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дубна

E2-89-333

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ANISOTROPIC SPACE-TIME

1989

INTRODUCTION

As is known, in the special theory of relativity the concept of simultaneity of events at different space points or, what is the same, the procedure of synchronization of distant clocks is based on sending light signals. It is commonly supposed that the times of travelling a light signal to a distant event (clock) and back are equal. If these ways are equal, it is equivalent to the assumption (agreement) of the equality of light velocities in any two opposite directions. However, the experiment can agree in a like manner with the supposition of the inequality of the above times and hence with the assumption of the inequality of corresponding velocities.

Probably, Poincare was the first who concentrated his attention on the fact that the synchronization of distant clocks was a fundamental problem. He has shown /1/ that the statement of constancy of the light velocity in different directions is a postulate without which one cannot measure this velocity and that this postulate can never be checked directly by experiment.

The statement that the velocity of light cannot be in principle measured without arbitrary assumptions, and thus we have the right to do arbitrary suggestions of the velocity of light (in opposite directions),can be found in Enstein's paper /2/.

From the philosophical point of view, a conditional, conventional character of the concept of simultaneity is considered in detail in Reichenbachs book /3/ where a numerical measure of this concept, i.e. time parameter, is introduced for the first time.

The transformations for coordinates which take into account explicitly (quantitatively) the conventional character of the definition of simultaneity and thus generalize usual Lorentz transformations were first obtained by W.Edwards in 1963 /4/ and quite independently by the author /5/ three years later. Since a variety of papers devoted to this subject has been published*. A.A.Logunov's monograph /11/ contains a detailed description of these questions.

*References to them can be found, e.g., in /6,7/. Among the recent papers see, in particular, /8-10/.

On the other hand, the concept of distance (space coordinate) in relativity theory is also defined from a radar experiment where the sum of distances, which a light signal covers in the positive and negative directions, is immediately measured. These distances are commonly supposed to be equal. However, the assumption of their inequality does not contradict the experiment. Consequently, it allows one to introduce anisotropic space which is achieved with the aid of spatial "gauge" /12/. In the general case we deal with anisotropic space-time embracing both possibilities /13/. The range of these questions will be considered below within the frame of a Minkowskian plane space. In addition, we are going to discuss the use of "right" and "left" Feynman clocks (made on the basis of parity nonconservation) for practical realization of anisotropic space-time. At the same time we shall discuss distinctive features of anisotropic (noncommutative) geometry.

1. ANISOTROPIC SPACE-TIME

1.1. Time Anisotropy. Lorentz-Edwards Transformations

Thus, the concept of simultaneity in the special theory of relativity is defined with the aid of the following experiment. An observer (at some point A) sends a light signal to B at time t_1; the signal reflected at B returns to A at time t_2. The instant of reflection of the signal is expressed as

$$t_B = t_1 + \epsilon_0 (t_2 - t_1), \qquad (1.0_t)$$

where ϵ_0 is Reichenbach's time parameter ($0 < \epsilon_0 < 1$). The traditional definition of simultaneity corresponds to $\epsilon_0 = 1/2$ which is apparently equivalent to the assumption of the equality of times of light propagation in two opposite directions. This commonly reveals the equality of corresponding velocities. of light. Simplicity and convenience of the traditional definition of simultaneity lies in this fact. However, as t_1 and t_2 are a direct result of the considered experiment, we cannot single out any specific value among $0 < \epsilon_0 < 1$ as corresponding to objective simultaneity. So, it is evident that the approach, in which ϵ_0 not fixed previously can assume any values within $0 < \epsilon_0 < 1$, represents the generalization of the traditional concept of simultaneity. For example, the time of light propagation "forth" is

$$t_{AB} = \epsilon_0 (t_2 - t_1) \quad . \qquad (1.1_t)$$

To obtain Lorentz transformations, making allowance for time anisotropy, from usual special ones, let us rewrite formula (1.0_t) as follows

$$t_B = \frac{t_1 + t_2}{2} + \delta_0 \, \frac{t_2 - t_1}{2} \,, \qquad (1.0_t')$$

where $\delta_0 = 2\epsilon_0 - 1$. Taking into account the fact that in the right part of $(1.0_t')$ the first term is an ordinary time coordinate and $(t_2 - t_1)/2 = x(c=1)$, we are led to the following substitution for time coordinate

$$t \to t - \delta_0 x \,. \qquad (1.2_t)$$

Introducing the velocities of light propagation in the positive $\hat{c}_1$ and negative $\hat{c}_2$ directions

$$\hat{c}_1 = (1+\delta_0)^{-1}, \quad \hat{c}_2 = (1-\delta_0)^{-1} \; (1/2 < \hat{c} < \infty), \qquad (1.3_t)$$

rewrite (1.2_t) in the form

$$t \to t - \frac{1}{2}\left(\frac{1}{\hat{c}_1} - \frac{1}{\hat{c}_2}\right) x \,. \qquad (1.2_t')$$

One can say that we deal with a kind of "gauge" transformation of the time coordinate /12/.

Using (1.2_t) and $(1.2_t')$, we get Lorentz-Edwards transformations

$$x' = [(1 + \delta_0 v)x - vt]\gamma \,,$$
$$t' = [(1 - \delta_0 v)x - (1 - \delta_0^2)vx]\gamma \,, \qquad (1.4_t)$$

or

$$x' = \left\{\left[1 + \frac{1}{2}\left(\frac{1}{\hat{c}_1} - \frac{1}{\hat{c}_2}\right)v\right]x - vt\right\}\gamma \,,$$
$$t' = \left\{\left[1 - \frac{1}{2}\left(\frac{1}{\hat{c}_1} - \frac{1}{\hat{c}_2}\right)v\right]t - \frac{v}{\hat{c}_1\hat{c}_2}x\right\}\gamma \,. \qquad (1.4_t')$$

Here $\gamma = (1 - v^2)^{-1/2}$ and v is the velocity of a material object in the usual definition of simultaneity.

Transformation ($1.4'_t$) keeps the quadratic form

$$\tau^2 = t^2 - \left(\frac{1}{\hat{c}_1} - \frac{1}{\hat{c}_2}\right) tx - \frac{1}{\hat{c}_1 \hat{c}_2} x^2 \qquad (1.5_t)$$

invariant.

For the velocities of motion v_1 (S´relative to S) and v_2 (S relative to S´) or, what is the same, for the velocities of motion of a material object in the positive and negative directions we have

$$\hat{v}_{\frac{1}{2}} = \frac{v}{1 \pm \delta_0 v} \; . \qquad (1.6_t)$$

1.2. Introduction of Space Anisotropy. Transformations for Coordinates

On the other hand, the above experiment can be also used (in accord with the radar method of measuring distances) to define the concept of distance from A to B. As t_1 and t_2 are again a direct result of the discussed experiment or the summary distance ($2X = t_2 - t_1$) covered by a light signal, the ways "forth" (X_{AB}) and "back" (X_{AB}) may be different. For example, for X_{AB} we have

$$X_{AB} = \epsilon_1 (t_2 - t_1) \; . \qquad (1.1_x)$$

Here ϵ_1 is the space parameter ($0 < \epsilon_1 < 1$). The traditional definition of distance corresponds to $\epsilon_1 = 1/2$. In the case of $\epsilon_1 \neq 1/2$ the "right" and "left" distances are different, i.e. we deal with experimentally admissible space anisotroipy*.

To get the "gauge" transformation for spatial coordinate similar to (1.2_t), let us consider the expression defining the space position of point B

$$x_B = -t_1 + \epsilon_1 (t_1 + t_2) = t_2 - (1 - \epsilon_1)(t_1 + t_2) \; . \qquad (1.0_x)$$

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*Earlier this problem was discussed in [14-16] although the change of standard scale was in question in [14].

Introducing $\delta_1 = 2\epsilon_1 - 1$, rewrite (1.0_x) in the form

$$x_B = \frac{t_2 - t_1}{2} + \delta_1 \frac{t_1 + t_2}{2}. \qquad (1.0_x')$$

From analogy between expressions $(1.0_x')$ and $(1.0_t')$, we obtain the following transformation for the spatial coordinate

$$x \to x - \delta_1 t. \qquad (1.2_x)$$

Introducing again the velocities of light propagation in the positive $\tilde{c}_1$ and negative $\tilde{c}_2$ directions

$$\tilde{c}_1 = 1 + \delta_1, \quad \tilde{c}_2 = 1 - \delta_1 \quad (0 < \tilde{c} < 2), \qquad (1.3_x)$$

let us rewrite (2.2) as follows

$$x \to x - \frac{1}{2}(\tilde{c}_1 - \tilde{c}_2)\, t. \qquad (1.2_x')$$

Substituting (1.2_x) and $(1.2_x')$ into the Lorentz formulae, we come to the generalized transformations for coordinates which take into account explicitly admissible space anisotropy[/17/]

$$\begin{aligned} x' &= [(1 - \delta_1 v)\, x - (1 - \delta_1)^2\, vt]\, \gamma, \\ t' &= [(1 + \delta_1 v)\, t - vx]\, \gamma, \end{aligned} \qquad (1.4_x)$$

or

$$\begin{aligned} x' &= \{[1 - \frac{1}{2}(\tilde{c}_1 - \tilde{c}_2)\, v]\, x - v\tilde{c}_1\tilde{c}_2 t\}\, \gamma, \\ t' &= \{[1 + \frac{1}{2}(\tilde{c}_1 - \tilde{c}_2)\, v]\, t - vx\}\, \gamma. \end{aligned} \qquad (1.4_x')$$

In this case for the interval squared we have

$$s^2 = x^2 - (\tilde{c}_1 - \tilde{c}_2)\, xt - \tilde{c}_1\tilde{c}_2 t^2. \qquad (1.5_x)$$

Using (1.4_x), for the velocities of motion of a material object in the positive and negative directions we find

$$\tilde{v}_{\frac{1}{2}} = \frac{v(1-\delta_1^2)}{1 \mp \delta_1 v}. \qquad (1.6_x)$$

The degree of space anisotropy is characterized by the ratio of the "right" and "left" distances which is equivalent to the ratio of the corresponding velocities. In this case for light we have

$$a = \frac{\tilde{c}_1}{\tilde{c}_2} = \frac{1+\delta_1}{1-\delta_1} \qquad (1.7_x)$$

and for material object

$$a^v = \frac{\tilde{v}_1}{\tilde{v}_2} = \frac{1+\delta_1 v}{1-\delta\ v}, \qquad (1.8_x)$$

i.e. the degree of anisotropy is maximum for a light signal and decreases with decreasing velocity. In the nonrelativistic limit of very small v $a^v \to 1$, i.e. the discussed phenomenon is purely relativistic in nature.

As the discussed possibility of anisotropic space is rather unusual, it is worth-while to turn to a geometric picture. Using formula (1.0_x), Figure 1 presents graphically the process of propagation of a light signal in the considered experiment at different values of space parameter δ_1. For simplicity $t_1 = 1s$ (origin of the t-axis) and $t_2 = 3s$. Points A and B correspond to the traditional definition of distance, indices A and

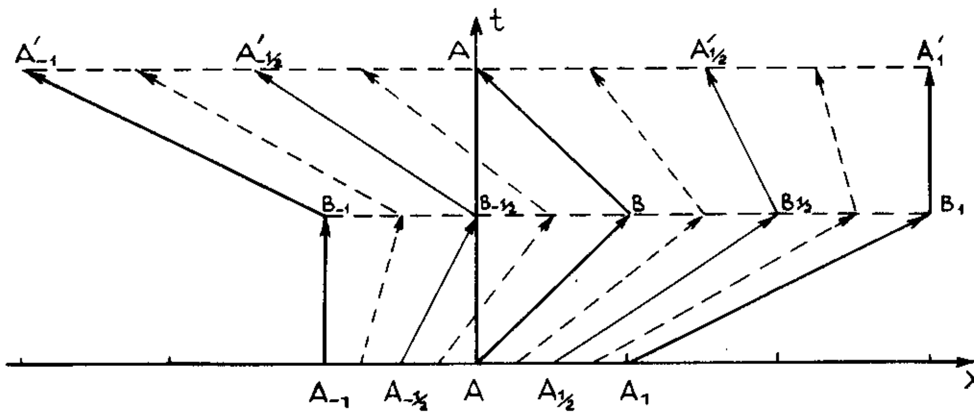

Fig.1. Diagram of a radar experiment.

B show the values of δ_1 and the broken arrows describe intermediate cases. For clearness and simplicity the position of B is fixed, and hence the points of emitting and receiving a light signal have different coordinates. If the point of standing an observer is fixed, point B will have two different positions (relative to A and A´).

1.3. ANISOTROPIC SPACE-TIME

1.3.1. Generalized Lorentz Transformations

Let us dwell on the consideration of the general case when both time and space anisotropy is admitted. For example, for the time of light propagation from A to B and the distance covered by it we have

$$t_{AB} = (1+\delta_0)(t_2 - t_1)/2. \; X_{AB} = (1+\delta_1)(t_2 - t_1)/2, \qquad (1.1)$$

Space-time anisotropy is introduced by two successive transformations

$$x_i \to x_i - \delta_{ik} x_k, \quad i,k = 0,1, \qquad (1.2a,b)$$

where

$$\delta_{00} = \delta_{11} = 0, \quad \delta_{01} = \delta_0, \quad \delta_{10} = \delta_1.$$

In addition, using (1.1), for velocities of light propagation in the positive and negative directions we obtain

$$c_1 = \frac{1+\delta_1}{1+\delta_0}, \quad c_2 = \frac{1-\delta_1}{1-\delta_0}, \qquad (1.3)$$

i.e. $0 < c_{1,2} < \infty$.

Substituting (1.2) into the special Lorentz transformations, we get the generalized transformations for coordinates taking into account explicitly a possible space-time anisotropy

$$x' = [(1 + \frac{\delta_0 - \delta_1}{1 - \delta_0 \delta_1} v) x - v \frac{1-\delta_1^2}{1-\delta_0\delta_1} t] \gamma,$$

$$t' = \left[\left(1 - \frac{\delta_0 - \delta_1}{1 - \delta_0 \delta_1} v\right) t - v \frac{1 - \delta_0^2}{1 - \delta_0 \delta_1} x \right] \gamma , \qquad (1.4)$$

or

$$x' = \left[\left(1 - \frac{c_1 - c_2}{c_1 + c_2} v\right) x - v \frac{2 c_1 c_2}{c_1 + c_2} t \right] \gamma ,$$

$$(1.4')$$

$$t' = \left[\left(1 + \frac{c_1 - c_2}{c_1 + c_2} v\right) t - v \frac{2}{c_1 + c_2} x \right] \gamma .$$

Thus, the conventional character of the concepts of simultaneity (for events at different points) and distance admits the transformations (1.2). One can say that we deal with "coordinate gauge" here*.

In essence, a certain relativity is available here. It means that a true physical configuration corresponds not to a single choice of parameters δ_0 and δ_1 but to a variety of gauge-equivalent configurations. In other words, the singled out values of $\delta_0(\delta_1)$ do not exist in an interval of -1,1. For convenience of practical work parametrization is usually performed, i.e. an additional condition destroying gauge arbitrariness is imposed. The traditional or standard gauge is the following: δ_0, $\delta_1 = 0$.

In general, a number of transformations (1.2a) and (1.2b) do not constitute a group because it is possible that the sum $(\delta + \delta') \notin -1,1$.

It should be also noted that the limiting values of $\delta_0 = -1$ and $\delta_1 = 1$ in formulae (1.2) lead to the known variables of light front

$$t_+ = t + x , \quad t_- = t - x , \qquad (1.5)$$

which in the considered case of (1+1)-space are simple coincident with the times of receiving and sending a light signal, t_2 and t_1, respectively.

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*What is like the Weyl scale deformation /18/, i.e. "gauge" in its original form as it must be said that the scale length changes with its rotation.

1.3.2. One-Way Velocities, "Right" and "Left" Distances for Material Bodies

Using (1.4), in the general case for the velocity of motion of a material body in the positive and negative directions we have

$$v_{\substack{1\\2}} = \frac{(1-\delta_1^2)\,v}{1-\delta_0\delta_1 \pm (\delta_0-\delta_1)\,v} . \qquad (1.6)$$

Let us write out two relations which connect the "right" and "left" distances covered by a light signal and a body (X^v) between the same points A and B in the positive and negative directions.

$$\frac{X^v_{AB}}{v_1} - \frac{X_{AB}}{c_1} = \frac{X^v_{BA}}{v_2} - \frac{X_{BA}}{c_2} , \qquad (1.7)$$

$$\frac{X_{AB}}{c_1} + \frac{X_{BA}}{c_2} = v\left(\frac{X^v_{AB}}{v_1} + \frac{X^v_{BA}}{v_2}\right) . \qquad (1.8)$$

These relations are a consequence of experiment. Adding and subtracting (1.7) and (1.8) with allowance for the fact that

$$\frac{X_{AB}}{c_1} + \frac{X_{BA}}{c_2} = t_{AB} + t_{BA} = t_2 - t_1 , \quad t_{AB} - t_{BA} = \delta_0\,(t_2 - t_1) , \qquad (1.9)$$

we find

$$X^v_{AB} = \frac{v_1}{v}(1+\delta_0 v)\,X , \qquad (1.10a)$$

$$X^v_{BA} = \frac{v_2}{v}(1-\delta_0 v)\,X , \qquad (1.10b)$$

where

$$2X = X_{AB} + X_{BA} = t_2 - t_1 . \qquad (1.11)$$

Using (1.6), we get

$$X^v_{AB} + X^v_{BA} = \frac{(1-\delta_1^2)[1-\delta_0\delta_1 - \delta_0(\delta_0 - \delta_1)v^2]}{(1-\delta_0\delta_1)^2 - (\delta_0 - \delta_1)^2 v^2} 2X . \qquad (1.12)$$

In other words, the sum of the "right" and "left" distances for a material body does not equal the corresponding sum for a light signal. In the specific case of purely space anisotropy ($\delta_0 = 0$) we have

$$X^v_{AB} + X^v_{BA} = \frac{1 - \delta_1^2}{1 - \delta_1^2 v^2} 2X , \qquad (1.12')$$

i.e. the summary distance covered by a nonlight signal in the positive and negative directions between A and B is larger than the corresponding distance for a light signal.

The degree of space anisotropy is defined as

$$a^v = \frac{X^v_{AB}}{X^v_{BA}} = \frac{(1+\delta_0 v)[1-\delta_0\delta_1 - (\delta_0 - \delta_1)v]}{(1-\delta_0 v)[1-\delta_0\delta_1 + (\delta_0 - \delta_1)v]} . \qquad (1.13)$$

i.e. if for a light signal (v = 1) it remains the same, for a material body it depends (rather unexpectedly) on time parameter as well. In this case for the degree of time anisotropy we have as before

$$a^v_t = \frac{t^v_{AB}}{t^v_{BA}} = \frac{1 + \delta_0 v}{1 - \delta_0 v} . \qquad (1.14)$$

Thus, the existence of the limiting velocity of interaction transfer has resulted in the ascertainment of the conventional character of commonly used coordinates. An explicit introduction of the time and space parameters into the usual formulae allows one to feel the degree of introduced arbitrariness in order to try to eliminate completely conditional agreements for the sake of larger simplicity of a physical description.

1.4. GENERALIZATION OF LOBACHEVSKY GEOMETRY TO ANISOTROPIC VELOCITY SPACE

As usual, to proceed to the Lobachevsky velocity space, a relative velocity of two bodies moving with infinitely close velocities v and v + dv is considered using the relativistic rule of velocity addition (see, e.g., Ref.[19]).

One can assume an apparently covariant way of introducing a length element into the Lobachevsky space. It consists in the calculation of the Lorentz-invariant interval element squared for infinitesimal 4-velocity du^i (i = 0,1,2,3).

For the components du^i we have

$$du^0 = \vec{v}d\vec{v}\gamma^3, \quad du^\alpha = dv^\alpha\gamma + v^\alpha(\vec{v}d\vec{v})\gamma^3, \tag{1.15}$$

with $\alpha = 1, 2, 3$, $\gamma = (1 - v^2)^{-1/2}$ and $v^2 = (\vec{v})^2$. Using (1.15) for the interval squared $d\tau^2 = du^i du_i$, it is easy to find

$$-d\tau^2 = [(1 - v^2)(d\vec{v})^2 + (\vec{v}d\vec{v})^2](1 - v^2)^{-2}. \tag{1.16}$$

In fact, this represents the known expression for the length element squared in the Lobachevsky velocity space.

By analogy with this fact, we arise from the interval squared in an anisotropic coordinate (1+2)-space which can be presented as

$$d\tau^2 = dt^2[1 - v_x(\frac{1}{c} - \frac{1}{c'}) - \frac{v_x^2}{cc'} - v_y^2] = dt^2\gamma^{-2}. \tag{1.17}$$

Here for simplicity $c = c_1$, $c' = c_2$, v_x and v_y are the components of the velocity of a material body, and anisotropy is related to the direction of the x-axis. In this case for the components of metric tensor g_{ik} which differ from zero it is evident that

$$g_{00} = 1, \quad g_{01} = -\frac{1}{2}(\frac{1}{c} - \frac{1}{c'}), \quad g_{11} = -\frac{1}{cc'}, \quad g_{22} = 1. \tag{1.18}$$

From (1.17) for the components du_i we find

$$du^0 = [(-g_{01} - g_{11}v_x)dv_x + v_y dv_y]\gamma^3,$$

$$du^1 = [(1 + g_{01}v_x - v_y^2)dv_x + v_x v_y dv_y]\gamma^3, \tag{1.19}$$

$$du^2 = [(1 + g_{01}v_x + g_{11}v_x^2)dv_y - (g_{01} + g_{11}v_x)v_y dv_x]\gamma^3.$$

The interval element squared corresponding to (1.17) takes the form

$$d\tau_u^2 = g_{ik}\, du^i\, du^k \,, \tag{1.20}$$

in anisotropic velocity space. Taking (1.18) and (1.19) into account, after cumbersome enough calculations we obtain

$$-d\tau_L^2 = \{(\frac{dv_x^2}{cc'} + dv_y^2)\,\gamma^{-2} + [\,(2v_x + c - c')\,\frac{dv_x}{2cc'} + v_y dv_y\,]^2\}\gamma^4. \tag{1.21}$$

The length element squared in generalized Lobachevsky velocity space is defined by expression (1.21).

In the specific case of one dimension, (1.21) goes to

$$-2d\tau_L^2 = \frac{(c+c')^2\, dv^2}{[\,cc' - v(c'-c) - v^2]^2}\,. \tag{1.21'}$$

2. ANISOTROPIC SPACE GEOMETRY

2.1. Models of Anisotropic Time-Space

Some time ago the problem of introduction anisotropic time-space (and studies of its properties) arises as a direct consequence of relativization of the concept of coordinate. However, this approach which is distinguished for its uncommonness and some complications did not meet with an understanding, and, moreover, it was considered as a needless one (because of "nonobservability" of an instant of signal reflection). The discovery of parity nonconservation in weak interactions was practically the manifestation of a similar type of anisotropy.

In fact, space parity nonconservation[20] makes it possible to realize time anisotropy. Now one can build "right-hand" and "left-hand" clocks[21]. It is possible, say, to make a Co-60 clock with a magnet and detectors which detect β-decay electrons and count them. For definiteness, let the magnet field be directed along the x-axis and the source be placed at the origin. Each time, when an electron is detected, a second hand moves slightly. Then the detector on the right realizes the "right" clock;and the mirror detector,the "left" clock. As a greater number of electrons comes to the mirror clock, it is evident that it goes faster. In particular, Fig.2 illustrates the case when the "left-hand" clock (located, say, at a unitary distance from the origin: x = -1) goes twice as fast as the "right-hand" clock (x = 1), i.e. $\Delta t_\ell = 2\Delta t_r$. If these clock are used to measure the velosity of light, c_r is found to be equal to $2c_\ell$. Taking into account that $c_r^{-1}=1+\delta_0$ and $c_\ell^{-1}=1-\delta_0$, we get $\delta_0 = -1/3$ for the time parameter.

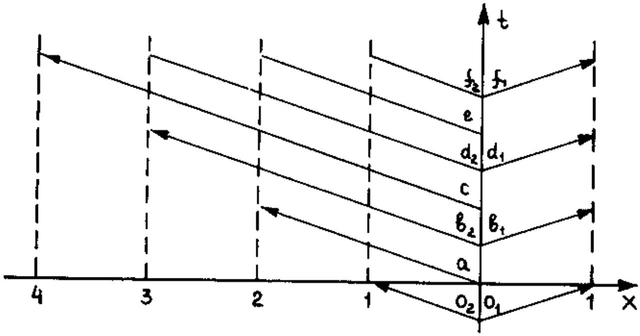

Fig.2. Diagram illustrating time-space asymmetry.

As follows from the experiment, the interactions conserving and violating parity make an equal contribution which means that in this case $\Delta t_\ell = 3\Delta t_r$ and $c_r = 3c_\ell$ ($\delta_0 = -0.5$).

On the other hand, from the considered construction one can obtain that the time on the left and right is counted synchronously. With this aim, e.g., the "left-hand" clock realizes a number of distant detectors. If the first detector on the left giving the time origin has x=-1 as before (see Fig.2),the detector counting the 2nd second has x=-2;the 3rd second, x=-3 and so on. Certainly, it is quite unusual but well admissible to assume that in this case the detector on the left counting 1s is also situated at a unitary distance. The standard of length has just changed with time: $\Delta x_\ell = 2\Delta x_r$ and so on. Of course, such a "nonstationary" model of anisotropic space cannot be thought to be successful, in particular for our first acquaintance. That is why we are going to consider in detail the simplest examples of stationary anisotropic space and distinctive features of the geometry related to it. According to the procedure of relativization, we believe that the difference between the distances forth and back or, what is the same, the "right" and "left" distances are not associated with changing standard scale*.

*Just as the distances, which a light signal travels in the direction of motion of a rod and back, differ although the standard of length is invariable.

2.2. Noncommutative Geometry

For simplicity we restrict ourselves to the consideration of plane geometry, i.e. planimetry.

In principle, all peculiarities of anisotropic geometry are determined by the behaviour of the length of a given segment characterizing the distance between fixed events. In accord with the radar experiment, for changing the length of the segment as a function of its slope angle, we have

$$r_n(\phi) = r[1 + \delta_1 f(\phi)] = rF(\phi), \tag{2.1}$$

where δ_1 is the space parameter $(-1 < \delta_1 < 1)$ and $f(\phi)$ satisfies the condition $f(\phi + \pi) = -f(\phi)$, $|\delta_1 f(\phi)|_{max} \leq 1$.

Figure 3 presents a variety of possible cases of choosing $f(\phi)$, namely:

1. $f_1(\phi) = \cos\phi$, 2. $f_2(\phi) = \cos^9\phi$, 3. $f_3(\phi) = \cos\phi + \sin\phi$

and $\delta_1 = -0.5$. As the obtained figures represent a geometric position of points spaced from the origin at the distance equal to the length of a given segment, they can be called "anisotropic circles" according to the definition of circle. Note that curve 2 is partically coincident with a usual circle for $f_2(\phi)$ in the interval of angles $\pi/4 < |\phi| < 3\pi/4$.

Thus, if $r_n(\phi)$ is the way "forth", then $r_n(\phi + \pi)$ is the way "back". In other words, point B of reflection of a light signal is practically fixed here in contradistinction to Fig.1 illustrating the radar experiment.

Nete that although the length of a segment varies as a function of its slope angle, the measure of calculating angles is unchangeable, i.e. it remains circular as before. Using (2.1), from usual formulae one can introduce Cartesian coordinates

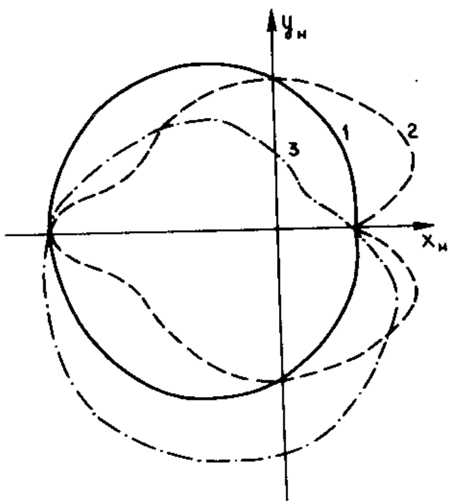

Fig.3. Anisotropic space models (change of the segment length as a function of its slope angle).

$$x_n = r_n \cos\phi, \qquad y_n = r_n \sin\phi. \tag{2.2}$$

As a result, e.g., for $f_1(\phi)$ the equation of "anisotropic circle" in terms of Cartesian coordinates (centre at the origin) takes the form

$$\left[\frac{x_n^2 + y_n^2}{(x_n^2 + y_n^2)^{1/2} + \delta_1 x_n} \right]^2 = R^2. \tag{2.3}$$

As a matter of fact, the principal peculiarity of the approach in question is that the second axiom of metric space is not valid (see, e.g., /22a/). According to this axiom, $\rho(A,B) = \rho(B,A)$, where ρ is the distance from A to B. In other words, the discussed geometry can be also referred to as non-commutative because already within its frame

$$\rho(A,B) - \rho(B,A) \neq 0. \tag{2.4}$$

As "... to date we have a common "tsar's way" in geometry which passes through the concepts of "vector space" and "scalar product"/23/ , let us use it. In fact, the use of a coordinate system already means the introduction of direction, that is the characteristic property of a vector and, finally, vector space determined, as known, by two groups of axioms (see, for instance, /22b/).

It is evident that the sum of AB and its opposite vector BA is not equal to a zero vector. Otherwise, the fourth addition axiom (I_4), which is also called an axiom of existence of a "back" element, does not hold. As far as the first axiom of multiplication (II_1) by number (λ) is concerned, in the case of a negative number, for example $\lambda = -1$, we, apparently, obtain no opposite vector. As the studied approach is the generalization of the traditional one, the following expressions for the above axioms can be written

$$I_4 \qquad \vec{a}_n F(\phi_a + \pi) + (-\vec{a}_n) F(\phi_a) = 0. \tag{2.5}$$

$$II_4 \qquad (\lambda' = -|\lambda|), \qquad \lambda\vec{a}_n \to \lambda' [F(\phi_a + \pi)/F(\phi_a)]\, \vec{a}_n. \tag{2.6}$$

To be sure, a formal multiplication of vector by negative number is written as before.

Let us now turn to the third group of scalar product axioms which allows Euclidean vector space to be constructed. Based on (2.2), for vector length $\vec{r}_n$ we have

$$r_n = \sqrt{x_n^2 + y_n^2} . \tag{2.7}$$

In this case the vector norm is defined as

$$||\vec{r}_n|| = \sqrt{x^2 + y^2} , \tag{2.8}$$

with

$$x = r_n \operatorname{csn}\phi , \quad y = r_n \operatorname{snn}\phi , \tag{2.9}$$

and

$$\operatorname{csn}\phi = \cos\phi \, F^{-1}(\phi) , \quad \operatorname{snn}\phi = \sin\phi \, F^{-1}(\phi) . \tag{2.10}$$

It is evident that in this case

$$||\vec{r}_n|| = r_n F^{-1}(\phi) . \tag{2.11}$$

The scalar product of vectors $\vec{a}_n$ and $\vec{b}_n$ is expressed as

$$\vec{a}_n \vec{b}_n = (a_n^x b_n^x + a_n^y b_n^y) \, F^{-1}(\phi_a) \, F^{-1}(\phi_b) , \tag{2.12}$$

or

$$\vec{a}_n \vec{b}_n = a_n b_n \operatorname{csn}(\phi_a - \phi_b) = a_n b_n \cos(\phi_a - \phi_b) \, F^{-1}(\phi_a) \, F^{-1}(\phi_b) . \tag{2.12'}$$

In the specific case $\vec{b}_n = \vec{a}_n$, the scalar squared is written as follows

$$(\vec{a}_k)^2 = a_n^2 \operatorname{csn} 0 = a_n^2 F^{-2}(\phi_a) = a^2 . \tag{2.13}$$

i.e. it equals the norm squared as before.

In conclusion let us consider one more classical geometric figure, namely a triangle. Figure 4 presents two of such corresponding anisotropic triangles (for the case of $f_1(\phi)$). In so doing, $A_1B_1C_1$ is the right

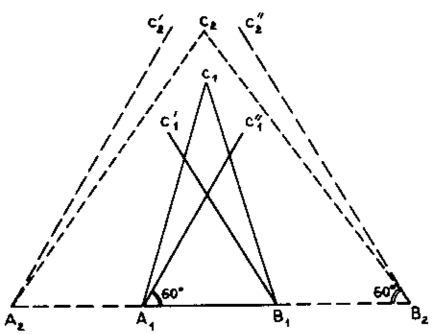

Fig.4. "Equilateral" triangles in anisotropic geometry. $A_1B_1C_1$ - right, $A_2B_2C_2$ - left.

"equilateral" triangle constructed of corresponding "equal" segments. It is evident that it is not equiangular and only isosceles (the angles at its base are 73°). The segments A_1C_1'' and B_1C_1' illustrate an attempt to construct the corresponding equiangular triangle. By analogy, $A_2B_2C_2$ is the left "equilateral" triangle ($\angle A_2$ = 54.5 °); accordingly, A_2C_2'' and B_2C_2'' show an attempt to construct an equiangular triangle. It should be noted that the right triangle is made up of positive or right segments as distinct from the commonly used definition (see, e.g., /24/). In other words, their projections on the axis are positive values. The left triangle is constructed of left segments.

CONCLUSION

At one time the Lorentz-Edwards transformations taking into account quantitatively admissible time anisotropy were introduced. The relativistic definition of distance based on the radar experiment is also associated with measuring the summary time of propagation of a light signal "forth" and "back", i.e. some arbitrariness is again admissible. The transformation of a spatial coordinate, which is similar to "time gauge", allows one in a simple way to obtain the corresponding transformations for coordinates keeping in mind explicitly admissible space anisotropy. Finally, the generalized Lorentz transformations describe the general case of anisotropic space-time which embraces both noted possiblities. The generalization of Lobachevsky geometry to anisotropic velocity space is of interest.

Possible models of a Minskowskian anisotropic space are discussed from nonconservation parity experiments (e.g., using "left" and "right" clocks). The basic properties of the geometry (planimetry) of an anisotropic Euclidean space, i.e. noncommutative geometry, are considered.

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Received by Publishing Department
on May 11, 1989.