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ON RENORMALIZATION OF AXIAL ANOMALY

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Recently, Anselm and Ioganeen /1/ have shown that the well-known old statement /2/ about nonrenormalizability of the axial anomaly /3/ is broken in a three-loop approximation. In this paper, we shall verify the result of /1/ for the QCD and shall correct it for a non-Abelian gauge theory (QCD). That correction however, is necessary for validity of the Adler--Bardeen theorem.

More than a year ago the evolution of the quark and gluon axial currents was investigated in connection with the so-called "spin-crisis" discovered by the EMC /4/.

$$A_\nu = \sum_f \bar{q}_f \gamma_\nu \gamma_5 q_f \quad , K_\nu = -\frac{\alpha N_f}{2\pi} \varepsilon_{\nu\mu\sigma\rho} B^a_\sigma \left(\partial_\rho B^a_\mu - \frac{1}{3} f_{abc} B^b_\mu B^c_\rho \right) \quad (1)$$

We changed the sign and included the flavour number N_f into the definition of K_ν as compared to the standard definition for to stress the conservation of the sum of the current

$$\partial_\nu \left(A_\nu + K_\nu \right) = 0 . \qquad (2)$$

The conservation of the total current $A_\nu + K_\nu$ (2) requires for it to be an eigenvector of evolution equations with a zero eigenvalue. Together with multiplicative renormalization of the axial current required by gauge invariance, $\dot{A}_\nu = -\gamma A_\nu$ this results in evolution equations /5,6,7/

$$\begin{pmatrix} \dot{A}_\nu \\ \dot{K}_\nu \end{pmatrix} = \begin{pmatrix} -\gamma & 0 \\ \gamma & 0 \end{pmatrix} \begin{pmatrix} A_\nu \\ K_\nu \end{pmatrix} , \qquad (3)$$

where the dot means the derivative with respect to the logarithm of an ultraviolet regularization parameter Λ.

Let us stress that this result is obtained without any appeal to perturbation theory.

Since in (3) the renormalization constant and not the operator itself is differentiated, this operation is commutative with the space-time differentiation. The divergence of both the sides of (3) gives immediately for axial anomaly $K = \partial_\nu A_\nu$

$$\dot{K} = -\gamma K, \tag{4}$$

i.e. renormalization of the axial anomaly is a simple consequence of the multiplicative renormalization of the axial current.

Turn now to the realization of constraints due to (3) on perturbation theory diagrams. Consider for this aim the following obvious consequence of (3) and (2)

$$\partial_\nu \dot{K}_\nu = -\gamma \partial_\nu K_\nu \tag{5a}$$

$$\partial_\nu \dot{K}_\nu = \gamma \partial_\nu A_\nu \tag{5b}$$

$$\partial_\nu \dot{A}_\nu = \gamma \partial_\nu K_\nu . \tag{5c}$$

Inspite of the equality of the operators $\partial_\nu A_\nu$ and $-\partial_\nu K_\nu$ it contains different fields. So we will obtain different Feynman diagrams in calculating anomalous dimensions of the operators, e.g. by computing matrix elements of (4) and (5) in free-particle states corresponding to the r.h.s. operators (i.e. quarks for the A's and gluons for the K's).

Compare the expansion of (4,5) in α , one can find that the same contribution comes from different diagrams: the diagrams for (4) and (5a) contain one extra loop and (5c) even two extra loops than diagrams of (5b). This is because K_ν contains an extra power of α . It naturally explains why $\gamma \sim \alpha^2$ and why anomaly renormalization starts at the three-loop level. For example, all diagrams of Fig. 1 in QED determine the same function $-\frac{3}{4}\left(\frac{\alpha}{\pi}\right)^2 \ln \Lambda^2$. With the two-loop diagram, Fig. 1a, it was calculated even in the pioneering work by Adler /3/, in three loops, Fig. 1c, in recent work /1/ and in one loop, Fig. 1b, in

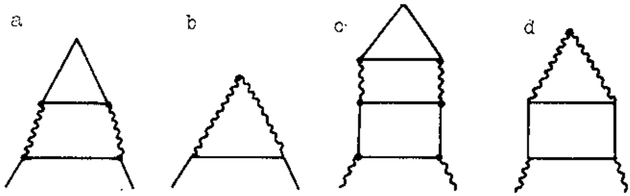

Fig.1

works /5,6/. The latter has been made in QCD which differs by the obvious group factor $T(R)C_2(R)=\frac{2}{3}N_f$. The same factor has to result from calculations of other diagrams of QCD.

From this point of view, an extra contribution in /1/ coming from the self-interaction of gluons in QCD seems erroneous. As it is known (see e.g. /5/), the one-loop contribution to (5a) which has to be absent, is cancelled by evolution of the coupling constant proportional to the β-function.

This cancellation, by all means, underlies supersymmetric properties of QCD which have initiated the investigation /1/. (see /10/).Indeed,the contribution to the gluon-gluon kernel of the Altarelli-Parisi Equation proportional to the β-function guarantees the energy-momentum tensor conservation which is thus connected with the axial current conservation.

Let us discuss now the infrared regularization problem which has played a crucial role in obscuring the renormalizability of the anomaly. The matter is that in the limit $K^2/m^2 \to 0$, where one can disregard the contribution of the light-by-light scattering /2/, the anomaly itself is absent /8,9/. (More strictly, as it can be easily shown, the anomaly contribution is cancelled by the normal term proportional to $2m$).
Both the triangle and box diagrams contain the same function of K^2/m^2 which is responsible for a point-like interaction of photons and gluons in the limit $K^2/m^2 \to \infty$ /8,9/.

Renormalization of the anomaly, however, does not mean breaking of the one-loop character of the Adler-Bardeen relation

$$\partial_\nu A_\nu = \frac{\alpha}{2\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} \qquad (6)$$

but rather a consistent change of the both sides of (6).

Use now the result of /2/ except of the wrong statement about the absence of light-by-light scattering. Then the diagrams determining the radiation corrections to the anomaly have the form of Fig. 2a. However, the contribution of the upper loop coincides when $m \to 0$ with the photon axial current, Fig. 2b. The latter diagram corresponds to Eq.(5a). So, contribution to (5a) the same as to (5c), contains (in all orders) one additional loop (triangle!). This proves the one-loop character of the anomaly in QED.

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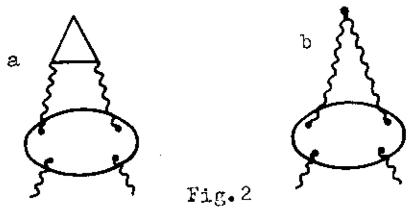

Fig.2

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