

# обьединөнный институт ЯдерНых исследований 

A.A.Akhundov ${ }^{1}$, D.Yu.Bardin, Č.Burdik, P.Ch.Christova, L.V.Kalinovskaja ${ }^{2}$

CONTRIBUTION OF THE RADIATIVE TAIL FROM THE ELASTIC PEAK TO DEEP INELASTIC SCATTERING AT HERA

Submitted to "Zeitschrift für Physik C"

1 Institute of Physics, Academy of Sciences of the Azerbaijan SSR, Baku, USSR
2 Gomel Polytechnical Institute, Gomel, USSR

## 1. INTRODUCTION

The electron-proton collider HERA is devoted to study neutral and charged current lepton-nucleon interactions in an energy range ( $\mathrm{E}_{\theta}=30 \mathrm{Gev}, \mathrm{E}_{\mathrm{p}}=820 \mathrm{Gev}$ ) where both photon and weak boson exchanges are present in a space-like region with comparable strength. In the HRRA physical programme [1] an important role will play the atudy of deep inelastic scattering [2].

In this article we continue our study of various radiative corrections to deep inelastic scattering at HERA started in refs [3]. Here we present some results for the contribution of the radiative tail from the elastic peak. In inclusive-type experiments when only the final lepton is detected, the process:

$$
\begin{equation*}
1+N \longrightarrow 1+\gamma+N \tag{1}
\end{equation*}
$$

cannot be distinguished from the main reaction

$$
\begin{equation*}
1+\mathrm{N} \longrightarrow 1+\mathrm{X} \tag{2}
\end{equation*}
$$

and hence the contribution of the elastic radiative tail (ERT), i.e. of process (1), may be considered as one of the corrections to the main reaction.

For deep inelastic $1 N$ - scattering with $Q^{2} \ll M_{z}^{2}$ the most important QKD radiative corrections to the lepton current have been calculated by Mo and Tsai [4] and later by Akhundov et. al [5]. In papers [5a,5c], in particular, exact formalae for the contribution of the ERT in the order $a^{3}$ and $a^{4}$ to the measured cross section were obtained, using general, model-independent hadronic tensor, but only $r$-exchange was taken into account. Recent progress in the theoretical study of the higher-order QKD processes in deep inelastic $1 \mathbb{N}$ - scattering has been achieved in refs [6].

Process (1) was studied also in the literature [7] in connection with the luminosity measurement for HKRA accelerator ( see [8]). Recently in ref.[9] many distributions in different kinematical variables have been calculated by the Monte-Carlo method for phase space integration.

It is well known that $Z$-exchange gives a negligible contribution to the cross section of the RRT [9], but this article is just devoted to the calculation of the contribution of the ERT to the measured cross section of deep inelastic lN - scattering taking into account both $\gamma$-exchange, $Z$ - exchange and their interference ( diagrams in Fig. 1 ). We have realised these, at first sight senseless calculations

to develop a proper treatment for subsequent calculations of the inelastic radiative tail, where the $Z$-exchange will give an essential contribution.


Fig.1. Feynman graphs corresponding to the ERT considered in this paper.

Here, as usual, we restrict ourselves to the consideration of lepton leg QED corrections as the real photon emission from the lepton line gives the most important contribution. It can amount up to dozens percent of this corrections and even larger at small $x$ and large $y$ with step decrease in the region of large $Q^{2}$. This behaviour of leptonic corrections can be understood from the following well-known reasons:
a) large logarithmic terms of the form

$$
\frac{a}{\pi}\left[\ln \frac{Q^{2}}{m^{2}}-1\right]
$$

with mbeing the lepton (electron) mass, are present due to the radiation of photons collinear with the emitting leptons,
b) the emission of a very energetic photon shifts the momentum in the propagator of the $r$-exchange to a value which is essentially smaller than determined from the energy and momentum of the final lepton.

The paper is organized as follows. In sect. 2 we repeat briefly the
kinematics of process (1). In sect. 3 we obtain the formulae for the invariant inclusive cross section of the ERT. applying simple but very effective trick. In sect. 4 the results for the contribution of the ERT to the measured cross section at HRRA energies are presented and discussed. The last section contains a summary. In Appendix A.we list the set of explicitly calculated integrals.

## 2. KINEMATICS OF THE PROCESS

Reaction (1) is characterized by five independent invariants which in the problem under consideration are taken as follows [5a]:

$$
\begin{gather*}
S=-2 p_{1} k_{i}, \quad X=-2 p_{1} k_{z}, \\
Y=\left(k_{1}-k_{2}\right)^{2}, \quad t=\left(p_{1}-p_{2}\right)^{2}, \quad z_{2}=-2 k_{z} k . \tag{3}
\end{gather*}
$$

It is convenient to get also the following variable

$$
\begin{equation*}
z_{1}=-2 k_{1} k=z_{2}+t-Y \tag{4}
\end{equation*}
$$

In terms of variables (3) the phase space element of ERT takes the form

$$
\begin{equation*}
d \Gamma=\frac{\pi}{4\left(\lambda_{s}\right)^{1 / 2}} d X d Y \frac{d t z_{z}}{\left(R_{z}\right)^{1 / 2}}, \tag{5}
\end{equation*}
$$

where $R_{z}$ is the positive definite guadratic trinomial
( the Gram determinant ):
$R_{z}=A_{2} z_{2}^{2}+2 B_{2} z_{2}-C_{2} \equiv A_{1} z_{1}^{2}+2 B_{1} z_{1}-C_{1}$,
$A_{2}=-\left(S_{s}^{2}+4 M^{2} Y\right) \equiv A_{1} \equiv-\lambda_{Y}$,
$B_{2}=-2 M^{2} Y t_{Y}+X t V_{2}+S Y\left(S_{x}-t\right) \equiv-B_{1}(S \longrightarrow-X)$,
$C_{2}=[X t-Y(S-t)]^{2}+4 m^{2}\left[t\left(S_{x}-t\right) V_{z}-M^{2} t_{Y}^{2}\right] \equiv-C_{1}(S \longrightarrow-X), \quad$ ( $\mathcal{S}$
with $H$ being the proton mass, $\lambda_{s}=S^{2}-4 m^{2} M^{2}, S_{x}=S-X, V_{2}=S_{X}-Y, t_{Y}=t-Y$. One can easily obtain [5a] the physical $z_{z}$ - and $t-$ region :

$$
\begin{gather*}
\left(z_{2}\right)_{\text {min,max }}=\frac{-B_{2} \pm\left(B_{2}^{2}+A_{z} C_{2}\right)^{1 / 2}}{A_{2}},  \tag{10}\\
t_{\max , \min }=\frac{1}{2 W^{2}}\left[\left(W^{2}-M^{2}\right)\left(S_{X} \pm\left(\lambda_{Y}\right)^{1 / 2}\right)+2 M^{2} Y\right], \tag{11}
\end{gather*}
$$

$$
\begin{equation*}
W^{2}=-\left(p_{1}+k_{1}-k_{2}\right)^{2}=M^{2}+S_{x}-Y . \tag{12}
\end{equation*}
$$

It is useful to introduce two quantities $t_{1}$ and $t_{2}$ which are arguments of $C_{1}$ and $C_{2}$, respectively, where they reach minimal values:

$$
\begin{equation*}
t_{1}=\frac{Y S}{X+Y}, \quad t_{2}=\frac{Y X}{S-Y} . \tag{13}
\end{equation*}
$$

The $t_{1}$ and $t_{2}$ always satisfy the inequality:

$$
\begin{equation*}
t_{\min }<t_{1}<Y<t_{2}<t_{\max } \tag{14}
\end{equation*}
$$

3. INCLUSIVE CROSS SECTION

The differential cross section of process (1) for the acattering of mpolarized leptons on unpolarized nucleons corresponding to the diagrams of Fig. 1 can be represented in a compact form when only one ultrarelativistic approximation is made, i.e. the lepton mass is neglected everywhere possible (for comparision see formulae (36)-(38) of ref. [5a]):

$$
\begin{equation*}
d \sigma^{E R T}=\frac{2 \alpha^{3}}{\pi S^{2}} S\left(t, z_{1}, z_{z}\right) d X d Y \frac{d t d z_{z}}{t^{2}\left(R_{z}\right)^{1 / 2}} \tag{15}
\end{equation*}
$$

$S\left(t, z_{1}, z_{2}\right)=t A_{1}(t) S_{1}\left(t, z_{1}, z_{2}\right)+A_{2}(t) S_{2}\left(t, z_{1}, z_{2}\right)+$

$$
\begin{equation*}
+A_{9}(t) S_{3}\left(t, z_{1}, z_{2}\right) \tag{16}
\end{equation*}
$$

where three factorized functions $S_{1}\left(t, z_{1}, z_{2}\right), S_{2}\left(t, z_{1}, z_{z}\right)$ and $S_{3}\left(t, z_{1}, z_{2}\right)$ are explicitly calculated expressions:

$$
\begin{aligned}
S_{1}\left(t, z_{1}, z_{2}\right)= & \frac{1}{2}\left(\frac{z_{2}}{z_{1}}+\frac{z_{1}}{z_{2}}\right)+t^{2} \frac{1}{z_{2} z_{1}}-t\left(\frac{1}{z_{2}}-\frac{1}{z_{1}}\right)- \\
& -m^{2} t\left(\frac{1}{z_{2}^{2}}+\frac{1}{z_{1}^{2}}\right),
\end{aligned}
$$

$$
S_{2}\left(t, z_{1}, z_{2}\right)=-M^{2}\left[\frac{z_{2}}{z_{1}}+\frac{z_{1}}{z_{2}}\right]+\frac{1}{z_{2} z_{1}}[t[S(S-t)+X(X+t)-
$$

$$
\left.\left.-2 M^{2} t\right]+4 m^{2} S X\right]-t\left[\frac{1}{z_{2}}\left(X-2 M^{2}\right)+\frac{1}{z_{1}}\left(S+2 M^{2}\right)\right]-
$$

$$
\begin{equation*}
-2 m^{2}\left[\frac{1}{z_{2}^{2}}\left(S(S-t)-M^{2} t\right]+\frac{1}{z_{1}^{2}}\left(X(X+t)-M^{2} t\right)\right] \tag{18}
\end{equation*}
$$

$$
S_{3}\left(t, z_{1}, z_{2}\right)=2 t\left[\frac{1}{2}\left[\frac{z_{1}}{\dot{z}_{2}}-\frac{z_{2}}{z_{1}}\right]+\frac{t}{z_{2} z_{1}}(S+X)-\frac{1}{z_{2}}(X+t)+\right.
$$

$$
\begin{equation*}
\left.+\frac{1}{z_{1}}(S-t)-\frac{m^{2}}{z_{2}^{2}}(2 S-t)-\frac{m^{2}}{z_{1}^{2}}(2 X+t)\right] \tag{19}
\end{equation*}
$$

All elastic form factors describing the nucleon vertex in the diagrams of Fig. 1 together with the $\gamma / Z$ propagator ratio $t /\left(t+M_{z}^{2}\right)$ can be absorbed in three functions which we call generalized elastic form factors of the nucleon:

$$
\begin{align*}
A_{1}(t)= & {\left[F_{1}+F_{2}+x \frac{t}{t+M_{z}^{2}} g_{v}\left(F_{1}^{0}+F_{z}^{0}\right)\right]^{2}+} \\
& +x^{2} \frac{t^{2}}{\left(t+M_{z}^{2}\right)^{2}}\left[\left(g_{v}^{2}+g_{a}^{2}\right)\left(F_{3}^{0}\right)^{2}\left(1+\frac{1}{\tau}\right)+g_{a}^{2}\left(F_{1}^{0}+F_{2}^{o}\right)^{2}\right],  \tag{20}\\
A_{2}(t)= & {\left[F_{1}+x \frac{t}{t+M_{z}^{2}} g_{v} F_{1}^{0}\right]^{2}+\tau\left[F_{z}+x \frac{t}{t+M_{z}^{2}} g_{v} F_{z}^{0}\right]^{2}+} \\
& +x^{2} \frac{t^{2}}{\left(t+M_{z}^{2}\right)^{2}}\left[\left(g_{v}^{2}+g_{a}^{2}\right)\left(F_{g}^{0}\right)^{2}+g_{a}^{2}\left(\left(F_{1}^{0}\right)^{2}+\tau\left(F_{2}^{0}\right)^{2}\right)\right]  \tag{21}\\
A_{3}(t)= & 2 x g_{a} F_{s}^{0} \frac{t}{t+M_{z}^{2}}\left[F_{1}+F_{2}+x \frac{t}{t+M_{z}^{2}} g_{v}\left(F_{1}^{0}+F_{z}^{0}\right)\right] \tag{22}
\end{align*}
$$

Here $F_{1}, F_{2}\left(F_{1}^{o}, F_{2}^{o}, F_{3}^{o}\right)$ are electromagnetic (weak) nucleon form factors, $g_{v}$ and $g_{a}$ are the vector and axial coupling constants of the lepton to the Z bozon:

$$
g_{v}=1-4 \sin ^{2} \theta_{v}, g_{a}=1
$$

( $\theta_{v}$ is the weak mixing angle),

$$
\begin{equation*}
x=\frac{G_{\mu}}{2^{1 / 2}} \frac{M_{z}^{2}}{B \pi \alpha} \tag{23}
\end{equation*}
$$

$G_{\mu}$ is the Fermi constant: $G_{\mu}=1.16637 \cdot 10^{-5} \mathrm{GeV}^{-2}$, and $\tau=\mathrm{t} / 4 \mathrm{M}^{2}$. The generalized elastic form factors (20)-(22) are in full accordance with [10].

The elastic form factors $\mathrm{F}_{1,2}, \mathrm{~F}_{1,2,3}^{\mathrm{o}}$ contain information about the nucleon structure. The weak form factors $F_{1}^{0}, F_{2}^{0}$ were related with proton and neutron electromagnetic form factors $F_{1,2}^{p, n}$ [11] and calculated as $F_{1}, F_{2}$ by using an experimental fit [12].

Integrating over invariant $z_{2}$ within limits (10), we derive from (15)-(19) the following expression for the invariant inclusive cross section of the ERT:

$$
\begin{equation*}
\frac{d^{2} d^{E R T}}{d x d y}=2 a^{2} y \int \frac{d t}{t^{2}} S(t) \tag{24}
\end{equation*}
$$

with $x$ and $y$ being the usual scalling variables.
The $S(t)$ reads
$S(t)=\frac{1}{\pi} \int \frac{d z_{2}}{\left(R_{z}\right)^{1 / 2}} S\left(t, z_{1}, z_{2}\right)=t A_{1}(t) S_{1}(t)+A_{2}(t) S_{2}(t)+$

$$
\begin{equation*}
+A_{3}(t) S_{3}(t) \tag{25}
\end{equation*}
$$

The functions $S_{i}(t)$ are expressed in terms of invariant variables $\mathbf{X}=S \cdot(1-\mathbf{y}), \mathbf{Y}=S \cdot \mathbf{x} \cdot \mathbf{y}$ and $t$ :

$$
\begin{aligned}
S_{1}(t) & =\left\{\frac{1}{\left(C_{2}\right)^{1 / 2}}\left[\frac{t_{Y}}{2}+\frac{t Y}{t_{Y}}\right]-m^{2} t \frac{B_{2}}{\left(C_{Z}\right)^{3 / 2}}\right\}- \\
& -\{S \longleftrightarrow-X\}+\frac{1}{\left(\lambda_{Y}\right)^{1 / 2}}
\end{aligned}
$$

$$
\begin{align*}
S_{2}(t) & =\left\{\frac{1}{\left(C_{2}\right)^{1 / 2}}\left[M^{2}(t+Y)-X t\right]+\frac{1}{t_{Y}\left(C_{2}\right)^{1 / 2}} \cdot\left[t\left[S(S-t)+X(X+t)-2 M^{2} t\right]+\right.\right. \\
& \left.\left.+4 m^{2} S X\right]-2 m^{2} \frac{B_{2}}{\left(C_{2}\right)^{3 / 2}}\left[S(S-t)-M^{2} t\right]\right\}- \\
& \left.-\{S \longleftrightarrow-X\}-\frac{2 M^{2}}{(\lambda}\right)^{1 / 2} \\
S_{9}(t) & =t\left\{(2 S \rightarrow Y-t) \frac{1}{\left(C_{2}\right)^{1 / 2}}+\frac{2}{t_{Y}\left(C_{2}\right)^{1 / 2}} Y(S+X)-2 m^{2}(2 S-t) \cdot \frac{B_{z}}{\left(C_{2}\right)^{3 / 2}}\right\}+ \\
& +\{S \longleftrightarrow-X\} . \tag{28}
\end{align*}
$$

The next step is the integration over the invariant $t$ in limits (11). Inside this region the integrand. $S(t) / t^{2}$ develops two very sharp peaks located at $t=t_{1}$ and $t=t_{2}$ which correspond to zeros of $C_{1}$ and $C_{2}$ in the lepton zero mass limit (the a- and, p-peaks in the Mo and Tsai's terminology [4]). The third peak is near $t=t_{m i n}$ (the so-called t-peak). These peaks correspond to the situation when most of the photons are radiated in the directions of the inaident or scattered lepton, and in the direction of the virtual photon or $Z$ bozon, respectively. Behaviour of peaks of the integrand in (24) is presented in Fig.2. The main contribution to the cross section (24) comes from the vicinities of points $t_{1}, t_{2}$ and $t_{m i n}$. To remove the collinear singularities at points $t_{1}$ and $t_{2}$ in the integrand, let us carry out the trivial identical tranaformation:

$$
\begin{align*}
& \int_{L_{\min }}^{t \max } d t A_{1}(t) \frac{S_{1}(t)}{t}=\int_{L_{\min }}^{Y} d t\left[A_{1}(t)-A_{1}\left(t_{1}\right)\right] \frac{S_{1}(t)}{t}+ \\
& \\
& +A_{1}\left(t_{1}\right) \int_{L_{\min }}^{Y} d t \frac{S_{1}(t)}{t}+  \tag{29}\\
& \quad+\int_{\dot{Y}}^{1 \max } d t\left[A_{1}(t)-A_{1}\left(t_{2}\right)\right] \frac{S_{1}(t)}{t}+A_{1}\left(t_{2}\right) \int_{Y}^{t \max } d t \frac{S_{1}(t)}{t},
\end{align*}
$$

inequality (14) being essentially used here.
The structure of this transformation of expression (29) is such that
the differences made of $A_{1}$ have zero instead of peaks at $t_{1}$ and $t_{2}$ and all of the singularities are entirely in the terms with factorized $A_{1}\left(t_{1}\right)$ and $A_{2}\left(t_{2}\right)$. Analogously, we proceed for the rest of terms in equation (24):

$$
\frac{S_{2}(t) A_{2}(t)}{t^{2}} \text { and } S_{3}^{\prime}(t) A_{3}^{\prime}(t)
$$

where $S_{3}^{\prime}(t)=S_{3}(t) / t$ and $A_{3}^{\prime}(t)=A_{3}(t) / t$.
After this transformation we arrive at the smooth integrand as is illustrated in Fig. 3.

Using transformation (29) we derived from (24) the following expression for the inclusive cross section of the ERT:

$$
\begin{aligned}
\frac{d^{2} \alpha^{E R T}}{d x d y}= & 2 \alpha^{3} y\left\{\int_{t_{\min }}^{Y} d t\left[\sum_{i=1}^{3}\left[A_{i}(t)-A_{i}\left(t_{1}\right)\right] S_{i}^{\prime}(t)\right]+\right. \\
& +\int_{Y}^{t} d t\left[\sum_{i=1}^{3}\left[A_{i}(t)-A_{i}\left(t_{2}\right)\right] S_{i}^{\prime}(t)\right]+ \\
& \left.\left.+\sum_{i=1}^{3} A_{i}\left(t_{1}\right) \int_{t_{\min }}^{Y} d t \cdot S_{i}^{\prime}(t)+\sum_{i=1}^{3} A_{i}\left(t_{2}\right)\right]_{Y}^{\max } d t S_{i}^{\prime}(t)\right\}
\end{aligned}
$$

with $S_{1}^{\prime}(t)=\frac{S_{1}(t)}{t}, S_{2}^{\prime}(t)=\frac{S_{2}(t)}{t^{2}}$, and for $A_{3}^{\prime}(t)$ here we use again the notation $A_{3}(t)$.

So, our calculations make use of the numerical (first two items) and analytical (the last items) integration methods. It is possible to rewrite the last items as follows:

$$
\begin{align*}
& \int_{i_{\text {mir }}}^{Y} d t \cdot S_{i}^{\prime}(t)=\sum_{j=1}^{19} C_{i j} I_{j},  \tag{31}\\
& \int_{Y}^{t}{ }^{\max } d t S_{i}^{\prime}(t)=\sum_{j=1}^{10} C_{i j} \bar{I}_{j}, \tag{32}
\end{align*}
$$

where $I_{j}$ and $\bar{I}_{j}$ are explicitly calculated by our (from (26)-(28)) integrals over $t$ in the intervals $\left[t_{\text {min }}, Y\right.$ ] and $\left[Y, t_{\text {max }}\right.$ ], respectively (see Appendix A).

## 4. NUMERICAL RESULTS

Using final formulae (30)-(32), we have performed the numerical calculation of the inclusive cross section of the reaction

$$
\begin{equation*}
e+p \longrightarrow e+\gamma+p \tag{33}
\end{equation*}
$$

at $S=10^{5} \mathrm{GeV}^{2}$. The results are depicted in Fig. 4 .
For the proton and neutron elastic form factors $F_{1,2}^{p, n}$ we have used there the experimental fit [12], and for the axial form factor $F_{9}^{a}-$ a simple formulae from [11]. As input values for the parameters, we have used $M_{z}=93 \mathrm{GeV}, \sin ^{2} \theta_{y}=0.23$.

As the dominant contribution to integral (24) comes from diagrams with $\gamma$-exchange, the result is practically independent of the contribution diagrams with Z-exchange to KIRT. It is less than one percent from the full integral (24).

At last, in Fig. 5 for the sake of illuatrations we present the ratio

$$
\begin{equation*}
\delta^{E R T}(x, y)=\frac{d^{2} \sigma^{E R T}}{d x d y} / \frac{d^{2} \sigma_{o}}{d x d y}, \tag{34}
\end{equation*}
$$

where $d^{2} \sigma_{o} / d x d y$ is the Born cross section of the main reaction (1) calculated within the quark-parton model with quark distributions from ref.[13].

## 5. CONCLUSION

In this paper, we have calculated the contribution of the ERT to the deep inelastic ep $\rightarrow$ eX scattering at HERA energies. The simple procedure to smooth integrands is suggested, which is very efficient for ultrahigh energy and light mass of the radiating particle ( very mall mo $\mathrm{m}_{0}^{2} / \mathrm{S} \sim 10^{-12}$ ). We would like to emphasize that this procedure differs essentially from peaking [4], and collinear [14] approximations combining analytic and numerical methods using only the ultrarelativistic approximation, i.e. neglecting terms vanishing with $\mathrm{m}_{\dot{\theta}}^{2} / \mathrm{S}$.

Into our calculations, we included besides $\gamma$-exchange, the

Z-exchange too, in spite of its negligible contribution, because we intend to use this procedure and some of formulae for calculating the contribution from the inelastic radiative tail where the $Z$-exchange is expected to be very important. This work is in progress now.
We wrote FORTRAN codes realising formulae of this paper and computed with their aid numerical results for ERT at HERA energies and, in particular, at mall $x$ where it is important.

## 6. APPENDI X

The expressions for integrals' $I_{j}, \bar{I}_{j}$ and coefficients $C_{i}$; of (31) and (32) are presented here.
The firat column of table 1 correaponds to integrand, and the next two colums results after integration in the intervals [ $t_{m i n} . Y$ ] and [Y, $\mathrm{t}_{\text {max }}$ ].

Table 1. Integrals $I_{j}$ and $\bar{I}_{j}$
( $S_{Y} \equiv S-Y, X_{Y} \equiv X+Y$ )

|  |  | $t_{\text {min }} \longrightarrow Y$ | $\mathbf{Y} \longrightarrow \mathrm{t}_{\text {max }}$ |
| :---: | :---: | :---: | :---: |
| 1 | $\frac{1}{\left(C_{1}\right)^{1 / 2}}$ | $\frac{1}{S_{Y}} \ln \frac{V_{2} S_{Y} Y}{m^{2} W^{2}\left(t_{\max }-t_{1}\right)}$ | $\frac{1}{S_{Y}} \ln \frac{S_{Y}\left(t_{\max }-t_{i}\right)}{V_{2} Y}$ |
| 2 | $\frac{1}{\left(C_{2}\right)^{1 / 2}}$ | $\frac{1}{X_{Y}} \ln \frac{\left(S X-Y M^{2}\right) V_{2}}{X_{Y} W^{2}\left(t_{\max }-t_{2}\right)}$ | $\frac{1}{X_{Y}} \ln \frac{X_{Y}^{9}\left(t_{\max }-t_{2}\right)}{m_{2}^{2} V\left(S X-Y M^{2}\right)}$ |
| 3 | $\frac{1}{t\left(C_{1}\right)^{1 / 2}}$ | $\frac{1}{X Y} \ln \frac{X^{2} Y S_{Y}\left(t_{1}-t_{\text {min }}\right)}{m^{2} V_{2} t_{m i n}\left(S X-Y M^{2}\right)}$ | $\frac{1}{X Y} \ln \frac{\left(S X-Y M^{2}\right) V_{2} t_{\text {min }}}{} \mathrm{YM}^{2} S_{Y}\left(t_{1}-t_{\text {min }}\right) \quad$ |
| 4 | $\frac{1}{t\left(C_{2}\right)^{1 / 2}}$ | $\frac{1}{S Y} \ln \frac{X_{Y}\left(t_{2}-t_{\text {min }}\right)}{t_{\text {min }} V_{2}}$ | $\frac{1}{S Y} \ln \frac{s^{2} t_{\min } V_{2}}{m^{2} M^{2} X_{Y}\left(t_{2}-t_{\min }\right)}$ |
| 5 | $\frac{1}{t^{2}\left(C_{1}\right)^{1 / 2}}$ | $\frac{1}{X Y}\left(\frac{-X-2 V_{2}}{X Y}+\frac{1}{t_{\text {min }}}\right)+\frac{1}{t_{1}} I_{3}$ | $\frac{1}{X Y}\left(\frac{1}{t_{\max }}-\frac{1}{Y}\right)+\frac{1}{t_{1}} I_{3}$ |
| 6 | $\frac{1}{t^{2}\left(C_{2}\right)^{1 / 2}}$ | $\frac{1}{S Y}\left(\frac{1}{t_{\text {min }}}-\frac{1}{Y}\right)+\frac{1}{t_{2}} I_{4}$ | $\frac{1}{S Y}\left(\frac{1}{t_{\max }}+\frac{2 V_{2}-S}{S Y}\right)+\frac{1}{t_{2}} \bar{I}$ |


| 7 | $m^{2} \frac{B_{1}}{\left(C_{1}\right)^{3 / 2}}$ | $\frac{1}{V_{z}}$ | 0 |
| :---: | :---: | :---: | :---: |
| 8 | $\mathrm{m}^{2} \cdot \frac{B_{2}}{\left(\mathrm{C}_{2}\right)^{3 / z}}$ | 0 | $\frac{1}{v_{2}}$ |
| 9 | $m^{2} \frac{B_{1}}{t\left(C_{1}\right)^{3 / 2}}$ | $\frac{1}{t_{1} v_{2}}$ | 0 |
| 10 | $\mathrm{m}^{2} \frac{B_{z}}{t\left(C_{2}\right)^{3 / 2}}$ | 0 | $\frac{1}{t_{2} V_{2}}$ |
| 11 | $\mathrm{m}^{2} \frac{\mathrm{~B}_{1}}{t^{2}\left(\mathrm{C}_{1}\right)^{3 / 2}}$ | $\frac{m^{2}\left(X S^{\prime}-2 M^{2} Y\right)}{X^{3} Y^{2} t_{\text {min }}}+\frac{1}{t_{1}^{2} V_{2}}$ | 0 |
| 12 | $m^{2} \frac{B_{2}}{t^{2}\left(C_{2}\right)^{3 / 2}}$ | $\frac{\mathrm{m}^{2}\left(S S_{x}+2 M^{2} Y\right)}{S^{3} Y^{2} t_{\text {min }}}$ | $\frac{1}{t_{2}^{2} v_{2}}$ |
| 13 | $\begin{aligned} & \frac{1}{t_{Y}}\left[\frac{1}{\left(C_{2}\right)^{1 / 2}}\right. \\ & \left.-\frac{1}{\left(C_{1}\right)^{1 / 2}}\right] \end{aligned}$ | $\frac{1}{Y V_{2}}$ |  |
| 14 | $\frac{t}{\left(C_{1}\right)^{1 / 2}}$ | $\frac{\left(t_{\text {min }}-2 t_{1}+Y\right)}{S_{Y}}+t_{1} \cdot I_{1}$ | $\frac{\left(t_{\max }-Y\right)}{S_{Y}}+t_{1} \cdot \bar{I}_{1}$ |
| 15 | $\frac{t}{\left(C_{2}\right)^{1 / 2}}$ | $\frac{\left(t_{\min }-Y\right)}{X_{Y}}+t_{2} \cdot I_{2}$ | $\frac{\left(t_{\max }-2 t_{2}+Y\right)}{X_{Y}}+t_{2} \bar{I}_{z}$ |
| 16 | $\mathrm{m}^{2} \frac{t B_{1}}{\left(C_{1}\right)^{3 / 2}}$ | $\frac{t_{1}}{v_{2}}$ | 0 |
| 17 | $m^{2} \frac{t B_{2}}{\left(C_{2}\right)^{3 / 2}}$ | 0 | $\frac{t_{2}}{v_{2}}$ |


| 18 | $\frac{1}{t}$ | $\ln \frac{Y}{t_{\text {min }}}$ | $\ln \frac{t_{\text {max }}}{Y}$ |
| :---: | :---: | :---: | :---: |
| 19 | $\frac{1}{t^{2}}$ | $\frac{1}{t_{\text {min }}}-\frac{1}{Y}$ | $\frac{1}{Y}-\frac{1}{t_{\text {max }}}$ |

2-Table 2. Coefficiente $C_{i j}$

| ${ }_{j}{ }^{i}$ | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| 1 | -1/2 | 0 | -Y-2X |
| 2 | 1/2 | 0 | -Y+2S |
| 3 | Y/2 | $-M^{2}-S+\left(X^{2}+S^{2}\right) / Y$ | 0 |
| 4 | -Y/2 | $M^{2}-\mathrm{X}-\left(\mathrm{X}^{2}+\mathrm{S}^{2}\right) / Y$ | 0 |
| 5 | 0 | $-Y M^{2}+4 m^{2} S X / Y$ | 0 |
| 6 | 0 | YM ${ }^{2}-4 m^{2} S X / Y$ | 0 |
| 7 | -1 | 0 | -4X |
| 8 | -1 | 0 | -4S |
| 9 | 0 | $-2\left(X-M^{2}\right)$ | 0 * |
| 10 | 0 | $2\left(S+M^{2}\right)$ | 0 |
| 11 | 0 | $-2 \mathrm{X}^{2}$ | 0 |
| 12 | 0 | $-2 s^{2}$ | 0 |
| 13 | Y | $-S_{x}-2 M^{2}+\left(X^{2}+S^{2}\right) / Y$ | $2(X+S) / Y$ |
| 14 | 0 | 0 | -1 |
| 15 | 0 | 0 | -1 |
| 16 | 0 | 0 | -2 |
| 17 | 0 | 0 | 2 |
| 18 | $\lambda^{-1 / 2}$ | 0 | 0 |
| 19 | 1. | $-2 M^{2} \lambda_{Y}^{-1 / 2}$ | 0 |

INTEGRAND OF T-INTEGRATION


Fig.2. Integrands (in arbitrary units) in Eg.(24) for $e+p \longrightarrow e+r+p$ at $S=10^{5} \mathrm{GeV}^{2} ; x=0.01$.


Fig.3. Integrands (in arbitrary units) in Eg.(30) for $\mathbf{e}+\mathbf{p} \longrightarrow \mathbf{e}+\gamma+\mathbf{p}$ at $\mathrm{S}=10^{5} \mathrm{GeV}^{2} ; \mathbf{x}=0.01$.


## RRFERENCES

1 Study on the proton-electron storage ring project HERA, ECFA 80/42 (1980);

HERA Proposal report DESY-HERA 81/10 (1981).
2 Proc. of the DESY Theory Workshop on Physics at HERA, Hamburg, 12-14 Oct. (1987).
3. D. Yu.Bardin, C.Burdik, P.Ch.Christova, T.Riemann, Berlin-Zeuthen prepr. PHE 88-15 (1988), Z.Phys.C 42 (1989) 1517. D.Yu.Bardin, C.Burdik, P.Ch.Christova, T.Riemann, JINR, E2-89-145, Dubna, (1989), Z.Phys.C 44 (1989) 677.
4. L_W.Mo and Y.S.Tsai, Rev.Mod.Phys., 41 (1969) 205; Y.S.Tsai, SLAG-PUB-848 (1971).

5a. A.A.Akhundov, D.Yu.Bardin, N.M.Shumeiko, JINR, E2-10147, Dubna, (1976).
b. A.A.Akhundov, D.Yu.Bardin, N.M.Shumeiko, JINR, E2-10205, Dubna, (1976); Sov.J.Nucl.Phys., 26 (1977) 660.
c. A.A.Akhundov, D.Yu.Bardin, N.M.Shumeiko, Sov.J.Nucl.Phys., 44 (1986) 988 .
6. E.A.Kuraev, V.S.Fadin, N.P.Merenkov, Yad. Fiz. 47 (1988) 1593; N.P.Merenkov, Yad.Fiz. 48 (1988) 1782.

7 G.L.Kotkin, S.I.Polityko, V.G.Serbo, Yad. Fiz. 42 (1985) 692, 925; G.L.Kotkin et al., Z.Phys. C 39 (1988) 61.

8 D.Barber et al., Proc. XIII Int. Conf. on High Energy Accelerators, Novosibirsk (1986); Nauka (1987), v.2, p-72.
9. R.J.F.Gaemers, M.van der Horst, NIKHEF-H188-13 (1988).
10. T.V.Kuchto, N.M.Shumeiko, T.Z.Nguen, Voprosy atomnoi nauki i tekhniki, S:Obshaj i jdernaj phizika, v2.(38) (1987) 39 .
11 S.M.Bilenky, Lectures on physics of neutrino and lepton-nucleon processes, M., Energoizdat (1981).
12 S.I.Bilenkaya et al., JETF Letters, 19 (1974) 613.
13. D.W.Duke and J.F.Owens, Phys.Rev. D30 (1984) 49.
14. J.Kripfgans and H.J.Moehring, Z.Phys. C 38 (1986) 653.

## Ахундов А.А. и др.

E2-89-405
Вклад радиационного хвоста от упругого пика
в глубоконеупругое рассеяние на ускорителе HERA
С использованием простой процедуры "сглаживания". подынтегральной функции, получены обобщенные формулы для вклада упругого радиационного хвоста (с учетом обмена фотоном и Z-бозоном) в измеряемое сечение глубоконеупругого ер $\rightarrow \mathrm{eX}$ рассеяния. Приведены результаты численных расчетов в кинематической области мальх $x$ при энергиях ускорителя HERA.

Работа выполнена в Лаборатории теоретической физики Оияи.

Препринт Объединенного института яцерных исследований. Дубна 1989

## Akhundov A.A. et al. <br> Contribution of the Radiative Tail <br> E2-89-405 <br> from the Elastic Peak to Deep Inelastic Scattering at HERA

The formulae for the contribution of the elastic radiative tail (photon- and Z-bozon exchange are included) to the measured cross section of deep inelastic ep $\rightarrow$ eX scattering are derived, using a simple procedure of smoothing the integrand. The numerical results in the kinematical region of small $x$ at HERA energies are presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Received by Publishing Department on June 6, 1989.

