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ACCURATE MULTIPLICITY SCALING IN ISOTOPICALLY CONJUGATE REACTIONS

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1. INTRODUCTION

The multiplicity distributions of negative particles (in fact, π^- mesons) in PP interactions at energies from zero and at least up to ISR ones are similar if the concept of similarity is formulated consistently. Then, instead of the asymptotic expression [1]

$$P_n = 1/\langle n\rangle\ \Psi(n/\langle n\rangle) \tag{1}$$

with the normalization conditions

$$\int\Psi(z)dz = 1, \qquad \int z\Psi(z)dz = 1\ , \tag{2}$$

which contradicts the equality $\Sigma P_n=1$ at finite energies (fig.1a), one should use an accurate formula (fig.1b) [2]

$$P_n = \int_n^{n+1} P(m)dm\ , \tag{3}$$

where

$$P(m) = 1/\langle m\rangle\ \Psi(m/\langle m\rangle)\ , \tag{4}$$

with

$$\langle m\rangle = \int mP(m)dm \tag{5}$$

under the same normalization conditions (2).

The formula, which is more general than (3), can be written as a consistent extrapolation of asymptotic expression (1) to finite energies (fig.1c)

$$P_n = \int_{n-\varepsilon}^{n+1-\varepsilon} P(m)dm\ . \tag{6}$$

It differs from (3) in the possibility of shifting the scale grid, which cuts the KNO invariant function P(m) into partial probabilities P_n. The shift ε must satisfy the following conditions: $\varepsilon\geq 0$ in order that P_{-1} may not appear and $\varepsilon<1$ if we want that P_0 should not be equal to zero.

Note, however, that, e.g. for the reaction $PP\to n_{ch}$, where $P_0=P_1=0$, ε satisfies the condition $2\leq\varepsilon<4$ and expression (6) should be rewritten as

$$P_n^{(PP\to n_{ch})} = \int_{n-\varepsilon}^{n+2-\varepsilon} P(m)dm\ . \tag{7}$$

Here the step of scale grid equals 2 as all odd probabilities are equal to zero due to charge conservation. At $\varepsilon= 2$ the result

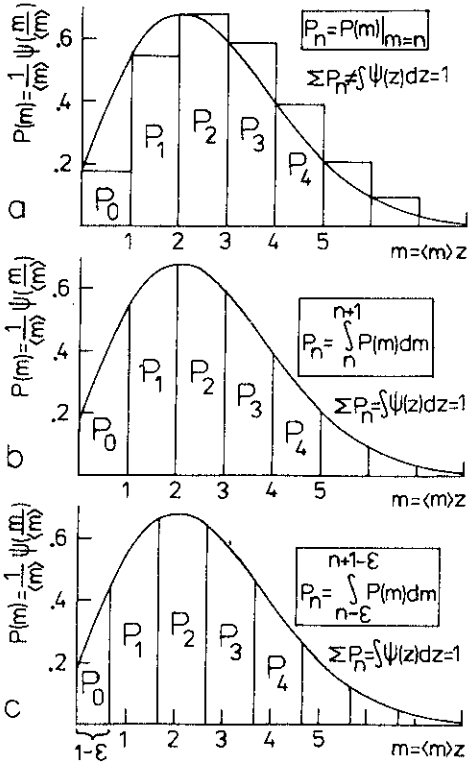

Fig.1. Obtaining of the discrete multiplicity distribution from the continuous normalized universal function $\Psi(z)$: (a)- according to the commonly used recipe $P_n=1/\langle n\rangle\ \Psi(n/\langle n\rangle)$ [1], then $\Sigma P_n \neq 1$; (b)- according to the accurate recipe [2]; (c)- according to the generalized accurate recipe which differs from the previous one in the possibility of shifting the scale grid which cuts the function P(m) into probabilities P_n.

coincides with the one of eq. (3) applied to PP→ n_{neg} ($n_{ch}=2n_{neg}+2$).

Such generalization (6) turns out to allow the description of the multiplicity distributions of π^+ and π^- mesons for PP, NP and NN interactions with the same function $\Psi(z)$ and energy dependence

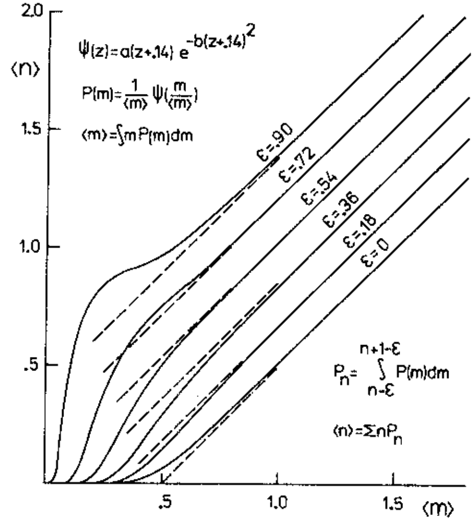

Fig.2. Dependence of the average value of the discrete distribution P_n on the one of the corresponding continuous distribution $P(m)$ for various shifts ε. The dashed lines correspond to the approximation $\langle m\rangle = \langle n\rangle+.5-\varepsilon$ (see (8)).

of $\langle m\rangle$. From charge symmetry it follows that the distributions in the PP, NP, NN→ π^+ reactions are identical to NN, PN, PP→ π^-, respectively. Therefore, 2 other parameters ε are needed to describe all these reactions (ε=0 for PP→ π^- [2]).

We have used for comparison the experimental data: PP→n_{neg} ($n_{neg}=(n_{ch}-2)/2$) at P_{lab}=1.5÷2000 GeV/c (see refs. in [2]); NP→n_{neg} ($n_{neg}=(n_{ch}-1)/2$) at 1.25÷400 GeV/c [3 - 13]; NN→n_{neg} ($n_{neg}=n_{ch}/2$) at 6.1 GeV/c [14]; PP→π^+ at 1÷69 GeV/c [15 - 21].

2. APPROXIMATE CONSEQUENCES

For not very small $\langle m\rangle$, from (5) and (6) one can obtain ($\langle n\rangle = \Sigma nP_n$):

$$\langle m\rangle = \int mP(m)dm = \sum_n \int_{n-\varepsilon}^{n+1-\varepsilon} mP(m)dm \approx \sum_n (n+.5-\varepsilon) \int_{n-\varepsilon}^{n+1-\varepsilon} P(m)dm =$$

$$= \sum_n (n+.5-\varepsilon)P_n = \langle n\rangle+.5-\varepsilon \ . \qquad (8)$$

As it follows from fig.2, this equality (dashed lines) works well already from $\langle m\rangle \geq .7 \div 1$ which corresponds, as seen below, to $P_{lab} = 3 \div 5$ GeV/c for nucleon-nucleon interactions.

The curves of fig.2 are obtained by formulae (4)-(6) with the function

$$\Psi(z) = a(z+.14)\, e^{-b(z+.14)^2} , \tag{9}$$

where a and b obtained from (2) are equal to 1.251 and .618, respectively. The PP→ π^- data are well described by this function [2]; in addition, the curves in fig.2 are nearly independent of the used function.

From formulae (4), (6) and (8) we get an approximate modified KNO scaling [22, 23]

$$P_n = \int_{n-\varepsilon}^{n+1-\varepsilon} P(m)dm \approx P(m)\big|_{m=n+.5-\varepsilon} = \frac{1}{\langle m\rangle}\, \Psi\left(\frac{n+.5-\varepsilon}{\langle m\rangle}\right) \approx$$

$$\approx \frac{1}{\langle n\rangle+.5-\varepsilon}\, \Psi\left(\frac{n+.5-\varepsilon}{\langle n\rangle+.5-\varepsilon}\right) . \tag{10}$$

The fitting of this expression for the NP data at 19÷300 GeV/c, performed in [6], yields $\varepsilon = .31 \pm .02$ (for negative particles in our designations).

For the central moments: $\mu_q = \int (m-\langle m\rangle)^q P(m)dm$ (continuous function P(m)) and $D_q^q = \sum (n-\langle n\rangle)^q P_n$ (discrete one P_n) one can obtain the following approximate equality

$$\mu_q = \sum_n \int_{n-\varepsilon}^{n+1-\varepsilon} (m-\langle m\rangle)^q P(m)dm \approx \sum_n (n+.5-\varepsilon-\langle n\rangle-.5+\varepsilon)^q \int_{n-\varepsilon}^{n+1-\varepsilon} P(m)dm =$$

$$= \sum_n (n-\langle n\rangle)^q P_n = D_q^q , \tag{11}$$

which results in an approximate proportionality for the discrete multiplicity distributions [22, 23]

$$D_q \propto (\langle n\rangle+.5-\varepsilon) , \tag{12}$$

taking into account (8) and the known proportionality for continuous KNO invariant functions: $\mu_q^{1/q} \propto \langle m\rangle$. The fitting of (12) (for D_2) for the NP data at 28÷400 GeV/c, conduct in [8], yields $\varepsilon = .33 \pm .015$ (for negative particles in our designations).

Comparing eqs. (3) and (6) (or figs.1a and 1b), one can obtain an approximate expression for the multiplicity distributions with $\varepsilon\neq0$ through the distribution with the same P(m) and $\varepsilon=0$

$$P_n^{(\varepsilon)} \approx \varepsilon P_{n-1}^{(o)} + (1-\varepsilon)P_n^{(o)} . \tag{13}$$

The same ratio between the multiplicity distributions for PP and NP interactions at equal energy has been obtained empirically in paper [9]. In our designations it looks like

$$P_n^{(NP\to n_{neg})} \approx \varepsilon P_{n-1}^{(PP\to n_{neg})} + (1-\varepsilon)P_n^{(PP\to n_{neg})} . \tag{14}$$

The parameter $\varepsilon= .36\pm.03$ for this ratio is obtained in paper [13] by fitting it for the data at $P_{lab}= 100\div400$ GeV/c. Further we use this value of ε for $NP\to n_{neg}$.

3. NORMALIZATIONS

It is customary to normalize multiplicity distributions to the inelastic cross section $P_n=\sigma_n/\sigma_{in}$. However, such normalization becomes not quite natural at very low energies. For example, the average multiplicity of π^+ mesons in PP interactions normalized to σ_{in} does not approach zero with decreasing energy and passes through a minimum ($\simeq.8$) at $P_{lab}\simeq 2$ GeV/c and even increases with a further decrease of energy down to threshold (even though one meson should be produced in order to realize inelastic collision). In some multiple production models, e.g. a statistical one [24], the part of the elastic cross section passing through the "intermediate state" enters into the multiplicity distribution in an equal in rights manner: the production probabilities of 0, 1, 2, ...n π mesons are calculated by a general formula.

In this paper the multiplicity distributions at very low energies are normalized to 28 mbarns. This is the value of σ_{in} in PP interactions at $P_{lab}=2.8\div6.6$ GeV/c, and the inelastic cross section decreases drastically with a further decrease of energy (elastic scattering begins dominating for the same impact parameters). It should be noted that it will be essential only for the conclusions of section 5.

The cross section of one-prong inelastic events (σ_0 in $NP \to \pi^-$) is often not measured in NP interactions at high energies because of experimental difficulties. The authors usually assume that it is equal to .6÷.67 of σ_0 in PP interactions following expression (14). However, the calculation by formulae (3), (6) and (9) with ε=.36 leads to $P_0^{(NP\to n_{neg})} = (.53\pm.01)\ P_0^{(PP\to n_{neg})}$ for the interval of multiplicities corresponding to energies of 12÷400 GeV/c. We are going to use this value for lack of an experimental one, the more so it is as a rule kept within the error presented by the authors.

4. COMPARISON WITH EXPERIMENT

Formula (6) can be presented in an integral form

$$\sum_n^{\infty} P_k = \int_{n-\varepsilon}^{\infty} P(m)dm = \int_{(n-\varepsilon)/\langle m\rangle}^{\infty} \Psi(z)dz = \Phi\left(\frac{n-\varepsilon}{\langle m\rangle}\right) , \qquad (15)$$

where $\Phi(z)=\int_z^{\infty}\Psi(z)dz$ is a universal function as $\Psi(z)$, which is independent of the energy and charge of interacting nucleons and the sign of charged π mesons normalized by the conditions arising from (2)

$$\Phi(0)= 1 \ ; \qquad \int\Phi(z)dz= 1 \ . \qquad (16)$$

The partial probabilities are expressed through this function simpler than through $\Psi(z)$

$$P_n= \Phi\left(\frac{n-\varepsilon}{\langle m\rangle}\right) - \Phi\left(\frac{n+1-\varepsilon}{\langle m\rangle}\right) \ . \qquad (17)$$

For the function $\Psi(z)$ (9)

$$\Phi(z)= \frac{a}{2b}\, e^{-b(z+.14)^2} \ . \qquad (18)$$

Unlike $\Psi(z)$, the function $\Phi(z)$ allows one to plot the multiplicity distributions (integral: $\sum_n^{\infty} P_k$) for a variety of energies and reactions on one curve according to (15) as shown in fig.3. For $NP\to n_{neg}$ ε=.36 and for $PP\to\pi^+$ $(NN\to n_{neg})$ ε=.72.

At very low energies (when only P_0 and P_1 are not equal to zero) these distributions satisfy the scaling (6) automatically and the points lie exactly on the curve independently of the used function $\Psi(z)$ and parameter ε. This is clear from fig.1c where one can always choose such a scale that the function $\Psi(z)$ is divided into areas which are equal to given P_0 and P_1. Therefore, these points are not presented in fig. 3.

Figure 4 presents a more sensitive comparison, namely the comparison of the ratios of the moments of the distributions for $NP \to \pi^-$ which should go fastly to the plateau with increasing $\langle n \rangle$

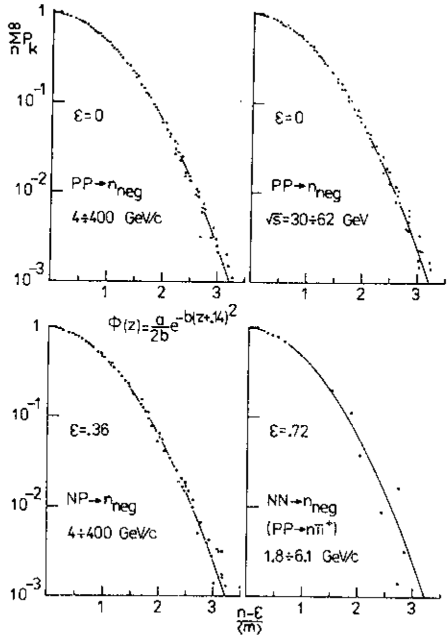

Fig. 3. Dependence of the integral probability $\sum\limits_{n}^{\infty} P_k$ on $(n-\varepsilon)/\langle m \rangle$ for various nucleon-nucleon reactions (see (15)-(18)).

(according to (12)). Such ratios for $PP \to n_{neg}$ are given in [2]. Figure 5 shows $f_2 = D_2^2 - \langle n \rangle$ versus $\langle n \rangle$. The curves in figs. 4 and 5 are obtained by eqs. (4), (6) and (9).

5. ENERGY DEPENDENCE

Note that only the validity of the scaling of multiplicity distributions allows us to say how "the number of produced particles" increases with energy without a detailed description of

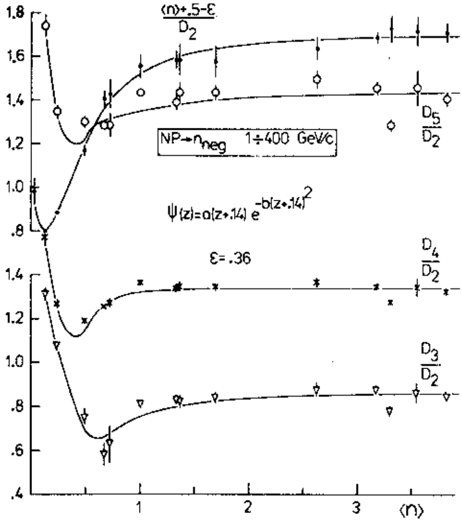

Fig. 4. Ratios which should go fastly to the plateau with increasing energy (according to (12)). The curves are obtained by eqs. (4)-(6) with $\Psi(z)$ presented in the figure. The coefficients a and b calculated from (2) are equal to 1.251 and .618, respectively.

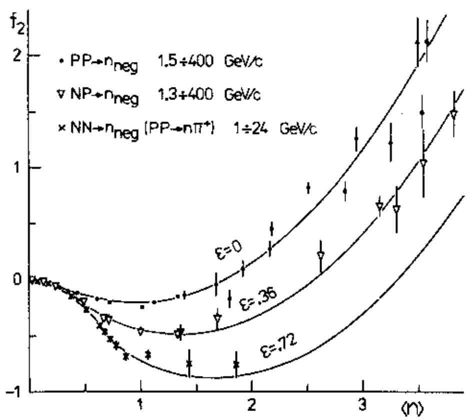

Fig. 5. Correlation function $f_2 = D_2^2 - \langle n \rangle$ versus $\langle n \rangle$. The curves are obtained as in fig. 4.

the distribution for each energy. If the scaling were not valid, the modal value of multiplicity could increase, e.g. as ln s, and the average value as $s^{1/4}$.

The scale parameter $\langle m\rangle(s)$ in formula (4) determines the stretching factor for the "unit" distribution $\Psi(z)$ to obtain the desired multiplicity distribution. Therefore, it is this parameter that is a natural, linear characteristic of the number of particles produced at a given energy. In the asymptotic formula (1), $\langle n\rangle(s)$ is such a scale parameter which coincides with $\langle m\rangle$ at asymptotic energies.

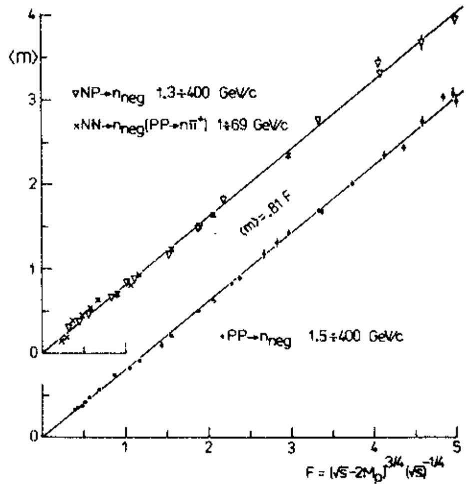

Fig.6. Energy dependence of $\langle m\rangle$ for various nucleon-nucleon reactions. In the Fermi thermodynamic model the multiplicity of π mesons is proportional to the quantity F.

Figure 6 presents the $\langle m\rangle$ dependence (obtained from $\langle n\rangle$ according to (8) and fig.2) on the quantity

$$F= (\sqrt{s}-2M_p)^{3/4} (\sqrt{s})^{-1/4} , \qquad (19)$$

with M_p the nucleon mass. In the Fermi thermodynamic model the multiplicity of π mesons in nucleon-nucleon interactions over an energy range of 10÷1000 GeV/c should be proportional to this quantity [25]. As seen, the scale parameter $\langle m\rangle$ for all our reactions PP, NP and NN→ π^- (π^+) in the interval of energies from threshold up to 400 GeV/c has the same energy dependence

$$\langle m \rangle = .81\ F\ \mathrm{GeV}^{-1/2}\ . \tag{20}$$

It is surprising that the quantity $s^{1/4}$ is commonly used in lieu of F and the multiplicity of charged particles instead of that of π mesons for comparison with the thermodynamic model although each of these errors is larger than the deviation of the experimental data from the model observed usually by the authors. It is true that in the first paper of Fermi [24] this formula has been also written with some error [26, 27].

The energy dependence of ($\langle n \rangle-\alpha$), the scale parameter of the modified KNO scaling $P_n = 1/(\langle n \rangle-\alpha)\ \Psi((n-\alpha)/(\langle n \rangle-\alpha))$, has been also obtained in paper [28]. The value of $\alpha \simeq +1$ (for all charged particles) allows the authors of [23] and [28] to interpret it as the number of leading particles and ($\langle n \rangle-\alpha$) as the number of actually produced particles. In our case such an interpretation leads to that the number of actually produced negative particles m, e.g. in PP interactions, is by approximately .5 larger than the number of produced particles. However, this could take place at the stage of particles production when they still interact between themselves (see [29] as well).

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