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A NEW QCD INSPIRED VERSION OF THE NAMBU-JONA-LASINIO MODEL

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В работе показано, что сепарабельное приближение релятивистской потенциальной модели, инспирированной КХД, ведет к новой версии модели Намбу-Йона-Лазинио с вполне определенной регуляризацией расходящихся интегралов. В рамках полученной модели описываются низколежащие резонансы в подходе Бете-Солпитера для трех ароматов кварков.

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A New QCD Inspired Version
of the Numbu-Jona-Lasinio Model

We use the relativistic generalization of the QCD-inspired nonrelativistic potential model to get, as separable approximation for low-lying resonances, a modified Nambu-Jona-Lasinio model in a regularized form. In the framework of the latter we describe low-energy resonance physics within the Bethe-Salpeter approach for the three-flavour case.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1.Introduction

During the last time one observes a revival of the interest in the Nambu - Jona - Lasinio (NJL) model [1]. The most important feature of this model is the spontaneous breakdown of chiral symmetry. Generalizations of the original NJL model allow one to reproduce the low-energy chiral phenomenology with a minimal number of parameters [2] - [7] .

On the other hand, NJL models have of course some significant shortcomings. They are not renormalizable and do not contain quark confinement. Furthermore, their relation to the gluon exchange in QCD is not clear so far, at least in the continuum field theory. And practically, NJL models do not describe heavy quarkonia and Regge phenomenology.

In this paper we give the foundation of a regularized new version of the NJL model obtained by means of a separable approximation [8] to a relativistic potential model [9] for small orbital momenta, $l = 0, 1$. Thereby, the latter is a generalization of the nonrelativistic QCD potential model with rising potential [10]. The reasons for the occurrence of the divergences in the NJL model as well as the impossibility of the description of heavy quarkonia become clear.

The modifications of the original NJL model concern the Lorentz structure of the four-quark interaction and the introduction of the model parameters (one for each flavour). We use it to describe quark-antiquark bound states within the Bethe - Salpeter approach. Thereby, we extend results of [6] as well as our earlier work [11] dealing with the two - flavour case to the three - flavour one.

2. Formulation of the model

In [9] a relativistic bilocal potential model has been put forward intended to describe hadrons within QCD. Thereby, two assumptions were used made indirectly in QED by considering atoms. These are, first, the invariant decomposition of the gauge fields into Coulomb and radiative parts and, secondly, the identification of the time axis of the gauge field quantization with the relative time axis of the bound states. (The latter means, that the Coulomb field moves together with the atom). According to Markov and Yukawa [12] the unit vector η_μ of the relative time is choosen as operator parallel to the operator of the total momentum $\hat{\mathcal{P}}_\mu$ of the bound state,

$$\mathcal{M}(x,y) \equiv \mathcal{M}(X = \frac{x+y}{2}|z = x - y)\,,$$

where

$$\hat{\mathcal{P}}_\mu \mathcal{M}(X|z) = -\frac{1}{i}\frac{\partial}{\partial X_\mu}\mathcal{M}(X|z)\,.$$

The relativistic invariant bilocal effective action obtained in [9], takes for the quark sector in the colour singlet channel the form

$$S_{eff} = \int d^4x \; \{ \; \bar{q}(x)\,(i\partial\!\!\!/ - \hat{m}^0)\, q(x) - \frac{1}{2N_c}\int d^4y \;\; q(y)\bar{q}(x) \;\; \mathcal{K}^\eta(x-y) \;\; q(x)\bar{q}(y) \;\; \} \;. \quad (1)$$

For simplicity we omitted in (1) all spinor, colour and flavour indices. N_c means the colour number. $\hat{m}^0 = diag \; (\; m_u^0 \; , \; m_d^0 \; , \; m_s^0 \;)$ denotes the bare quark mass matrix. The kernel $\mathcal{K}^\eta$ has the form

$$\mathcal{K}^\eta(z) = \eta\!\!\!/ \;\; V(z^\perp) \;\; \delta(z\cdot\eta)\, \eta\!\!\!/ \;.$$

Here η_μ is the time axis of quantization [9], [13] with $\eta\!\!\!/ = \eta_\mu\gamma^\mu$, $\eta^2 = 1$ and $z_\mu^\perp =$ $= z_\mu - z_\mu^{||}, z_\mu^{||} = \eta_\mu(z\cdot\eta)$. $V(z^\perp)$ denotes the sum of Coulomb and oscillator potentials :

$$V(r) = \frac{4}{3}(-\frac{\alpha_s}{r} + V_0 r^2), \quad r = |z^\perp|\,.$$

In short - hand notation action (1) can be written as [14]

$$S_{eff} = (q\bar{q}, -G^{-1}) - \frac{1}{2N_c}(q\bar{q}, \mathcal{K}^\eta q\bar{q})\,.$$

After quantization over the quark fields it takes the form

$$S_{eff}[\mathcal{M}] = N_c \;\; \{\frac{1}{2}(\mathcal{M}, \; (\mathcal{K}^\eta)^{-1}\mathcal{M}) - i\, Tr\; ln\; (-G_{\hat{m}^0}^{-1} + \mathcal{M})\} \qquad (2)$$

with $\mathcal{M} = \mathcal{M}(x,y)$ being the bilocal field; Tr means both the integration over continuous variables and trace over discrete indices.

The extremum condition for action (2) coincides with the Schwinger - Dyson equation for the quark mass operator Σ :

$$\Sigma(z) = \hat{m}^0\, \delta^4(z) + i\, \mathcal{K}^\eta(z)\cdot G_\Sigma(z)\,, \qquad (3)$$

where

$$G_\Sigma^{-1}(z) = i\, \partial\!\!\!/\delta^4(z) - \Sigma(z)\,.$$

This equation defines the quark mass spectrum.

The Bethe - Salpeter equation in the ladder approximation for the modified bound state field $\mathcal{M}'(X|z) = \mathcal{M}(X|z) - \Sigma(z)$ is given by

$$\mathcal{M}'(X|z) = -i\mathcal{K}^\eta \int dz_1 dz_2 \;\; G_{\Sigma_1}(z_1 - z_2)\; \mathcal{M}'(\frac{z_1+z_2}{2}|z_1 - z_2)\; G_{\Sigma_2}(z_2 - z_1)\,. \qquad (4)$$

It follows from variation of the free part of effective action (2) over fluctuations $\mathcal{M}'$. Equation (4) defines the bound state mass spectrum.

Let us now introduce the bound state vertex functions Γ^H via a plane wave expansion of $\mathcal{M}'$:

$$\mathcal{M}'(X|z) = \sum_H \int \frac{d\mathbf{P}}{(2\pi)^3}\frac{1}{\sqrt{2\omega_H}}\,[e^{i\mathcal{P}_H X} a_H^+(\mathcal{P})\Gamma^H(\mathcal{P}) + e^{-i\mathcal{P}_H X} a_H^-(\mathcal{P})\bar{\Gamma}^H(\mathcal{P})]\,,$$

where $\omega_H = \sqrt{M_H^2 + \mathbf{P}_H^2}$ is the bound state energy and $\mathcal{P}_H = (\omega_H, \; \mathbf{P}_H)$ the total momentum. a_H^+ and a_H^- denote the corresponding creation and annihilation operators. Then, from (4) we get the Bethe - Salpeter equation for Γ

$$\Gamma(\mathbf{p}^\perp) = -\int \frac{d\mathbf{q}^\perp}{(2\pi)^3} \;\; \underline{\mathrm{V}}(\mathbf{p}^\perp - \mathbf{q}^\perp) \;\; \mathcal{D}_0(\mathbf{q}^\perp) \otimes \Gamma(\mathbf{q}^\perp) \qquad (5)$$

with

$$\mathcal{D}_0(\mathbf{q}^\perp) \otimes \Gamma(\mathbf{q}^\perp) = \int \frac{dq_0}{2\pi} \;\; \eta\!\!\!/ \;\; G_{\Sigma_1}(\mathbf{q}^\perp) \;\; \Gamma(\mathbf{q}^\perp) \;\; G_{\Sigma_2}(\mathbf{q}^\perp) \;\; \eta\!\!\!/ \;\; \stackrel{\text{def}}{=} \;\; \psi(\mathbf{q}^\perp)\,. \qquad (6)$$

The Bethe - Salpeter equation for the wave function ψ reads

$$\psi(\mathbf{q}^\perp) = -\mathcal{D}_0(\mathbf{p}^\perp)\int \frac{d\mathbf{q}^\perp}{(2\pi)^3} \;\; \underline{\mathrm{V}}(\mathbf{p}^\perp - \mathbf{q}^\perp) \;\; \psi(\mathbf{q}^\perp)\,. \qquad (7)$$

Now, in the case of a short - range potential ($m << \mu$) we make use of the separable approximation [8] ,

$$\underline{\mathrm{V}}(\mathbf{p}^\perp - \mathbf{q}^\perp) = f(\mathbf{p}^\perp) f(\mathbf{q}^\perp)\,. \qquad (8)$$

Then, after integrating (7) over $\mathbf{q}^\perp$ with the weight $f(\mathbf{q}^\perp)/(2\pi)^3$ we get

$$\begin{aligned}\Gamma' \;\; &\stackrel{\text{def}}{=} \;\; \int \frac{d\mathbf{q}^\perp}{(2\pi)^3} f(\mathbf{q})\psi(\mathbf{q}) = \\ &= \;\; -\int \frac{d\mathbf{p}^\perp}{(2\pi)^3} f^2(\mathbf{p})\mathcal{D}_0(\mathbf{p}^\perp)\otimes\Gamma' \simeq -\frac{1}{\mu^2}\int_0^L \frac{d\mathbf{p}^\perp}{(2\pi)^3}\mathcal{D}_0(\mathbf{p}^\perp)\cdot\Gamma'\,. \qquad (9)\end{aligned}$$

The last relation of (9) is obtained after substituting the formfactor $f^2(\mathbf{p})$ by a Θ - function (corresponding parameters μ^2 and L can ever be found). Analogiously the Schwinger - Dyson equation (3) has in momentum space the form

$$m_i = m_{0i} + \int \frac{d\mathbf{q}^\perp}{(2\pi)^3} f^2(\mathbf{q})\frac{m_i}{2E(q)} \simeq m_{0i} + \frac{N_c}{\mu_i^2}\int_0^L \frac{d\mathbf{q}^\perp}{(2\pi)^3}\frac{m_i}{2E(q)}\,, \;\; (i = u, d, s)\,. \qquad (10)$$

Integration in (10) leads to

$$8\pi^2\mu_i^2 = N_c\frac{m_i}{m_i - m_i^0}[L^2 - m_i^2 ln\frac{2L}{m_i}] \ . \tag{11}$$

Relations (9) and (10) show that in the separable approximation (8) the underlying potential model is equivalent to a regularized NJL model with the kernel

$$\mathcal{K}^{\eta}_{NJL}(z) = \not\eta\ \delta^4(z)\ \not\eta\ .$$

Then, action (1) becomes the effective action of a generalized NJL model :

$$S^{NJL}_{eff} = \int d^4x\{\ \bar{q}(x)\,(i\not\partial - \hat{m}^0)\,q(x) - \frac{1}{2N_c}\,[\,\bar{q}(x)\not\eta\,\hat{\mu}^{-1}\,q(x)\,]^2\ \}\ . \tag{12}$$

Here the parameter matrix $\hat{\mu}^{-1} =\ diag\ (\ \mu_u^{-1}\ ,\ \mu_d^{-1}\ ,\ \mu_s^{-1}\ )$ has been introduced for phenomenological reasons. The difference to the original NJL model lies in the choise of the four - quark interaction term, which in our case depends on the time axis of quantization and breaks flavour symmetry.

3. Meson properties

From our model with effective action (12) we calculate the low - lying meson masses M_H within the Bethe - Salpeter approach [15] . Thereby, it is sufficient to consider the underlying equation (9) in the rest frame ($\not\eta = \gamma_0$ and $\mathcal{P}_{H\mu} = (M_H,\ 0\ ,\ 0\ ,\ 0\ )$)

$$\Gamma^H = -i\frac{N_c}{\mu^2}\int\frac{d^4p}{(2\pi)^4}\,\gamma_0\,G_a(p + \frac{M_H}{2})\,\Gamma^H\,G_b(p - \frac{M_H}{2})\,\gamma_0\,. \tag{13}$$

Now, we decomppse Γ^H as

$$\begin{cases} \Gamma^H = \Gamma^H_1 + \gamma_0\cdot\Gamma^H_2 \\ \Gamma^H_l = \gamma^S L^S_l + \gamma^P L^P_l + \gamma^V_i L^V_{li} + \gamma^A_i L^A_{li} \end{cases} \tag{14}$$

where $\gamma^S = 1\ ,\ \gamma^P = \gamma_5\ ,\ \gamma^V_i = \gamma_i\ ,\ \gamma^A_i = \gamma_i\gamma_5$ and $l = 1,\ 2;\ \ i = 1,\ 2,\ 3$. Then, the Bethe - Salpeter equation (13) decouples into the following sets of algebraic equations

$$\begin{aligned} -\frac{\mu_a\mu_b}{N_c}L^I_1 &= C^I L^I_1 + B^I L^I_2 \\ -\frac{\mu_a\mu_b}{N_c}L^I_2 &= D^I L^I_2 + B^I L^I_1\ . \end{aligned} \tag{15}$$

The coefficients B^I, C^I, and D^I, which are in general complex quantities, are given by

$$\begin{cases} B^S = B^A = M_H A_-(M_H)\ , \\ B^P = B^V = M_H A_+(M_H)\ ; \end{cases}$$

$$\begin{cases} C^S = C^A + \frac{1}{3}\beta = D^S + \beta\ , \\ C^P = C^V + \frac{1}{3}\beta = D^P + \beta\ ; \end{cases}$$

$$\begin{cases} D^S = D^A - \frac{1}{3}\beta = (m_1 - m_2)A_-(M_H)\ , \\ D^P = D^V - \frac{1}{3}\beta = (m_1 + m_2)A_+(M_H)\ ; \end{cases}$$

$$\begin{aligned} A_\pm(M_H) &= \frac{1}{2}\ [\ m_1\alpha_1(M_H) \pm m_2\alpha_2(M_H)\ ]\ ; \\ \beta(M_H) &= \frac{1}{2}\ [\ \beta_1(M_H) + \beta_2(M_H)\ ]\ ; \end{aligned}$$

$$\begin{pmatrix} \alpha_1(M_H) \\ \beta_1(M_H) \end{pmatrix} = \int_0^L \frac{d\mathbf{k}}{(2\pi)^3}\cdot\frac{1}{E_1}\cdot\frac{1}{(E_1 + E_2)^2 - M_H^2 + i\epsilon}\cdot\begin{pmatrix} 1 \\ \mathbf{k}^2 \end{pmatrix}$$

(α_2, β_2 are obtained from α_1, β_1 by interchanging indices 1 and 2).

System (15) has an unambiguous solution if

$$\begin{aligned} &(ReC^I - \frac{\mu_a\mu_b}{N_c})(ReD^I - \frac{\mu_a\mu_b}{N_c}) - (ReB^I)^2 = 0 \\ &(ImC^I)\cdot(ImD^I) - (ImB^I)^2 = 0\ . \end{aligned} \tag{16}$$

We will use just these equations to determine the mass spectrum.

Let us now briefly explain how to calculate the pion and kaon decay constants. According to (14) the Bethe - Salpeter vertex function for the pseudoscalar sector can be represented as $\Gamma^P = \gamma_5 L^P_1 + \gamma_0\gamma_5 L^P_2$. F_π and F_K are defined by the pseudoscalar part of the axial - vector coupling in the second term of action (1) which reads

$$S^{(2)\,P}_{free}(M_P) = \frac{4N_c}{2}tr_{fl}[\ C^P(M_P)(L^P_1)^2 + D^P(M_P)(L^P_2)^2 + 2B^P(M_P)(L^P_1)(L^P_2)\ ]\ . \tag{17}$$

From the last term on the r.h.s. of (17) we obtain the decay constants in the form

$$F_{P_i} = 2\ N_c\ A_+(M_{P_i})\ L_1^{P_i}\ ,\ \ (i = u, s) \tag{18}$$

with $F_{P_u} \equiv F_\pi, F_{P_s} \equiv F_K$. Here the value of $L_1^{P_i}$ follows from the normalization condition for the Bethe - Salpeter vertex function :

$$\sum_H\int\frac{d^4\mathcal{P}}{(2\pi)^4}(\Gamma^H(\mathcal{P}))^+\Gamma^H(\mathcal{P})(\mathcal{P}_0 - \omega_H)M_H = 1\ . \tag{19}$$

As has been shown in [11] from (19) one gets

$$\frac{1}{\mathcal{P}_P}\frac{\partial}{\partial\mathcal{P}_P}S^{(2)\,P}_{free}(\mathcal{P}_P)|_{\mathcal{P}_P = M_P} = 1\ \ ,\ \ \mathcal{P}_P = \sqrt{\mathcal{P}_P^2}\ .$$

Then, with the help of (12) we obtain

$$L_1^{P\,i} = [\ \frac{4N_c}{\mathcal{P}_P}\frac{\partial}{\partial\mathcal{P}_P}C^P(\mathcal{P}_P)|_{\mathcal{P}_P = M_P}\ ]^{-\frac{1}{2}}\ .$$

4. Numerical results

Here we present the results for the isospin symmetric case $m_{0u} = m_{0d}$, $m_u = m_d$. Our theory contains five parameters : the bare quark masses $m_{0u} = m_{0d}$, m_{0s}, the parameters $\mu_u^2 = \mu_d^2$, μ_s^2, and the cut - off L . μ_i^2 and L are connected by relation (11).

To fix the parameters we take, as input data, $m_u = m_d = 330 MeV$, $m_s = 400 MeV$, $M_\pi = 140 MeV$, $M_K = 494 MeV$ and $M_\rho = 770 MeV$. Then one obtains for $N_c = 3$, $L = 1590 MeV$, $\mu_u^2 = \mu_d^2 = 8.7 \cdot 10^4 (MeV)^2$, $\mu_s^2 = 9.7 \cdot 10^4 (MeV)^2$ with the help of (10): $m_{0u} = m_{0d} = 2.1 MeV$, $m_{0s} = 56 MeV$, $M_\sigma = 660 MeV$, $M_{a_1} = (1132 - i654) MeV$, $M_{K^*} = 896 MeV$, $M_\Phi = 970 MeV$. From (18) one gets for the decay constants $F_\pi = 108 MeV$ and $F_K = 149 MeV$, so that $F_K/F_\pi = 1.38$. For the quark condensates we find $<\bar{q}_u q_u> = (-234 MeV)^3$, $<\bar{q}_s q_s> = (-246 MeV)^3$.

5.Conclusion

A new QCD- inspired version of the NJL - model with effective action (12) has been obtained (as a separable approximation for the low - lying resonances) from a relativistic potential model. Thereby it became clear how to continiously turn from the spectroscopy of light quark to that of heavy quarks. But for the latter, the QCD potential is a long - - range one and the simplest separable approximation is not applicable, so that the NJL model does not work in this case.

From our model we calculated for the three - flavour case the low - lying meson spectra within the Bethe - Salpeter approach. In distinction to the usual procedure [2] - [4], [7], we did not expend in energy. Therefore, in our treatment no tachyons appeared, whereas their occurrence was claimed in [16] for the QCD low - energy expansion. Furthermore, we took into account the $P - A$, $V - T$, and $S - V$ mixings from the very beginning by using decomposition (9) of the Bethe - Salpeter vertex function Γ. With the help of the normalization condition (19) for Γ we were able to determine the pion and kaon decay constants. The quark condensates have been analyzed, too. The numerical results reported in sect. 4 are in reasonable agreement with experiment.

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