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THE INFLUENCE OF QUARK MASSES ON THE INFRARED BEHAVIOR OF $\bar{\alpha}_s(Q^2)$ IN QCD

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1. INTRODUCTION

Asymptotic freedom, i.e. decreasing of the running coupling constant at small distances, is one of the most important properties of quantum chromodynamics. Usually this phenomenon is connected with the properties of the renormalization group β-function that is renormalization scheme (RS) dependent. At the present time we know the β - function up to three loops for minimal subtraction (MS) schemes /1/ and for some specific massless momentum subtraction (MOM) schemes /2/.

In practical applications MOM schemes turn out to be more preferable than MS ones. But as is well known /3/ at the two-loop level the β- function for massless MOM schemes is gauge dependent. It has been recently observed /2,4/ that gauge dependence is essential and can lead to the violation of asymptotic freedom. In MOM schemes with nonzero quark masses gauge dependence of the β- function calculated by using the quark-quark-gluon vertex to define the coupling constant can appear even at the one-loop level. So it is very interesting to investigate the influence of this gauge and the mass dependence on the behavior of the running coupling constant. But up to now the β- function in a MOM scheme with massive fermion was calculated on ghost-ghost-gluon /5/ and 3-gluon vertices /6/. These β- functions at the one- loop level are gauge independent. In all above mentioned renormalization schemes $\bar{\alpha}$ has a singularity at low energies in any known order of the perturbation theory.

In this article we give the description of one-loop calculations of the gauge dependent β- function with masses defined by the quark - quark - gluon vertex. We considered the MOM scheme where radiative corrections to this vertex are absent at a symmetric momentum point. When masses of the fermion particles are equal to zero our results coincide with the results given in /7/. For infinite fermion masses according to /8/ only massless particles are contributing to β. We analysed the behaviour of the running coupling constant in this scheme for the model with five massive quarks in the framework of the "stopping" gauge formalism /9/ (see also /10/). It is found that due to the mass dependence this coupling constant for some values of the gauge parameter have no pole singularity in the whole range of momentum.

The outline of this paper is as follows. In Sec.2 we describe our renormalization prescription. In Sec.3 we present results of the one-loop calculations of the quark-quark-gluon vertex. In

Sec.4. the analysis of the behavior of the running coupling constant is given. In Appendix A all one-loop integrals needed for calculation of diagrams are given. In Appendix B for the comparison we write β functions determined by ghost-gluon and three-gluon vertices. In Appendix C asymptotics of some one-dimensional integrals entering into the β-function are given.

2. THE MODEL AND RENORMALIZATION PRESCRIPTION

We shall consider the Yang - Mills theory with massive fermions belonging to the representation R of the gauge group G:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu a} F^{\mu\nu a} - \frac{1}{2\alpha} (\partial^{\mu} A_{\mu a})^2 - \partial^{\mu} \bar{\eta}_a \partial_\mu \eta_a + \\ - g f^{abc} \partial_\mu \bar{\eta}_a A_b^{\mu} \eta_c + \sum_{j=1}^{N_f} \bar{\psi}_i^j (i\gamma^{\mu} \mathcal{D}_\mu - m_j) \psi_i^j , \qquad (1)$$

$$F_{\mu\nu a} = \partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} + g f_{abc} A_{\mu b} A_{\nu c},$$

$$\mathcal{D}_\mu \psi_i^j = \partial_\mu \psi_i^j - i g R^a_{ik} \psi_k^j A_{\mu a} ,$$

where $A^{\mu a}, \eta_a$ and ψ_i^j are gauge, ghost and fermion fields respectively, α is the gauge parameter, and f^{abc} are the totally antisymmetric structure constants of the gauge group G. The indices of the fermion field ψ_i^j specify color (i) and flavor (j), respectively. The matrices R^a obey the following relations:

$$[R^a, R^b] = i f^{abc} R^c, \qquad f^{acd} f^{bcd} = C_A \delta^{ab}$$

$$R^a R^a = C_F I, \qquad tr(R_a, R_b) = T\, \delta_{ab}.$$

In particular, the group invariants C_A, C_F and T in the fundamental (quark) representation of SU(N) are:

$$C_A = N, \quad C_F = \frac{N^2-1}{2N} , \quad T = \frac{1}{2} .$$

The lagrangian (1) is renormalized by subtracting from each of its terms a counterterm of the same type. So, the renormalized lagrangian can be written as:

$$\mathcal{L}_R = -\frac{1}{4} Z_3 (\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a})(\partial^{\mu} A^{\nu}_a - \partial^{\nu} A^{\mu}_a) \\ - \frac{1}{2} Z_1 g f^{abc} (\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a}) A^{\mu}_b A^{\nu}_c - \frac{1}{4} Z_5 g^2 f^{abc} f^{ade} A_{\mu b} A_{\nu c} A^{\mu}_d A^{\nu}_e \\ - \tilde{Z}_3 \partial_\mu \bar{\eta}_a \partial^{\mu} \eta_a - \tilde{Z}_1 g f^{abc} \partial_\mu \bar{\eta}_a A_b^{\mu} \eta_c - \frac{1}{2\alpha} Z_6 (\partial_\mu A_a^{\mu})^2 \qquad (2) \\ + \sum_{j=1}^{N_f} \Big[i Z_{2Fj} \bar{\psi}_i^j \gamma^{\mu} \partial_\mu \psi_i^j + Z_{1Fj} g\, \bar{\psi}_i^j (R^a)_{ik} \gamma^{\mu} \psi_k^j A_{\mu a} - Z_{4j} m_j \bar{\psi}_i^j \psi_i^j \Big].$$

The counterterms can be absorbed into the parameters and fields of the lagrangian by redefining them. The new lagrangian written in terms of redefined "bare" fields:

$$A_{\mu aB} = Z_3^{1/2} A_{\mu a}, \qquad \eta_{aB} = \tilde{Z}_3^{1/2} \eta_a, \qquad \psi_{jB} = Z_{2Fj}^{1/2} \psi_j,$$

and "bare" parameters:

$$g_B = Z_g g, \qquad m_{jB} = Z_{mj} m_j, \qquad \frac{1}{\alpha_B} = Z_\alpha \frac{1}{\alpha} , \qquad (3)$$

may be regarded as the initial one with the same gauge symmetry provided the following identities hold:

$$Z_1/Z_3 = \tilde{Z}_1/\tilde{Z}_3 = Z_{1Fj}/Z_{2Fj} = Z_5/Z_1. \qquad (4)$$

These relations must be satisfied for arbitrary RS and for arbitrary flavor j. From (2) and (3) one can find that

$$Z_1 = Z_g Z_3^{3/2} , \qquad Z_5 = Z_3^2 Z_g^2 , \qquad \tilde{Z}_1 = Z_g \tilde{Z}_3 Z_3^{1/2} , \\ Z_{1Fj} = Z_g Z_{2Fj} Z_3^{1/2} , \qquad Z_4 = Z_{mj} Z_{2Fj}, \qquad Z_6 = Z_\alpha Z_3. \qquad (5)$$

In this paper we shall present the results of the one-loop calculation of the β- function defined as:

$$\beta(g) = \mu^2 \frac{\partial \alpha}{\partial \mu^2} \Big|_{m_B, g_B, \alpha_B, \varepsilon\text{-fixed}} = -2\alpha \frac{\partial \ln Z_g}{\partial \ln \mu^2} \Big|_{m_B, g_B, \alpha_B, \varepsilon\text{ -fixed}} \qquad (6)$$

where

$$Z_g = Z_{1Fj} Z_{2Fj}^{-1} Z_3^{-1/2} ,$$

$\alpha = g^2/4\pi$ and $\varepsilon = (4-n)/2$, n is the space-time dimension, i.e. the dimensional regularization is used.

Our renormalization prescriptions are the following. The gluon and ghost fields are normalized by requiring their propagators to take on free field values at a specific Euclidean point characterized by a mass $\mu>0$. This means that for the gluon propagator

$$\Delta^{ab}_{\mu\nu}(k) = \delta^{ab}\left\{\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right]\Delta_T(k^2) + \frac{k_\mu k_\nu}{k^2}\Delta_L(k^2)\right\}$$

in our renormalization prescription:

$$\Delta_T(-\mu^2) = -\frac{1}{\mu^2}.$$

Renormalization constant for the ghost propagator $D^{ab}(k^2)$ is determined from the requirement:

$$D^{ab}(k^2)\Big|_{k^2=-\mu^2} = \delta^{ab}\frac{1}{\mu^2}.$$

The conditions on the fermion propagator

$$S^{ij}_{kl}(k) = \frac{\delta^{ij}\,\delta_{kl}}{\hat{k}\,A(k^2,\{m_i^2\}) - m_j B(k^2,\{m_i^2\})}$$

are chosen so that at $k^2=-\mu^2$ the residue of the pole of the fermion propagator is not changed by interaction i.e.

$$A(k^2=-\mu^2,\{m_i^2\}) = 1,$$

and at $k^2=m_j^2$

$$B(m_j^2,\{m_i^2\})=A(m_j^2,\{m_i^2\})$$

i.e m_j in our consideration is a pole mass.

The fermion-fermion-vector vertex function can be written as:

$$\Gamma^{\mu a}_{ij}(p,p_1) = \gamma^\mu g(R^a)_{ji} + gG^{\mu a}_{ij}(p,p_1) \qquad (7)$$

where $G^{\mu a}_{ij}(p,p_1)$ is a contribution from 1PI diagrams. In this paper we used the following representation for this 1PI vertex function:

$$G^{\mu a}_{ij}(p,p_1)=(R^a)_{ji}\Big\{\gamma^\mu\Gamma_1+p^\mu\Gamma_2+p_1^\mu\Gamma_3+\gamma^\mu\hat{p}\Gamma_4+\gamma^\mu\hat{p}_1\Gamma_5+p^\mu\hat{p}\,\Gamma_6+$$
$$p^\mu\hat{p}_1\Gamma_7+p_1^\mu\hat{p}\,\Gamma_8+p_1^\mu\hat{p}_1\Gamma_9+p^\mu\hat{p}_1\hat{p}\Gamma_{10}+p_1^\mu\,\hat{p}_1\hat{p}\,\Gamma_{11}+\hat{p}_1\gamma^\mu\,\hat{p}\,\Gamma_{12}\Big\} \qquad (8)$$

where $\Gamma_i(i=1,2,\dots,12)$ are scalar functions depending on p^2, p_1^2 and $q^2=(p-p_1)^2$. From (7) and (8) we get:

$$\Gamma^{\mu a}_{ij}(p,p_1) = g\gamma^\mu(R^a)_{ji}[1+\Gamma_1(p^2,p_1^2,q^2)] + \tilde{\Gamma}^{\mu a}_{ij}(p,p_1). \qquad (9)$$

Only Γ_1 has ultraviolet divergences. We define our renormalization prescription for the 1PI vertex $\Gamma^{\mu a}_{ij}$ so that at some symmetric momentum point $p_1^2=p^2=q^2=-\mu^2$ the renormalized function Γ_1 obeys the condition:

$$\Gamma_1(p^2,p_1^2,q^2)\Big|_{p^2=p_1^2=q^2=-\mu^2} = 0. \qquad (10)$$

From this requirement we can express Z_{1F} in terms of the non-renormalized function Γ_{1B}:

$$Z^{-1}_{1F} = 1 + \Gamma_{1B}(-\mu^2,-\mu^2,-\mu^2). \qquad (11)$$

In fact we can show that for our renormalization prescription in all orders of perturbation theory the infrared behavior of $\bar{\alpha}(Q^2)$ is connected with the infrared behavior of the gluon propagator. To prove this, we shall use the fact /11,12/ that for our RS, the invariant charge constructed from the quark-gluon vertex taken at the symmetry point $q^2=p^2=p_1^2$ will coincide with the effective charge $\bar{\alpha}(Q^2)$. So, we can write: ($\bar{\Gamma}_1=1+\Gamma_1$):

$$\bar{\alpha}(Q^2)= \alpha\bar{\Gamma}_1^2(Q^2,\{m_i\},a,\alpha)\ \Delta_T(Q^2,\{m_i\},a,\alpha)\ A^{-2}(Q^2,\{m_i^2\},a,\alpha). \qquad (12)$$

Feynman diagrams contributing to the fermion-gluon vertex Γ_1 and fermion propagator A due to nonzero masses of external fermion particles are finite for small Q^2. This means that in (12) only Δ_T is infrared dangerous. As is well known Δ_T is gauge dependent and we may hope that for some gauges $\bar{\alpha}(Q^2)$ will be nonsingular. In this point our definition of $\bar{\alpha}$ crucially differs from other definitions widely used in applications. Usually these definitions were based upon the vertices where all external particles were massless. So, including higher radiative corrections we get a more singular behavior at small Q^2.

At the one-loop level nonsingular gauges can be found in the region $a\geq 13/3$. We shall demonstrate such possibilities below using concrete examples with different definitions of renormalized coupling constant.

3. RESULTS OF THE ONE-LOOP CALCULATIONS

The vertex renormalization constant Z_{1F} in the one-loop approximation is determined by two diagrams presented in fig 1.

a) b)

Fig 1. diagrams contributing to Z_{1F}.

To define Z_{1F}, we calculate the coefficient of γ^{μ} in (9) for $p^2 = p_1^2 = (p-p_1)^2$. Intermediate calculations for Γ_1 were rather complicated. To evaluate Z_{1F} and β, the algebraic manipulation program SCHOONSCHIP /13/ written by M.Veltman was intensively used. Our result for the renormalization constant Z_{1F} is:

$$Z_{1F}=1-\frac{\alpha}{4\pi}\left\{C_F[A_a+\alpha B_a]-\frac{1}{2}C_A[A_a+A_b+\alpha(B_a+B_b)+\alpha^2 C_b]\right\}. \qquad (13)$$

Here

$$A_a = -2+\frac{4}{3}(1+\lambda)\ln\frac{\lambda}{1+\lambda}-\frac{2}{3}(1+\lambda)M+\frac{4}{3}F(\lambda)$$

$$B_a = N_\varepsilon+2+\frac{2}{3}(1-\lambda^2)M-\ln\lambda+\frac{2}{3}(1-\lambda^2)\ln\frac{\lambda}{1+\lambda}-\frac{1}{3}(1+2\lambda)F(\lambda),$$

$$A_b=-\frac{3}{2}N_\varepsilon-\frac{9}{2}-\frac{2\lambda^2+5\lambda+8}{6}H+\left[\frac{5-2\lambda}{6}-\frac{\lambda-1}{2\Lambda}\right]\ln\lambda+\left[\frac{\lambda^2-6\lambda-1}{3}+\frac{\lambda-1}{\Lambda}\right]\ln\frac{\lambda}{1+\lambda}$$

$$B_b=-\frac{3}{2}N_\varepsilon-2-\frac{1}{3}(\lambda^2+4)H+\frac{1}{3}\left[5-\lambda+\frac{\lambda-1}{\Lambda}\right]\ln\lambda+\frac{1}{3}\left[\lambda^2-\lambda-6-2\frac{\lambda-1}{\Lambda}\right]\ln\frac{\lambda}{1+\lambda}$$

$$C_b=\frac{1}{2}+\frac{1}{6}\lambda H+\frac{(2+\lambda)\lambda}{6\Lambda}\ln\lambda+\frac{1}{3}(\lambda-1)\left[1-\frac{1}{\Lambda}\right]\ln\frac{\lambda}{1+\lambda}, \qquad (14)$$

where $\lambda = m^2/\mu^2$, $\Lambda=1+\lambda+\lambda^2$, $N_\varepsilon=\frac{1}{\varepsilon}-\gamma+\ln\frac{4\pi\nu^2}{\mu^2}$, ($\gamma$-Euler's constant), m is the pole mass of the external fermion particle of the chosen vertex, ν is massive parameter introduced to make coupling constant dimensionless and

$$F(\lambda)=\sqrt{1+4\lambda}\,\ln\frac{\sqrt{1+4\lambda}+1}{\sqrt{1+4\lambda}-1}$$

$$H=\int_0^1\frac{dy}{1-y+y^2}\ln\frac{y(1+\lambda-y)}{1+\lambda y} \qquad (15)$$

$$M=\int_0^1\frac{dy}{1-y+y^2}\ln\frac{y(1-y)+\lambda}{1+\lambda}.$$

Subscripts a and b in (13),(14) are related to the contributions from the diagrams a) and b), respectively. H and M can be expressed in terms of special functions but we find that formulas written in terms of these quantities are more compact. Asymptotics of F(λ), H, M for small and large λ are given in Appendix C.

Renormalization constants Z_{2F}, Z_3 have been taken from [6,7]:

$$Z_{2F} = 1-\frac{\alpha}{4\pi}C_F\,\alpha\,[N_\varepsilon+1-\lambda-\lambda^2\ln\lambda-(1-\lambda^2)\ln(1+\lambda)] \qquad (16)$$

$$Z_3 = 1+\frac{\alpha}{4\pi}\left\{C_A\left[\left(\frac{13-3\alpha}{6}\right)N_\varepsilon+\frac{97}{36}+\frac{\alpha}{2}+\frac{1}{4}\alpha^2\right]+\right.$$

$$\left.+\frac{4}{3}T\sum_{i=1}^{N_f}\left[-N_\varepsilon-\frac{5}{3}+4\lambda\rho_i+\ln\lambda\rho_i+(1-2\lambda\rho_i)F(\lambda\rho_i)\right]\right\}, \qquad (17)$$

where $\rho_i = m_i^2/m^2$, m_i is the pole mass of ith quark.

Now one can determine the β-function. From (6),(13),(16) and (17) we get:

$$\beta(\alpha,\lambda)=\frac{\alpha^2}{\pi}\left\{\frac{\lambda}{3}C_F\left[M-2\ln\frac{\lambda}{\lambda+1}-\frac{2F(\lambda)}{1+4\lambda}+\alpha[-3+2\lambda M-\lambda\ln\frac{\lambda}{\lambda+1}+\frac{8\lambda F(\lambda)}{1+4\lambda}]\right]\right.$$

$$+\frac{1}{12}C_A\left[(\alpha-1)(\alpha+3)\left[\frac{3\lambda^2}{2\Lambda}+[1+2\frac{1-\lambda}{\Lambda}-\frac{3}{\Lambda}2](\frac{1}{2}\ln\lambda-\ln\frac{\lambda}{\lambda+1})\right]+\lambda\left[-2(1+2\lambda\alpha)M+\right.\right. \qquad (18)$$

$$[\frac{\alpha^2-5}{2}-2\lambda(1+\alpha)]H+(1-4\alpha\lambda)\frac{4F(\lambda)}{1+4\lambda}-(1+\alpha)(2+\frac{3}{\Lambda})\ln\lambda+[\alpha(\alpha+1)+2\lambda(1-\alpha)+$$

$$\left.\left.\left.+\frac{6(1+\alpha)}{\Lambda}]\ln\frac{\lambda}{\lambda+1}\right]\right]\right\}-\frac{\alpha^2}{4\pi}\left\{\frac{11}{3}C_A-\frac{4}{3}T\sum_{i=1}^{N_f}\left[1-6\lambda\rho_i+\frac{12\lambda^2\rho_i^2F(\lambda\rho_i)}{1+4\lambda\rho_i}\right]\right\}.$$

At λ=0 our result coincides with the well known one-loop formula of asymptotic freedom

$$\beta(\alpha) = -\frac{\alpha^2}{4\pi}\left[\frac{11}{3}C_A - \frac{4}{3}TN_f\right].$$

For $\lambda\neq 0$ there is a nontrivial contribution depending on the gauge parameter. When $\lambda\to\infty$ and $\rho_i\neq 0$, we get:

$$\beta(\alpha,\lambda) = -\frac{\alpha^2}{8\pi}C_A\left[\frac{13}{3} - a + \frac{\ln\lambda}{\lambda}\left(\frac{5}{6} + \frac{10}{3}a + \frac{1}{2}a^2\right)\right] + O\left(\frac{\ln\lambda}{\lambda^2}\right). \qquad (19)$$

Thus, the leading contribution to β is gauge dependent and, as can be seen from (14),(16),(17), is determined only by massless particles. When $a>13/3$, asymptotical values of the β function for $\lambda\to 0$ and for $\lambda\to\infty$ have different signs. As we shall see later, this obstacle will be very essential for the infrared behavior of the running coupling constant.

4. RENORMALIZATION GROUP ANALYSIS

The effective running coupling $\bar{\alpha}$ in MOM schemes is determined from the system of coupled renormalization group equations for running $\bar{\alpha}$, $\bar{a}$ and $\bar{m}$. As was formulated in our renormalization prescription we used the pole mass, which means that this parameter does not run i.e. it is a constant. To simplify the renormalization group analysis we shall use the fixed gauge formalism [9,10] i.e. $\bar{a}$ in our consideration will be also a constant parameter. At the one-loop level this prescription will not change Z_g and consequently β. The fixed gauge formalism releases us from the difficulties with asymptotic freedom /4/.

The renormalization group equation for the running coupling $\bar{\alpha}(Q^2)$:

$$\frac{d\bar{\alpha}(Q^2)}{d\ln(Q^2/\mu^2)} = \beta\left(\bar{\alpha}(Q^2),\frac{m^2}{Q^2}\right) = -\beta_1\left(\frac{m^2}{Q^2}\right)\left[\bar{\alpha}(Q^2)\right]^2 - \beta_2\left(\frac{m^2}{Q^2}\right)\left[\bar{\alpha}(Q^2)\right]^3 - \ldots \qquad (20)$$

in the one-loop approximation with constant a and m can be integrated explicitly [11]:

$$\bar{\alpha}(Q^2) = \frac{\alpha}{1 + \frac{\alpha}{4\pi}\left[\left[\frac{11}{3}C_A - \frac{4}{3}TN_f\right]\ln\left(\frac{Q^2}{\mu^2}\right) + \phi\left(\frac{m^2}{Q^2}\right) - \phi\left(\frac{m^2}{\mu^2}\right)\right]}, \qquad (21)$$

where

$$\phi(\lambda) = -\frac{4}{3}T\sum_{i=1}^{N_f}\left[4\lambda\rho_i + \ln\lambda + (1-2\rho_i\lambda)F(\lambda\rho_i)\right] + \left(C_F - \frac{1}{2}C_A\right)\left\{\frac{2a+4a\lambda-8}{3}F(\lambda)\right.$$
$$\left. - \frac{2}{3}(1+\lambda)(4-a+a\lambda)\ln\frac{\lambda}{\lambda+1} + \frac{4}{3}(1+\lambda)(1-a+a\lambda)M - 2a\lambda\right\} + \qquad (22)$$
$$+C_A\left\{\left[\frac{\lambda^2-6\lambda-1}{3} - \frac{3+\lambda+2\lambda^2}{3}a + \frac{\lambda-1}{3}a^2\right]\ln\frac{\lambda}{\lambda+1} + \frac{5-2\lambda+4a-2a\lambda+a^2}{6}\ln\lambda -\right.$$
$$\left. - \left[\frac{8+5\lambda+2\lambda^2}{6} + \frac{\lambda^2+4}{3}a - \frac{\lambda}{6}a^2\right]H - \lambda a + \frac{1-\lambda}{6\lambda}(3-2a-a^2)\left[\ln\lambda - 2\ln\frac{\lambda}{\lambda+1}\right]\right\}.$$

In what follows we shall need limiting values of $\phi(\lambda)$ for $\lambda\to 0$ and $\lambda\to\infty$. When $\lambda\to 0$ and $\rho_i\neq 0$

$$\phi(\lambda)\underset{\lambda\to 0}{\simeq} \frac{2}{3}R(1)\left[2C_F(a-1) + (a+3)C_A\right] + \frac{4}{3}T\sum_{i=1}^{N_f}\ln\rho_i + O(\lambda\ln\lambda), \qquad (23)$$

where /3/ $R(1) = -2\int_0^1\frac{\ln x\,dx}{1-x+x^2} = 2.343907238690$. In the consideration $\bar{\alpha}(Q^2)$ at small Q^2 asymptotic for $\phi(\lambda)$ when $\lambda\to\infty$ will be needed:

$$\phi(\lambda)\underset{\lambda\to\infty}{\simeq}\left[\frac{a+3}{2}C_A - \frac{4}{3}TN_f\right]\ln\lambda + \frac{21-3a-2a^2}{4}C_A + (2a-4)C_F - \frac{20}{9}TN_f + O\left(\frac{\ln\lambda}{\lambda}\right). \qquad (24)$$

In the case of quantum chromodynamics the SU(3) gauge group must be taken with quarks belonging to the fundamental representation of this group. We shall consider five-quark model with the following quark masses [6]:

$m_u = m_d = 40$ MeV, $m_c = 1400$ MeV
$m_s = 500$ MeV, $m_b = 4500$ MeV.

At first we determine $\bar{\alpha}(Q^2)$ by the quark-gluon vertex with the fourth external quark. The behavior of $\bar{\alpha}(Q^2)$ with the initial condition $\bar{\alpha}(Q^2{=}10\,\mathrm{Gev}^2)\simeq 0.19$ for different values of the gauge parameter is shown in fig.2.

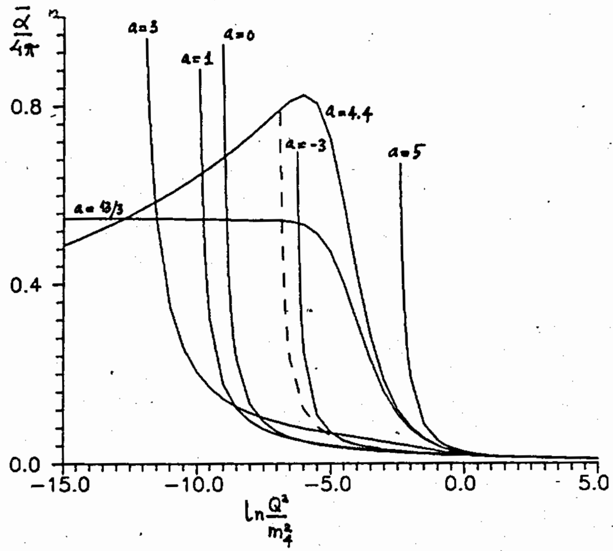

Fig.2. The Q^2 dependence of the effective running coupling (determined by the quark-gluon vertex with the fourth external quark) for different values of the gauge parameter. Dotted line corresponds to the massless case.

In the region $Q^2 \leqslant m^2$

$$\bar{\alpha}(Q^2) \text{ for } \begin{cases} a < \frac{13}{3} \text{ and } a > a_0 & \text{has a pole at some } Q^2 = Q_p^2 \\ a = \frac{13}{3} & \text{goes to a const} \neq 0, \text{ when } Q^2 \to 0, \\ \frac{13}{3} < a < a_0 & \text{goes to } 0 \text{ when } Q^2 \to 0 \end{cases}$$

where a_0 can be determined numerically from (21). For the case under consideration $a_0 \simeq 4.55$. To underline the significance of mass dependence, we have shown in fig.2 $\bar{\alpha}(Q^2)$ for the case when all masses are equal to zero (dotted line). One can observe also that when $\bar{\alpha}(Q^2)$ has a pole, its location depends on the value of the gauge parameter. To illustrate this dependence, we present in the Table the location of the singularity for several values of the gauge parameter a.

Table

a	-1.0	0.0	1.0	2.0	2.5	3.0	3.5	4.0
$\ln Q_p^2/m^2$	-8.25	-9.15	-10.15	-11.25	-11.75	-12.45	-13.45	-16.05

Next we consider $\bar{\alpha}(Q^2)$ determined by the quark-gluon vertex with the fifth external quark. Putting in (21) $m=m_5$ with the same initial condition $\bar{\alpha}(Q^2=10\text{Gev}^2) \simeq 0.19$ we found that qualitatively the behavior of the running coupling is similar to the previous case. The Q^2 dependence of $\bar{\alpha}(Q^2)$ is shown in Fig.3.

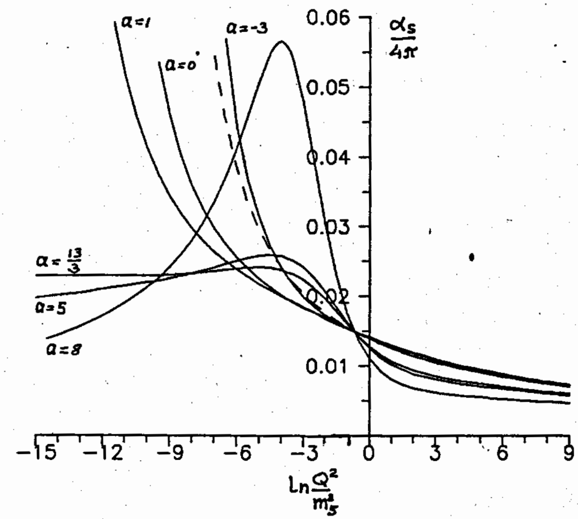

Fig.3. The Q^2 dependence of the effective running coupling (determined by the quark-gluon vertex with the fifth external quark) for different values of the gauge parameter.

There is the region $13/3 \leqslant a \leqslant a_0$ where $\bar{\alpha}(Q^2)$ has no infrared singularity but for $a > a_0$ and $a < 13/3$ it has a pole. In comparison with the previous case there are also significant differences. First of all, here $a_0 \simeq 9.75$ i.e. the region of a where $\bar{\alpha}(Q^2)$ has no singularity for small momenta is wider. Secondly, in some region of a close to $a=13/3$, the maximum value of $\bar{\alpha}(Q^2)$ is rather small. For example, when $a=13/3$, this maximum value is approximately 0.30 at $Q^2=0.136$ Gev^2.

Some remarks about the renormalization group with running gauge. In this case the behavior of $\bar{\alpha}$ as $Q^2 \to 0$ will essentially depend on the initial value for a. When the initial value $a=13/3$ at the one loop level effective charge will be the same as in the fixed gauge formalism with the same a. For other initial values running $\bar{a}$ will go to zero or to infinity and consequently this will lead to the singularity in $\bar{\alpha}(Q^2)$ as $Q^2 \to 0$. At the two-loop level and higher the situation may be more complicated.

When Q^2 goes to infinity, at the one-loop level in both the cases $\bar{\alpha}(Q^2)$ decreases for any value of the gauge parameter. For large Q^2 we can approximately write:

$$\bar{\alpha}(Q^2) \simeq \frac{4\pi}{(11-\frac{2}{3}N_f)\ln\frac{Q^2}{\Lambda^2} + O(\frac{m^2}{Q^2}\ln\frac{m^2}{Q^2})} \cdot \tag{25}$$

The value of Λ is gauge dependent. For example, when $a=13/3$ and $\bar{\alpha}$ is defined by the quark gluon vertex with the fourth external quark then $\Lambda \simeq 24$ Mev, if it is defined by the fifth quark, then $\Lambda \simeq 36$Mev. Of course, these Λ do not correspond to the location of the pole for $\bar{\alpha}$ at small Q^2, they characterize its ultraviolat asymptotics.

From the definition (12) we can obtain some general information about the behavior of $\bar{\alpha}$ in the whole region Q^2. When $Q^2 \gg m^2$

$$\beta_t(\frac{m^2}{Q^2}) \simeq -\beta_t(0) + O(\frac{m^2}{Q^2}\ln\frac{m^2}{Q^2}) \cdot \tag{26}$$

In the "stopping" gauge formalism [9] $\beta_1(0)=\beta_1^{MS}$ and $\beta_2(0)=\beta_2^{MS}$, i.e. at that level $\beta<0$ and $\bar{\alpha}$ in this region is asymptotically free. When $Q^2 \leqslant m^2$, the behavior of $\bar{\alpha}$, as mentioned above, is determined by Δ_T, and consequently we can conclude that

$$\beta(\bar{\alpha},\frac{m^2}{Q^2}) \simeq -\bar{\alpha}\,\gamma_3(\bar{\alpha},a) + O(\frac{Q^2}{m^2}\ln\frac{Q^2}{m^2}), \tag{27}$$

where $\gamma_3(\alpha,a)$ is the anomalous dimension of the gluon propagator that is gauge dependent. At the two loop level in the "stopping" gauge formalism we get:

$$\gamma_3(\alpha,a)=\left[\frac{13-3a}{6}C_A-\frac{4}{3}TN_f\right]\frac{\alpha}{4\pi}+\left[\left(\frac{38a-32+3a^2-3a^3}{12}+\frac{39+4a-3a^2}{9}R(1)\right]C_A^2+\right.$$

$$+\left(\frac{26-19a+3a^2}{3}+\frac{-26+32a-6a^2}{9}R(1)\right]C_AC_F+\left(\frac{-28+8a}{3}+\frac{16-16a}{9}R(1)\right]C_FTN_f+$$

$$\left.+\left(\frac{1-a-2a^2}{3}-\frac{24+8a}{9}R(1)\right]C_A TN_f\right]\frac{\alpha^2}{16\pi^2} \cdot \tag{28}$$

In the region under consideration N_f should be considered as a number of massless quarks. At least in this approximation choosing a we can make $\gamma_3(\alpha,a)<0$ and therefore for small Q^2 our β will be positive, i.e. $\bar{\alpha}$ will decrease as $Q^2 \to 0$. But as was seen at the one-loop level the intermediate region $Q^2 < \mu^2$ is dangerous. As Q^2 decreases, the term proportional to $\beta_1 \ln Q^2/\mu^2$ in the denominator of $\bar{\alpha}$ gives a negative contribution, and before the leading term from $\phi(m^2/Q^2)$ cancels it, the denominator becomes zero. This is the reason for the singularity when $a>a_0$. Choosing a and taking vertex with the heaviest external quark one can try to avoid the singularity.

Using nonlocal terms to fix the gauge we can improve infrared properties of $\bar{\alpha}$. Anomalous dimension of the gluon propagator in all orders of perturbation theory can be made proportional to its one-loop coefficient. This gauge-fixing term will differ from that proposed in /9/ to "stop" the gauge. For example, in our case the nonlocal gauge fixing term proportional to α will be determined from the two-loop condition on γ_3. In /9/ the term of the same order was obtained from the one-loop correction to the gluon propagator. As a consequence, we get running $\bar{\alpha}$, but the gauge

$$a_* = \frac{13}{3} - \frac{8}{3}\frac{TN_f}{C_A} \tag{29}$$

will not run because in this gauge $\gamma_3(\alpha,a_*)=0$ for all orders of the perturbation theory. So, according to (27) when $a=a_*$

$$\beta_t\left[\frac{m^2}{Q^2}\right] \underset{Q^2 \to 0}{\simeq} O\left[\frac{Q^2}{m^2}\ln\frac{Q^2}{m^2}\right],$$

and possibly $\bar{\alpha}$ will go to a constant in all orders of perturbation theory. Note here that at the one-loop level in both the considered examples $\bar{\alpha}$ has the smallest maximum value for $a=a_*$. Possibly, to improve infrared behavior of $\bar{\alpha}$, it will be useful to introduce an appropriate dependence of the gauge-fixing term on m^2/μ^2.

5. CONCLUSIONS

As follows from the above consideration, the singular behavior of perturbatively defined $\bar{\alpha}(Q^2)$ in the infrared region is not an inevitable property of QCD. Unfortunately, at the present moment we can rigorously confirm this statment only at the one-loop level. But we do not see any essential difficulties at higher orders. In the near future we shall present a detailed two-loop analysis. One-loop investigation showed that for two considered cases $\tilde{\alpha}$ is smaller when a heavier mass is taken. From this point of view we can improve the situation using running masses because they increase as $Q^2 \to 0$.

Gauge dependence of $\tilde{\alpha}$ used for the perturbative decomposition of physical quantities can make a feeling of dissatisfaction. But in this connection we must remind that QCD has no natural physical definition of $\tilde{\alpha}$ and in this sense all RS are equal. Nevertheless we argue that our RS is more equal than others. The first argument is that all physical quantities at small Q^2 are limited. So it is rather unnatural to use the singular decomposition parameter.

Of course at small Q^2 nonperturbative effects are essential and must be taken into account. One sort of nonperturbative effects is caused by the difference between the perturbative vacuum and physical one. To include this kind of nonperturbative corrections, one should add terms proportional to the quark and gluon condensates. The factors of these condensates are calculated perturbatively. This means that going to small Q^2 we must be confident that radiative corrections in that region are small enough. So, our second argument is that for a self-consistent treatment of such nonperturbative effects it is desirable to be in the weak coupling regime. From this point of view our RS can be also more useful and preferable than others.

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Appendix A

We present here integrals needed for the calculation of diagrams given on fig.1 (below $d^n\hat{k} = -i(2\nu)^{4-n}\pi^{2-n}d^nk$).

$$J_\alpha = \int \frac{d^n\hat{k}}{[(k+p)^2-m^2](k^2)^\alpha}$$

$$J_{\alpha\mu} = \int \frac{d^n\hat{k}\; k_\mu}{[(k+p)^2-m^2](k^2)^\alpha} \equiv p_\mu J^0_\alpha$$

$$J_{\alpha\mu\nu} = \int \frac{d^n\hat{k}\; k_\mu k_\nu}{[(k+p)^2-m^2](k^2)^\alpha} \equiv g_{\mu\nu}J^3_\alpha + p_\mu p_\nu J^4_\alpha$$

$$I_\alpha(m,m_1) = \int \frac{d^n\hat{k}}{[(p+k)^2-m^2][(p_1+k)^2-m_1^2](k^2)^\alpha}$$

$$I_{\alpha\mu}(m,m_1) = \int \frac{d^n\hat{k}\; k_\mu}{[(p+k)^2-m^2][(p_1+k)^2-m_1^2](k^2)^\alpha} \equiv p_\mu I^0_\alpha(m,m_1) + p_{1\mu}I^1_\alpha(m,m_1)$$

$$I_{\alpha\mu\nu}(m,m_1) = \int \frac{d^n\hat{k}\; k_\mu k_\nu}{[(p+k)^2-m^2][(p_1+k)^2-m_1^2](k^2)^\alpha}$$

$$\equiv g_{\mu\nu}I^3_\alpha(m,m_1) + p_\mu p_\nu I^4_\alpha(m,m_1) + p_{1\mu}p_{1\nu}I^5_\alpha(m,m_1) + (p_\mu p_{1\nu} + p_{1\mu}p_\nu)I^6_\alpha(m,m_1)$$

$$I_{\alpha\beta} = \int \frac{d^n\hat{k}}{[(p+k)^2-m^2][(k+p-p_1)^2]^\alpha(k^2)^\beta}$$

$$I_{\alpha\beta\mu} = \int \frac{d^n\hat{k}\; k_\mu}{[(p+k)^2-m^2][(k+p-p_1)^2]^\alpha(k^2)^\beta} \equiv pI^0_{\alpha\beta} + p_{1\mu}I^1_{\alpha\beta}$$

$$I_{\alpha\beta\mu\nu} = \int \frac{d^n\hat{k}\; k_\mu k_\nu}{[(p+k)^2-m^2][(k+p-p_1)^2]^\alpha(k^2)^\beta}$$

$$\equiv g_{\mu\nu}I^3_{\alpha\beta} + p_\mu p_\nu I^4_{\alpha\beta} + p_{1\mu}p_{1\nu}I^5_{\alpha\beta} + (p_{1\mu}p_\nu + p_\mu p_{1\nu})I^6_{\alpha\beta}.$$

To find Γ_1, we calculated in the Euclidean space-time the following coefficients taken for $p^2=p_1^2=(p-p_1)^2=-\mu^2$.

$$J_1 = N_\varepsilon + 2 + \lambda\, ln\lambda - (1+\lambda)\, ln(1+\lambda)$$

$$J_2 = \frac{1}{\mu^2}\frac{1}{(1+\lambda)}[N_\varepsilon - \lambda\, ln\lambda + (\lambda-1)\, ln(1+\lambda)]$$

$$J^0_1 = \frac{1}{2}[-N_\varepsilon - 2 - \lambda - (\lambda+2)\lambda\, ln\lambda + (1+\lambda)^2 ln(1+\lambda)]$$

$$J^0_2 = -\frac{1}{\mu^2}[1 + \lambda\, ln\frac{\lambda}{1+\lambda}]$$

$$I_0(m,m) = N_\varepsilon + 2 - F(\lambda) - ln\lambda$$

$$I_1(m,m) = -\frac{1}{\mu^2}M$$

$$I_2(m,m) = \frac{1}{\mu^4}\frac{1}{(1+\lambda)^2}[-N_\varepsilon - F(\lambda) - \ln\lambda + 2\ln(1+\lambda)]$$

$$I_2(m,o) = I_2(o,m) = \frac{1}{\mu^4}\frac{1}{(1+\lambda)}[-N_\varepsilon + \frac{\lambda}{\Lambda}\ln\lambda - \frac{2\lambda}{\Lambda}\ln(1+\lambda)]$$

$$I^0_\alpha(m,m) = I^1_\alpha(m,m), \quad I_{\alpha\beta} = I_{\beta\alpha}, \quad I^3_{\alpha\beta} = I^3_{\beta\alpha}, \quad I_{o\alpha} = J_\alpha$$

$$I^0_1(m,m) = -\mu^2(1+\lambda) I^0_2(m,m) = \frac{1}{3\mu^2}[-(1+\lambda)M - F(\lambda) - (1+\lambda)\ln\frac{\lambda}{1+\lambda}]$$

$$I^3_1(m,m) = -\frac{1}{2}(1+\lambda)\mu^2 I^0_1(m,m) + \frac{1}{4} I_0(m,m) + \frac{1}{4}$$

$$I^3_2(m,m) = I^1_1(m,m) + \frac{1}{2} I_1(m,m)$$

$$I_{11} = \frac{1}{\mu^2}H, \quad I_{12} = I_2(m,o), \quad I^0_{20} = J_2 + J^0_2, \quad I^1_{21} = I_{21} + I^0_{12}$$

$$I^0_{11} + I^1_{11} = \frac{1}{3\mu^2}[-(1+2\lambda)H + 2\lambda\ln\lambda - 2(1+\lambda)\ln(1+\lambda)]$$

$$I^0_{12} = \frac{1}{3\mu^2}[2H + \frac{\lambda-1}{\Lambda}\lambda\ln\lambda + \frac{1+3\lambda-\lambda^2}{\Lambda}\ln(1+\lambda)]$$

$$I^3_{12} = \frac{1}{4}(I_{11} + I^0_{11} + I^1_{11}) = \frac{1}{6\mu^2}[(1-\lambda)H + \lambda\ln\lambda - (1+\lambda)\ln(1+\lambda)]$$

$$I^3_{11} = \frac{1}{4}[N_\varepsilon + 3 + \frac{2}{3}\lambda(1-\lambda)\ln\lambda - \frac{2}{3}(1-\lambda^2)\ln(1+\lambda) + \frac{2}{3}\Lambda H]$$

$$I^3_{22} = \frac{1}{6\mu^2}[-H - \frac{(1+2\lambda)\lambda}{\Lambda}\ln\lambda - \frac{2(1-\lambda^2)}{\Lambda}\ln(1+\lambda)].$$

Appendix B

In this appendix, for completeness we present the renormalization group β-function with masses for MOM schemes where the renormalized coupling constant is determined by ghost-ghost-gluon and three-gluon vertices in the one-loop approximation.

a) β-function on the three- gluon vertex [6]:

$$\beta = -\frac{\alpha^2}{4\pi}\left[\frac{11}{3}C_A - \frac{4}{3}T\sum_{i=1}^{N_f}\left[1 - h(\lambda\rho_i)\right]\right],$$

where

$$h(\lambda) = 18\lambda\int_0^1 dx\frac{x(1-x)}{x(1-x)+\lambda} + 2\int_0^{1/3} dy\,\rho(y)\left[\frac{\lambda}{\lambda+y} - \frac{3\lambda^2}{(\lambda+y)^2}\right],$$

$$\rho(y) = \begin{cases} 2\sqrt{3}\,\mathrm{arctg}\sqrt{3}\,\frac{1-\sqrt{1-4y}}{1+3\sqrt{1-4y}} & \text{for } 0 \leqslant y \leqslant \frac{1}{4} \\ \frac{2\pi}{\sqrt{3}} & \text{for } \frac{1}{4} \leqslant y \leqslant \frac{1}{3} \end{cases}$$

b) β- function on the ghost-ghost-gluon vertex [5]:

$$\beta = -\frac{\alpha^2}{4\pi}\left[\frac{11}{3}C_A - \frac{4}{3}T\sum_i\left[1 - 6\lambda\rho_i + \frac{12\lambda^2\rho_i^2 F(\lambda_i)}{1+4\lambda\rho_i}\right]\right].$$

Appendix C

Asymptotics for the quantities $F(\lambda)$, H and M (see (14)).

When $\lambda\to\infty$ up to the order $O(\ln\lambda/\lambda)$

$$F(\lambda) \simeq 2\left[1 + \frac{1}{12\lambda} - \frac{1}{120\lambda^2} + \frac{1}{840\lambda^3} - \frac{1}{5040\lambda^4} + \frac{1}{27720\lambda^5}\right]$$

$$H \simeq -\frac{\ln\lambda}{\lambda}\left[1 - \frac{1}{2\lambda} + \frac{1}{4\lambda^3} - \frac{1}{5\lambda^4}\right] - \frac{1}{\lambda}\left[1 + \frac{3}{4\lambda} - \frac{1}{\lambda^2} + \frac{23}{48\lambda^3} + \frac{19}{150\lambda^4}\right]$$

$$M \simeq -\frac{1}{\lambda} + \frac{7}{12\lambda^2} - \frac{2}{5\lambda^3} + \frac{169}{560\lambda^4} - \frac{2672}{11025\lambda^5}.$$

When $\lambda\to 0$ up to the order $O(\lambda^6\ln\lambda)$

$$F(\lambda) \simeq -\left[1+2\lambda-2\lambda^2+4\lambda^3-10\lambda^4+28\lambda^5\right]\ln\lambda + \lambda\left[2+\lambda-\frac{10}{3}\lambda^2+\frac{59}{6}\lambda^3-\frac{449}{15}\lambda^4\right],$$

$$H \simeq -R(1) - \lambda\ln\lambda\left[1-\frac{1}{2}\lambda+\frac{1}{4}\lambda^3-\frac{1}{5}\lambda^4\right] + \lambda\left[1+\frac{3}{4}\lambda-\lambda^2+\frac{23}{48}\lambda^3+\frac{19}{150}\lambda^4\right],$$

$$M \simeq -R(1) - 2\lambda\ln\lambda\left[1-\frac{3}{2}\lambda+3\lambda^2-\frac{29}{4}\lambda^3+\frac{99}{5}\lambda^4\right] +$$

$$+ 2\lambda\left[1+\frac{1}{4}\lambda-2\lambda^2+\frac{317}{48}\lambda^3-\frac{1027}{50}\lambda^4\right].$$

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Влияние масс кварков на инфракрасное поведение $\bar{\alpha}_s(Q^2)$ в КХД

Проведено вычисление ренормгрупповой β-функции в однопетлевом приближении в МОМ схеме с массивными кварками. Для определения ренормированной константы связи использована кварк-глюонная вершина. β-функция содержит нетривиальную зависимость от масс кварков и калибровочного параметра $-\alpha$. Инфракрасное поведение эффективной константы связи $\bar{\alpha}(Q^2)$, в предложенной схеме ренормировки, существенным образом зависит от выбора α. Проведен ренормгрупповой анализ для модели с пятью сортами кварков. Обнаружено, что для некоторых калибровок бегущая константа связи не имеет сингулярностей во всем диапазоне Q^2. Этот эффект существенным образом связан с нетривиальной зависимостью β-функции от масс кварков. Мы полагаем, что используемая нами схема ренормировки имеет преимущества по сравнению с другими схемами, поскольку в ней $\bar{\alpha}$ отражает поведение физических величин, которые конечны при малых импульсах.

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The Influence of Quark Masses on the Infrared Behavior of $\bar{\alpha}(Q^2)$ in QCD

Renormalization group β-function in the MOM scheme is calculated in the one loop approximation by using the quark-quark-gluon vertex to define the renormalized coupling constant. It has nontrivial mass and gauge dependence. The infrared behavior of the effective coupling constant $\bar{\alpha}$ in this scheme essentially depends on the choice of the gauge parameter. We have analysed the situation with five flavours. It was found that for some gauges the running coupling constant does not have pole singularity in the whole range of momentum. This effect is essentially connected with mass dependence of the β-function. We suppose that our renormalization prescription is preferable than others because here $\bar{\alpha}$ reflects the behavior of physical quantities which are finite at low energies.

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