\rangle= \bar{u}(p_2)\gamma_{\mu}\gamma^{5}u(p_1)F^{o}_{A}(Q^2). \tag{24}$$

§2. Factorization and elimination of the nucleon structure dependence

Let us turn to the main task of the paper and find the combinations of cross sections for different processes where the dependence upon the structure functions and formfactors of the nucleon characterizing its structure are cancelled.

Let us begin with deep inelastic ℓN-scattering and introduce the following quantities:

$$\Delta^{\ell}_{\pm\ L,R}= \frac{d^2\sigma^{NC}}{dxdy}(\ell_{L,R}n)\ \pm\ \frac{d^2\sigma^{NC}}{dxdy}(\ell_{L,R}p), \tag{25}$$

$$\bar{\Delta}^{\ell}_{\pm\ L,R}= \frac{d^2\sigma^{NC}}{dxdy}(\bar{\ell}_{R,L}n)\ \pm\ \frac{d^2\sigma^{NC}}{dxdy}(\bar{\ell}_{R,L}p), \tag{26}$$

$$\Delta_{\pm}= \frac{d^2\sigma^{CC}}{dxdy}(\nu n)\ \pm\ \frac{d^2\sigma^{CC}}{dxdy}(\nu p)\ =\ \frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{L}p)\ \pm\ \frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{L}n), \tag{27}$$

$$\bar{\Delta}_{\pm}= \frac{d^2\sigma^{CC}}{dxdy}(\bar{\nu}n)\ \pm\ \frac{d^2\sigma^{CC}}{dxdy}(\bar{\nu}p)\ =\ \frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{R}p)\ \pm\ \frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{R}n). \tag{28}$$

They can also be expressed through the cross sections of scattering on nuclei:

$$\Delta^{\ell}_{+\ L,R}= \frac{d^2\sigma^{NC}}{dxdy}(\ell_{L,R}N), \qquad \bar{\Delta}^{\ell}_{+\ L,R}= \frac{d^2\sigma^{NC}}{dxdy}(\bar{\ell}_{R,L}N),$$

$$\Delta_{+}= \frac{d^2\sigma^{CC}}{dxdy}(\ell_{L}N), \qquad \bar{\Delta}_{+}= \frac{d^2\sigma^{CC}}{dxdy}(\bar{\ell}_{R}N)$$

$$\Delta^{\ell}_{-L,R}= \frac{1}{\beta}\left[\frac{1}{A_1}\frac{d^2\sigma^{NC}}{dxdy}(\ell_{L,R}A_1)\ -\ \frac{1}{A_2}\frac{d^2\sigma^{NC}}{dxdy}(\ell_{L,R}A_2)\right], \tag{29}$$

$$\bar{\Delta}^{\ell}_{-L,R}= \frac{1}{\beta}\left[\frac{1}{A_1}\frac{d^2\sigma^{NC}}{dxdy}(\bar{\ell}_{R,L}A_1)\ -\ \frac{1}{A_2}\frac{d^2\sigma^{NC}}{dxdy}(\bar{\ell}_{R,L}A_2)\right],$$

$$\Delta_{-}= \frac{1}{\beta}\left[\frac{1}{A_1}\frac{d^2\sigma^{CC}}{dxdy}(\nu A_1)\ -\ \frac{1}{A_2}\frac{d^2\sigma^{CC}}{dxdy}(\nu A_2)\right]\ =\ \frac{1}{\beta}\left[\frac{1}{A_2}\frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{L}A_2)\ -\ \frac{1}{A_1}\frac{d^2\sigma^{CC}}{dxdy}(e^{-}_{L}A_1)\right],$$

$$\bar{\Delta}_{-}= \frac{1}{\beta}\left[\frac{1}{A_1}\frac{d^2\sigma^{CC}}{dxdy}(\bar{\nu}A_1)\ -\ \frac{1}{A_2}\frac{d^2\sigma^{CC}}{dxdy}(\bar{\nu}A_2)\right]\ =\ \frac{1}{\beta}\left[\frac{1}{A_2}\frac{d^2\sigma^{CC}}{dxdy}(e^{+}_{R}A_2)\ -\ \frac{1}{A_1}\frac{d^2\sigma^{CC}}{dxdy}(e^{+}_{R}A_1)\right],$$

where N is the isoscalar target, A_1,A_2 are the nuclei with different isospin and atomic weights A_1,A_2, $\beta=n_1/A_1-n_2/A_2$, $n_{1,2}$ is the number of neutrons in nucleus $A_{1,2}$.

Using formulae (13)-(15),(19), we find:

$$\overset{(-)}{\Delta}{}^{\ell}_{-L}= \sigma_o f(x,Q^2)x\{|E^{\ell d}_{(L)L,R}|^2-|E^{\ell u}_{(L)L,R}|^2+$$

$$+\ (1-y)^2(|E^{\ell d}_{(L)R,L}|^2-|E^{\ell u}_{(L)R,L}|^2)\}, \tag{30}$$

$$\overset{(-)}{\Delta}{}^{\ell}_{-R}= \sigma_o f(x,Q^2)x\{|E^{\ell d}_{(R)R,L}|^2-|E^{\ell u}_{(R)R,L}|^2+$$

$$+\ (1-y)^2(|E^{\ell d}_{(R)L,R}|^2-|E^{\ell u}_{(R)L,R}|^2)\}, \tag{31}$$

$$\Delta_{-}= \sigma_o xf(x,Q^2), \qquad \bar{\Delta}_{-}= -\sigma_o xf(x,Q^2)(1-y)^2, \tag{32}$$

$$\Delta^{\ell}_{-i}+\ \bar{\Delta}^{\ell}_{-i}=\ -\sigma_o xf(x,Q^2)(1+(1-y)^2)\tfrac{1}{2}(\alpha^{\ell}_{i}\ \gamma^{\ell}_{i}\ +\ \beta^{\ell}_{i}\ \delta^{\ell}_{i}), \tag{33}$$

$$\Delta^{\ell}_{-i}-\ \bar{\Delta}^{\ell}_{-i}= e_i\sigma_o xf(x,Q^2)(1-(1-y)^2)\tfrac{1}{2}(\alpha^{\ell}_{i}\ \delta^{\ell}_{i}\ +\ \gamma^{\ell}_{i}\ \beta^{\ell}_{i}), \tag{34}$$

$$\Delta_{-}\pm\ \bar{\Delta}_{-}= \sigma_o xf(x,Q^2)(1\mp(1-y)^2), \tag{35}$$

$$\Delta^{\ell}_{+i}+\ \bar{\Delta}^{\ell}_{+i}=\ \sigma_o x\mathcal{A}_1(x,Q^2)(1+(1-y)^2)\tfrac{1}{4}((\alpha^{\ell}_{i})^2+(\beta^{\ell}_{i})^2+(\gamma^{\ell}_{i})^2+(\delta^{\ell}_{i})^2), \tag{36}$$

$$\Delta^{\ell}_{+i}-\ \bar{\Delta}^{\ell}_{+i}= -e_i\sigma_o x\mathcal{A}_2(x,Q^2)(1-(1-y)^2)\tfrac{1}{2}(\alpha^{\ell}_{i}\ \beta^{\ell}_{i}\ +\ \gamma^{\ell}_{i}\ \delta^{\ell}_{i}), \tag{37}$$

$$\Delta_{+}\pm\ \bar{\Delta}_{+}= \sigma_o x\mathcal{A}_{1,2}(x,Q^2)(1\pm(1-y)^2), \tag{38}$$

where

$$f(x,Q^2)= f^{p}_{u}(x,Q^2)-\ f^{p}_{d}(x,Q^2), \tag{39}$$

$$\mathcal{A}_{1,2}(x,Q^2)= f^{p+n}_{u}(x,Q^2)+f^{p+n}_{c}(x,Q^2)\pm f^{p+n}_{c}(x,Q^2)\pm f^{p+n}_{u}(x,Q^2), \tag{40}$$

and f^{p}_{q},f^{n}_{q} are the DF of quarks in the proton and neutron respectively, $f_q^{p+n} \equiv f_q^p + f_q^n$, $e_L=-1$, $e_R=1$. The dependence upon the nucleon structure is accumulated in f and Λ_i, which are common factors in formulae (30)-(38).

Now we obtain the similar relations for (quasi-)elastic ℓN-scattering. Let us introduce the differences

$$\mathcal{N}^{\ell}_{L(p,n)} = \frac{d\sigma}{dQ^2}(\ell_L p,n \to \ell_L p,n) - \frac{d\sigma}{dQ^2}(\bar{\ell}_R p,n \to \bar{\ell}_R p,n) \qquad (41)$$

$$\mathcal{N}^{e}_{R(p,n)} = \frac{d\sigma}{dQ^2}(e^+_L p,n \to e^+_L p,n) - \frac{d\sigma}{dQ^2}(e^-_R p,n \to e^-_R p,n) \qquad (42)$$

$$\mathcal{E} = \frac{d\sigma}{dQ^2}(\nu n \to e^-_L p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p \to e^+_R n) = \qquad (43)$$

$$= \frac{d\sigma}{dQ^2}(e^-_L p \to \nu n) - \frac{d\sigma}{dQ^2}(e^+_R n \to \bar{\nu} p). \qquad (44)$$

We also consider (quasi-)elastic scattering of leptons on the nuclear target of deuteron. For this purpose we shall use the following relations between the cross sections:

$$\mathcal{D}^{\ell}_L \equiv \mathcal{N}^{\ell}_{L(p)} + \mathcal{N}^{\ell}_{L(n)} = \frac{d\sigma}{dQ^2}(\ell_L d \to \ell_L np) - \frac{d\sigma}{dQ^2}(\bar{\ell}_R d \to \bar{\ell}_R np), \qquad (45)$$

$$\mathcal{D}^{\ell}_R \equiv \mathcal{N}^{\ell}_{R(p)} + \mathcal{N}^{\ell}_{R(n)} = \frac{d\sigma}{dQ^2}(e^+_L d \to e^+_L np) - \frac{d\sigma}{dQ^2}(e^-_R d \to e^-_R np), \qquad (46)$$

$$\mathcal{E} = \frac{d\sigma}{dQ^2}(\nu d \to e^-_L pp) - \frac{d\sigma}{dQ^2}(\bar{\nu} d \to e^+_R nn) = \frac{d\sigma}{dQ^2}(e^-_L d \to \nu nn) - \frac{d\sigma}{dQ^2}(e^+_R d \to \bar{\nu} pp). \qquad (47)$$

Following formulae (16)-(18),(20), we obtain:

$$\mathcal{N}^{\ell}_{i(p,n)} = \omega(E,Q^2)\ (F^{\ell}_{Vi(p,n)} + F^{\ell}_{Mi(p,n)}) F^{\ell}_{Ai(p,n)}, \qquad (48)$$

$$\mathcal{E} = \cos^2\theta_c\ \omega(E,Q^2)\ (F^{CC}_V + F^{CC}_M) F^{CC}_A, \qquad (49)$$

$$\omega(E,Q^2) = \frac{G^2}{\pi} \frac{Q^2}{ME} \left(1 - \frac{Q^2}{4ME}\right).$$

Lets us use the scaling law for the FFN:

$$G^p_M/\mu_p \approx G^n_M/\mu_n, \qquad (50)$$

$$F^0_A \approx \frac{\lambda}{2} F^{CC}_A, \qquad (51)$$

where $\mu_p = 2.79$, $\mu_n = -1.91$ are the magnetic moments of the proton and neutron.

Relation (50) is well-known and valid with a high accuracy in a wide interval of Q^2. The scale law for the axial FFN (51) is less reliable. It may have some grounds, for instance, in QCD, based on local duality [19] or dipole extrapolation of the results of perturbative calculations [20]. However, the experimental status of this relation is not quite clear. The normalization constant λ is calculated or taken from experiment. In the non-relativistic quark SU_6-model $\lambda=0.6$. Some difference in values of λ in different approaches as well as deviation from scale law (51) do not lead to noticeable influence on the effects under discussion. This is explained by the small contribution of δF^0_A to initial formulae (23) due to the small value of parameter δ. One should remember that in the SM at tree level $\delta=0$.

On the basis of formulae (22),(23),(50),(51) we transform (48) into:

$$\mathcal{N}^{\ell}_{i(p,n)} = \frac{1}{4}\, \omega(E,Q^2)\ (F^{CC}_V + F^{CC}_M) F^{CC}_A (3\gamma^{\ell}_i\, \mu \pm \alpha^{\ell}_i)(\lambda\delta^{\ell}_i \pm \beta^{\ell}_i), \qquad (52)$$

$$\mathcal{D}^{\ell}_i = \frac{1}{2}\, \omega(E,Q^2)\ (F^{CC}_V + F^{CC}_M) F^{CC}_A (\alpha^{\ell}_i \beta^{\ell}_i + 3\gamma^{\ell}_i \delta^{\ell}_i \lambda\mu), \qquad (53)$$

where $\mu = (\mu_p+\mu_n)/(\mu_p-\mu_n)$.

Formulae (30)-(38),(49),(52),(53) are initial ones for obtaining the relations independent of DF and FFN. From the cross sections combinations $\Delta, \mathcal{N}, \mathcal{E}$ one should choose two combinations so that DF and FFN would be cancelled if the combinations are divided by one another. For example, it is seen from (30) and (32) that in the ratio $\Delta^{\ell}_{-L}/\Delta_-$ the DF $f(x,Q^2)$ is cancelled. Thus, one can easily obtain all the relations of this kind from formulae (30)-(38),(49),(52),(53). To save place, we do not write them down in the general form, confining ourselves to the approximation $Q^2 \ll M^2_Z \approx 10^4\ \mathrm{GeV}^2$ and $M_{Z'} \gg M_Z$ ($M_2 \gg M_1$). The latter conditions corresponds to the SM limit. If the given approximation is inapplicable, e. g. in HERA experiments where $Q^2 \approx 10^{3-5}\,\mathrm{GeV}^2$, one should use the formulae in the general form.

Since electron scattering is due to both weak interaction W and electromagnetic interaction EM, the structure of the relevant formulae is $EM^2 + EM \cdot W + W^2$. In the Q^2 region considered the leading term is the one with the maximum power of EM. As to the neutral current parameters and the dependence upon the Z′-boson contribution, they all are included in W. So, below we give the relations that do not involve the dominating term EM^2 which is of little physical importance. In the formulae we shall only retain the leading term of the EM·W type corresponding to electroweak interference. In this case the accuracy is worse by no more than 1-2%, for $Q^2 < 200\ \mathrm{GeV}^2$. In the given approximation the desired relations have the following form:

-for νN-scattering

$$\frac{\overset{(-)}{\Delta}{}^{\nu}_{-L}}{\Delta_-} = -(1-y)^2\,\frac{\overset{(-)}{\Delta}{}^{\nu}_{-L}}{\bar{\Delta}_-} = \frac{1}{\sigma}\,\rho^2 X_W\{[1+(1-y)^2](1-2X_W) \pm [1-(1-y)^2]\}, \tag{54}$$

$$\frac{\bar{\Delta}^{\nu}_{-L}}{\Delta^{\nu}_{-L}} = \frac{(1-X_W)(1-y)^2 - X_W}{(1-X_W) - X_W(1-y)^2}, \tag{55}$$

$$\frac{\Delta^{\nu}_{\pm L} + \bar{\Delta}^{\nu}_{\pm L}}{\Delta_{\pm} \pm \bar{\Delta}_{\pm}} = \left\{ \begin{array}{l} \rho^2(1-2X_W + \frac{20}{9}X_W^2)/2 \\ \rho^2 X_W(1-2X_W)/3 \end{array} \right., \tag{56}$$

$$\frac{\Delta^{\nu}_{\pm L} - \bar{\Delta}^{\nu}_{\pm L}}{\Delta_{\pm} \mp \bar{\Delta}_{\pm}} = \left\{ \begin{array}{l} \rho^2(1-2X_W)/2 \\ \rho^2 X_W/3 \end{array} \right., \tag{57}$$

$$\frac{\mathscr{N}^{\nu}_{L(p,n)}}{\mathcal{E}} = \frac{\rho}{4\cos^2\theta_C}\{1-2X_W(1\pm\mu)\}, \quad \frac{\mathscr{N}^{\nu}_{L(p)} + \mathscr{N}^{\nu}_{L(n)}}{\mathcal{E}} = \frac{\rho}{\cos^2\theta_C}\left(\frac{1}{2} - X_W\right), \tag{58}$$

-and for eN-scattering

$$\frac{\Delta^{e}_{\pm i} - \bar{\Delta}^{e}_{\pm i}}{\Delta_{\pm} \mp \bar{\Delta}_{\pm}} = \frac{e_i}{1\mp 2}\,\chi\,\frac{m_P^2}{Q^2}\,\rho\,\varepsilon^Z_i(e), \tag{59}$$

$$\frac{\mathscr{N}^{e}_{i(p,n)}}{\mathcal{E}} = -\frac{\chi(1\pm\mu)}{2\cos^2\theta_C}\,\frac{m_P^2}{Q^2}\,\rho\,\varepsilon^Z_i(e), \tag{60}$$

$$\frac{d^{e}_{i}}{\mathcal{E}} = -\frac{\chi}{\cos^2\theta_C}\,\frac{m_P^2}{Q^2}\,\rho\,\varepsilon^Z_i(e), \tag{61}$$

$$\mathcal{R}^{e}_{NC} = \frac{\mathscr{N}^{e}_{L(p,n)}}{\mathscr{N}^{e}_{R(p,n)}} = \frac{d^{e}_{L}}{d^{e}_{R}} = -\frac{\Delta^{e}_{\pm L} - \bar{\Delta}^{e}_{\pm L}}{\Delta^{e}_{\pm R} - \bar{\Delta}^{e}_{\pm R}} = \frac{\varepsilon^Z_L(e)}{\varepsilon^Z_R(e)} = \frac{2X_W - 1}{2X_W}, \tag{62}$$

(i =L,R). The double differential cross sections $\frac{d^2\sigma}{dxdy}$, entering into Δ (25)-(38), can be replaced in formulae (59),(60) by the differential cross sections $\frac{d\sigma}{dQ^2}$, and in formulae (55)-(57), (62) they can be replaced both by $\frac{d\sigma}{dy}$, $\frac{d\sigma}{dQ^2}$, and by the total cross sections $\Delta\sigma$ taken in any region of variables x,y. According to the definition

$$\Delta\sigma^a = \int_{X_{min}}^{X_{max}} dx \int_{y_{min}}^{y_{max}} dy\, \frac{d^2\sigma^a}{dxdy}, \qquad a = NC, CC\,. \tag{63}$$

Relations (54)-(62) can be used for extractions of the SM parameters ρ and X_W ($X_W = \sin^2\theta_W$) from the experimental data on deep inelastic and (quasi-)elastic $\overset{(-)}{\nu}$N- and eN-scattering.

Specific difficulties of νN- experiments associated with relative normalization of ν and $\bar{\nu}$ beam flows make it more preferable to use relations without combinations of cross sections from different beams, i.e. combinations of νN- and $\bar{\nu}$N- scattering cross sections. These are relations (54) $\Delta^{\nu}_{-L}/\Delta_-$ и $\bar{\Delta}^{\nu}_{-L}/\bar{\Delta}_-$. Their disadvantage is that they contain differential cross sections $\frac{d^2\sigma}{dxdy}$, and consequently, require data on $\overset{(-)}{\nu}$N-scattering in narrow-band beam (NBB). This limits the statistics for extraction of the parameters (ρ, X_w). The situation is quite opposite for relations (56)-(58), among which there is the Paschos-Wolfenstein relation (formula (57)). They are formulated for total cross sections, which is much more favourable for gathering statistics, but these are combinations of νN- and $\bar{\nu}N$-scattering cross sections. As a result, there are uncertainties related to different normalization of ν and $\bar{\nu}$ beams.

Among relations (59)-(62) for eN-scattering we'd like to single out three last relations for $\mathcal{R}^e_{NC}$ (62). Their definition does not include the cross sections for charged current eN-scattering, which is a rare process occuring only due to weak interaction. These relations do not involve the parameter ρ. As a result the extraction of the remaining parameter X_w from the data becomes more reliable. We also notice, that $\mathcal{R}^e_{NC}$ is very sensitive to X_w:

$$K = \frac{d}{dX_w} \log \mathcal{R}^e_{NC} = \frac{1}{X_w(1-2X_w)} \approx 8 \text{ for } X_w=0.23 . \tag{64}$$

So, in the error ΔX_w induced by the error $\Delta\mathcal{R}^e_{NC}$ in the measurement of $\mathcal{R}^e_{NC}$ is suppressed by a large factor k=8:

$$\Delta X_w = \frac{1}{8} \frac{\Delta\mathcal{R}^e_{NC}}{\mathcal{R}^e_{NC}} . \tag{65}$$

The sensitivity of other relations, including the Paschos-Wolfenstein relation, is much lower and does not exceed K=1.

A question of electroweak corrections to the relations obtained has been left unconsidered in the paper. It requires special study similar to that in Ref [21] applied to the Paschos-Wolfenstein relation, where the authors showed that the contribution of the corrections is small and can be neglected in a good approximation. Similar properties could be typical of the electroweak corrections to all relations (54)-(62), whose particular case is the Paschos- Wolfenstein relation. There are some reasons for that based on the similarity in the structure of all relations obtained. But this is still a hypothesis.

Table. Chiral constants $\varepsilon^{Z'}_{L,R}$, parameterizing fermion- Z' boson interactions (Here $\zeta = \frac{1}{2}\sin\theta_{E6}/\sqrt{10}$, $\lambda = \frac{1}{2}\cos\theta_{E6}/\sqrt{6}$).

fermion	$\varepsilon^{Z'}_L$	$\varepsilon^{Z'}_R$
ν	$\lambda + 3\zeta$	$5\zeta - \lambda$
e	$\lambda + 3\zeta$	$\zeta - \lambda$
u	$\lambda - \zeta$	$\zeta - \lambda$
d	$\lambda - \zeta$	$-\lambda - 3\zeta$

The authors are thankful to D.Yu.Bardin, C. Burdik, Yu.P.Ivanov and B.Z.Kopeliovich for the useful discussions.

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Received by Publishing Department
on July 6, 1989.