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ON PARTICLE CREATION BY A TIME-DEPENDENT SCALAR FIELD

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1. Introduction

The interest in particle creation from vacuum, by an external scalar field, is provoked primarily by comsomological inflationary models. In all the models considered (see e.g. reviews /1/) exponential expansion is driven by a scalar field $\chi(t)$, providing the effective equation of state $p=-\varepsilon$. At the end of inflation the only nonvanishing field in the Universe is the scalar field $\chi(t)$, while the density of other forms of matter is exponentially damped. The ordinary matter is created due to the coherent oscillations of χ around its equilibrium point. That leads to the customary fridman universe. This process is called the Universe heating up or reheating. It was discussed in refs. /2-4/ ,but detailed calculations of the particle creation probability were not performed there.

The aim of this work is to present simple analytical formulae for particle creation probability for different cases, which could be realized in the early Universe, and also to discuss some cosmological implications, such as the Universe reheating, bariogenesis, the effect of particle creation on the Universe evolution.

Particle creation from vacuum by an external field is a well investigated problem of the quantum field theory (see e.g. the book /5/). Electromagnetic fields of different kinds, configurations and intensity have been considered in literature, while the case of an external scalar field is not yet well studied. It differs from the vector field case by some specific features, as for example, the fact that the interaction of a particle with scalar field can be regarded as a contribution to the particle mass. Hence, on one hand, the creation probability must increase with rising field strength, and on the other hand it decreases as the mass increases. This complex behaviour is reflected in the results obtained below.

The Bogolubov canonical transformation method can be used for the description of the particle creation processes (see e.g. ref. /5/).

In the next section we will apply this method for the scalar field case in analogy with the electromagnetic one. This formalism reduces the calculation of creation probabilities to solution of second order ordinary differential equations.

In sec. 3 the creation probability is calculated in the case when perturbation theory is valid.

In sec. 4 another method for the description of particle creation processes is presented based on calculation of the imaginary part of the action functional . This method is especially suitable for quasiclassical approximation in the imaginary time formalism. As will be shown, the results, obtained in quasiclassical approximation are in good accordance with those obtained in the opposite case, when the perturbation theory is valid.

2. Method of Bogoliubov transformation.

We will consider the case of time dependent space-homogeneous external scalar field $\mathcal{X}(t) = \mathcal{X}_0 f(t)$, interacting with fermions $\Psi(x)$ (or bosons $\varphi(x)$). The Lagrangian density and the corresponding field equations have the form:

$$\mathcal{L} = \frac{i}{2}\left[\bar{\Psi}(x)\gamma^{\alpha}\partial_{\alpha}\Psi(x) - \partial_{\alpha}\bar{\Psi}(x)\gamma^{\alpha}\Psi(x)\right] - (m_0 + g\mathcal{X})\bar{\Psi}\Psi \qquad (1)$$

$$\left[i\gamma^{\nu}\partial_{\nu} - (m_0 + \mathcal{X})\right]\Psi(x) = 0 . \qquad (2)$$

It is convenient to express the solution of the equation (2) through the solution of the second order differential equation:

$$\left[\partial_{\nu}\partial^{\nu} - i\gamma^0 \frac{\partial \mathcal{X}(t)}{\partial t} + (m_0 + g\mathcal{X})^2\right]\varphi(x) = 0 ,$$

where $(i\gamma_{\mu}\partial^{\mu} + m_0 + \mathcal{X})\varphi(x) = \Psi(x)$, $\varphi_{\vec{p}s}(x) = e^{i\vec{p}\vec{x}}\varphi(\vec{p},t)S_s$.

Substituting $\varphi_{\vec{p}s}(x) = e^{i\vec{p}\vec{x}}\tilde{\varphi}(\vec{p},t)S_s$, where S_s are the eigen-vectors of the matrix γ^0 , we obtain for $\tilde{\varphi}(\vec{p},t)$ an equation of the form:

$$\tilde{\varphi}(\vec{p},t)'' + \Omega^2(t)\tilde{\varphi}(\vec{p},t) = 0 \,.. \qquad (3)$$

where $\Omega^2(t) = \vec{p}^2 + (m_0 + g\chi)^2 + i\frac{\partial\chi}{\partial t} = \Omega_0^2(t) + i\frac{\partial\chi}{\partial t}$.

Equation (3) is an oscillator type equation with a complex time-dependent frequency. Analogous equation is obtained for the boson creation, but in that case the frequency is real. The field operator expansion in terms of creation and annihilation operators with definite momentum has the form:

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\tau=1,2} \int d^3p \left[a^{(-)}_{\vec{p}\tau} \psi^{(-)}_{\vec{p}\tau}(x) + a^{(+)}_{\vec{p}\tau} \psi^{(+)}_{-\vec{p}\tau}(x) \right]. \quad (4)$$

Here $\psi^{(\pm)}_{\vec{p}\tau}(x)$ are correspondingly positive and negative frequency solutions of the Dirac equation defined in such a way that they coincide with positive and negative-frequency asymptotics of the solutions when $t \to -\infty$ and the particle creating part of the field switches off (i.e. when the field becomes static).

After some transformations the hamiltonian of the fermion (boson) field in external scalar field can be written in the form:

$$\mathcal{H}^{1/2}(t) = \sum_{\tau=1,2} \int d^3p\, \omega(\vec{p},t) \Big[E(\vec{p},t) \big(\overset{*}{a}{}^{(+)}_{\vec{p}\tau} a^{(-)}_{\vec{p}\tau} \mp \overset{*}{a}{}^{(-)}_{\vec{p}\tau} a^{(+)}_{-\vec{p}\tau} \big) +$$

$$+ F(\vec{p},t) \overset{*}{a}{}^{(+)}_{\vec{p}\tau} a^{(+)}_{-\vec{p}\tau} + F^*(\vec{p},t) \overset{*}{a}{}^{(-)}_{-\vec{p}\tau} a^{(-)}_{\vec{p}\tau} \Big], \quad (5)$$

where $E(\vec{p},t)$ and $F(\vec{p},t)$ can be expressed through $\tilde{\varphi}^{(+)}, \tilde{\varphi}^{(-)}$:

$$E(\vec{p},t) = -i\vec{p}^2 \left[\tilde{\varphi}^{(+)} \varphi'^{(+)} - \overset{*}{\tilde{\varphi}}{}^{(+)\prime} \tilde{\varphi}^{(+)} \right] / \Omega_0(\vec{p},t) + (m_0 + \chi)/\Omega_0$$

$$E(\vec{p},t)^2 + |F(\vec{p},t)|^2 = 1 . \quad (6)$$

At infinitely large negative t the Hamiltonian is a diagonal operator, while in the presence of nonstationary field it is not. It can be diagonalized at any given time by the Bogoliubov transformations realizing the transition from operators $\overset{*}{a}{}^{(\pm)}, a^{(\mp)}$ to time dependent operators $\overset{*}{b}{}^{(\pm)}(t)$, $b^{(\pm)}(t)$ with conserved anticommutator (in fermionic case) or commutator (in bosonic case) relations.

$$a^{(-)}_{\vec{p}s} = \alpha^{*}_{p}(t)\, b^{(-)}_{\vec{p}s}(t) - \beta_{\vec{p}}(t)\, b^{(+)}_{-\vec{p}s}(t) \tag{7}$$

$$\overset{*}{a}{}^{(-)}_{\vec{p}s} = \alpha^{*}_{-\vec{p}}(t)\, \overset{*}{b}{}^{(-)}_{\vec{p}s}(t) \overset{+}{(-)} \beta_{-\vec{p}}(t)\, \overset{*}{b}{}^{(+)}_{-\vec{p}s}(t),$$

where $|\alpha|^2 \overset{+}{(-)} |\beta|^2 = 1$. Minus sign is for the bosonic case. The Hamiltonian can be diagonalized in terms of $b^{(\pm)}_{\vec{p}s}(t)$ and $\overset{*}{b}{}^{(\pm)}_{\vec{p}s}(t)$, provided the following conditions are fulfilled:

$$\frac{\beta_{\vec{p}}(t)}{\alpha_{\vec{p}}(t)} = \pm\frac{1-E(\vec{p},t)}{F^{*}(\vec{p},t)}, \quad |\beta_{\vec{p}}(t)|^2 = \pm\frac{1-E(\vec{p},t)}{2}, \quad E^2+|F|^2 = 1. \tag{8}$$

Minus sign is for bosons.
In terms of operators $b^{(\pm)}(t)$ the Hamiltonian takes the form:

$$\mathcal{H}^{(\prime)}(t) = \sum_{s=1,2}\int d^3p\, \Omega_0(\vec{p},t)\left[\overset{*}{b}{}^{(+)}_{\vec{p}s}(t)\, b^{(-)}_{\vec{p}s}(t) \mp \overset{*}{b}{}^{(-)}_{\vec{p}s}(t)\, b^{(+)}_{-\vec{p}s}(t)\right]$$

$b^{\pm}(t)$ are operators of the physical particles creation or annihilation at moment t . If $|0_t>$ is physical vacuum at moment t then

$$b^{(-)}_{\vec{p}r}(t)|0_t> = \overset{*}{b}{}^{(-)}_{\vec{p}r}(t)|0_t> = 0 .$$

In other words, an interacting quantum field (fermionic or bosonic one) can be represented in the same way as a noninteracting field, provided the necessary redefinition of the concepts of particles and the vacuum state is carried out. Diagonalization of the Hamiltonian is equivalent to exact solution of the Heisenberg equation of motion. It allows to express any matrix element through the Bogoliubov transformation coefficients. Thus, the absolute probability of the pair creation is $W_{\vec{p}}(t) = |\beta_{\vec{p}}(t)|^2$, while $\Gamma_{\vec{p}} = |\beta_{\vec{p}}/\alpha_{\vec{p}}|^2$ is the relative probability for creation of particle pair with a definite momentum.

The nuclear density of the created particle-antiparticle pairs in the state, that was a vacuum one at $t \to -\infty$ is $n^{1/2}(t) = \frac{2}{(2\pi)^3} \int d^3p \, |\beta_p|^2$.

Consequently, as can be seen from eqs.(6) and (8), the calculation of the pair creation probability may be reduced to a determination of the asymptotics of the solutions of oscillator equation (3) with a complex frequency. Similarly, the calculation of the probability for the boson pair creation reduces to a determination of the asymptotics of the solutions of equation (3) but with a real frequency.

The asymptotic solutions of the equation (3) are designated as

$$\tilde{\varphi}(t) = \begin{cases} \exp(-i\Omega_- t) & t \to -\infty \\ C_1(t)\exp(-i\Omega_+ t) + C_2(t)\exp(+i\Omega_+ t) & t \to +\infty \end{cases}$$

where $\Omega_\mp(\vec{p}) = \lim_{t \to \mp\infty} \Omega(\vec{p}, t) = \vec{p}^{\,2} + (m + \chi_\mp)^2$.

The impossibility to construct a solution, which both at $t \to +\infty$ and at $t \to -\infty$ indudes just one definite frequency, is a result of the pair production by the field. The quantity $\rho = |C_2/C_1|^2$ defines the degree of the oscillator excitation due to changes in its frequency. Consequently the particle creation probability can be expressed through ρ.

The problem of particle creation is reduced to the problem of parametric excitation of oscillator with variable frequency, or to the problem of flat wave reflection from the potential barrier /5,7,9-11 /. Indeed, using the formal substitution x instead of t in equation (3) we obtain the Schrodinger equation describing one dimensional **movement** of particle in potential $\sim g\chi(t)$. ρ in this case plays the role of reflection coefficient from the barrier). Due to this link we can express creation probabilities through the solutions of ordinary differential equations, avoiding the search of the exact solutions of the relativistic equations and the Green's functions. Some methods for solution of equations of type (3) are discussed in ref./ 7, 11 /.

Employing the Lagrange method to the oscillator equation (3) we obtain a set of first order differential equations:

$$\dot{a}_1(t) = -\frac{\dot{\chi}}{2\Omega_0(t)} a_2(t) \exp(-2i\varphi)/(\Omega_0+Q)$$

$$\dot{a}_2(t) = \frac{\dot{\chi}}{2\Omega_0(t)} \cdot a_1(t) \exp(+2i\varphi)(\Omega_0+Q),$$

where $a_1(0) = 1/\sqrt{2(\Omega_0^2-Q^2)}$, $a_2(0)=0$, $Q=m+\chi$, $\varphi(t)=\int_0^t \Omega_0(t')dt'$.

The solution of (3) reads:

$$\tilde{\varphi}(t) = \sqrt{\Omega_0+Q}\, a_1(t)\exp(i\varphi) - a_2(t)\exp(-i\varphi)/\sqrt{\Omega_0+Q}$$

$$\rho = |a_2(t)/[a_1(t)(\Omega_0+Q)]|^2.$$

Using the variable phase method we obtaine a solution for the boson fields in the form:

$$\tilde{\varphi}(t) = A(t)\left[\exp(-i\varphi) + R(t)\exp(+i\varphi)\right],$$

where $\varphi(t) = \int_0^t \Omega_0(t')dt'$, and $A(t)$ and $R(t)$ satisfy the equations:

$$A'(t) = \frac{\Omega_0'}{2\Omega_0} A(t)\left[R(t)\exp(+2i\varphi) - 1\right], \quad A(0) = 1$$
$$R'(t) = \frac{\Omega_0'}{2\Omega_0}\left[\exp(-2i\varphi) - R^2(t)\exp(+2i\varphi)\right], \quad R(0)=0. \tag{9}$$

Particle creation probability is $\Gamma = |R(+\infty)|^2$.

Thus the calculation particle creation probability can be reduced to the calculation of the asymptotic solutions of the oscillator equations with a complex or a real frequency. In most cases it is difficult to find the exact solution. There is a lot of problems when the approximate methods of calculation fit well enough. In what follows we will obtain the explicit analytic formulae for the pair production probabilities in two limiting cases: when perturbation theory is applicable and when the quasiclassical methods can be employed. The interaction of time dependent scalar field $\chi(t) = \chi_0 f(t)$ with the fermions $g\chi\bar{\Psi}\Psi$ is equivalent to a variable contribution to the fermion mass, $m_\Psi(t) = m_0 + g\chi_0 f(t)$. So the fermion mass becomes time-dependent.

If $\omega >> m_\psi$ the perturbation theory can be used, while for $\omega << m_\psi$ the quasiclassical approximation workes.

3. The perturbation theory calculations

The lowest order amplitude for the pair creation of fermions with momenta $\vec{p}_1$ and $\vec{p}_2$ is:

$$A(\vec{p}_1, \vec{p}_2) = g\int d^4x\, \chi(t) < p_1, p_2 | \bar{\psi}\psi | 0 > =$$

$$= (2\pi)^{-1/2} g\, \delta(\vec{p}_1 + \vec{p}_2)\, \bar{V}_\sigma^{\nu,+}(\vec{p}_1)\, V_\ell^{\nu,+}(\vec{p}_2)\, \tilde{\chi}(E_1 + E_2)/\sqrt{4E_1E_2} \;,$$

where ψ and $\bar{\psi}$ are expressed through the creation and annihilation operators, according to: $\psi = \psi^+(x) + \psi^-(x)$

$$\psi_\sigma^\pm(x) = \frac{1}{(2\pi)^{3/2}}\int dp\; \exp(\pm ipx)\, \delta(p^2 - m^2)\, \psi_\sigma^\pm(p)$$

$$\bar{\psi}_\sigma^\pm(p) = \sum_{\nu=1,2} \bar{V}_\sigma^{\nu,\pm}(p)\, \overset{*}{a}{}_\nu^{(\pm)}(p) \qquad \psi_\sigma^\pm(p) = \sum_{\nu=1,2} V_\sigma^{\nu,\pm}\, a_\nu^\pm(p)$$

$\bar{V}$ and V are Dirac spinors, $E_i = \sqrt{m_i^2 + \vec{p}_i^{\,2}}$,

$$\tilde{\chi}(E) = \int dt\; \exp(iEt)\, \chi(t)$$

The fermion creation rate per unit volume is:

$$\dot{N} = \frac{1}{(2\pi)^4}\int \frac{|M|^2}{\sqrt{4E_1E_2}}\; \delta(\vec{p}_1 + \vec{p}_2)\, d^3p_1\, d^3p_2 \;, \tag{10}$$

where $|M|^2 = g^2|\tilde{\chi}|^2(p_{10}p_{20} - \vec{p}_1\vec{p}_2 - m_0^2)$, $E_1 + E_2 = \omega$.

The coefficient proportional to the volume V appears as usually, from the square of the δ-function:

$$[\delta(\vec{k}_1 + \vec{k}_2)]^2 = V\, \delta(\vec{k}_1 + \vec{k}_2)\, / \,(2\pi)^3 .$$

The integral over time is taken assuming that $\omega >> t^{-1}$.
Now, let us take a concrete external field $\chi(t) = \chi_0 \cos(\omega(t)\, t)$.
Then, using the expression for the χ-particles number density and taking the integrals over momenta in eq.(10) we obtain for the fermion creation rate:

$$\Gamma_\chi = \frac{\dot{N}_f}{N_\chi} = \frac{g^2}{4\pi} \frac{(\omega^2 - 4m_0^2)^{3/2}}{\omega^2},$$

where $N_\chi = \chi_0^2\, \omega$

In approximation $m_0 << \omega$:

$$\Gamma_\chi \approx \frac{g^2}{4\pi} \omega = \frac{g^2}{4\pi} m_\chi^{eff} \qquad (11)$$

as one may have expected. Γ_χ is the decay width of the χ-meson.

Similar result can be obtained starting from equation (9). Provided that $\omega >> m_0 + g\chi_0$ and $\varrho >> (2m + g\chi)\, g\chi$, where $\varrho = \sqrt{m^2 + \vec{p}^{\,2}}$, we derive

$$\frac{dR_p}{dt} = \frac{1}{2\varrho} \frac{d\Omega_0}{dt} \left[\exp(-2i\varrho t) - R^2(t) \exp(+2i\varrho t) \right]$$

$$R_p(\infty) = \frac{1}{2\varrho} \int_{-\infty}^{+\infty} \exp(-2i\varrho t) \frac{d\Omega_0}{dt} dt \qquad \text{if} \qquad |R(t)| << 1$$

$$\Gamma \simeq \frac{g^2 \Omega}{4\pi} .$$

However, in these calculations we have not accounted for the damping of $\chi(t)$ caused by particle creation. In the case $\Gamma_\chi << \omega$ this can be done using the substitution $\chi(t) \longrightarrow \chi(t) \exp(-\int_{t_0}^{t} \Gamma_\chi\, dt')$.

4. Functional approach. Quasiclassical approximation.

Now, let us study particle creation processes in the other limiting case, when the oscillation frequency of the χ-field is small in comparison with the fermion mass: $\omega << m_\psi(t)$. Then, the quasiclassical approximation is valid.

The classical Lagrangian for the relativistic particle with a variable mass $m_\psi(t)$ has the form:

$$L = - m_\psi(t)\sqrt{1-\vec{v}^2} .$$

The corresponding Hamiltonian equals: $\mathcal{H} = \sqrt{\vec{p}^2 + m_\psi^2(t)} = \sqrt{\Omega_0^2(t)}$. Quantum description can be realized by path integral method. Particle Green function is (see, for example /12/):

$$G(\vec{x}_f t_f ; \vec{x}_i t_i) = \int \mathcal{D}\vec{p}\, \mathcal{D}\vec{x}\, exp\left\{ i \int_{t_i}^{t_f} dt\, (\vec{p}\, \dot{\vec{x}} - \mathcal{H}) \right\} .$$

In the case discussed, the functional integral can be easily taken and the result is

$$G(\vec{x}_f t_f ; \vec{x}_i t_i) = \int \frac{d^3p}{(2\pi)^3}\, exp\left\{ i\vec{p}\,(\vec{x}_f - \vec{x}_i) - i \int_{t_i}^{t_f} dt \sqrt{\vec{p}^2 + m_\psi^2(t)} \right\} .$$

The pair creation amplitudes are of the form:

$$A_c(\vec{k}_1, \vec{k}_2) = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2)\, exp\left\{ -i \int_C dt \sqrt{\vec{k}_1^2 + m_\psi^2(t)} \right. , \qquad (12)$$

where $\vec{k}_1$ and $\vec{k}_2$ are the momenta of the created particles. The integration contour C ,encircles the branching points $k_1^2 + m^2(t) = 0$. A passage along C from one side of the cut to the other changes the sign of the energy $\sqrt{\vec{k}^2 + m^2(t)}$. In order to have a correct description of the creation of particles from vacuum, the movement along contour C must be carried out in the increasing energy direction, i.e. the movement should correspond to a transfer from the negative energy continuum states (Dirac sea) to the positive energy ones.

Then the transition probability, corresponding to only one branching point, is:

$$W \sim exp\left(-2\, \mathrm{Im} \int_C dt \sqrt{\vec{p}^2 + m^2(t)} \right) . \qquad (13)$$

This result can be obtained without exploiting Green's functions, but using the oscillatory equation (3), as it was already stated in sec. 2. Substiting $t \to x$ into equation (3) we can reduce the problem of particle creation probabilities to the quantum mechanical problem of barrier penetration, i.e. to calculating the refraction coefficient R. So, using the well known results from quantum mechanics (see for example /13/) we can as well obtain expression (13) for the probability. We can also solve the problem using the analogy with the quantum mechanical transition caused by a slowly changing perturbation.

For periodical fields it is convenient to carry out quasiclassical calculations in the imaginary time approach (see refs. /6,7/). When $m_\psi(t)$ is a periodical function, $\sqrt{\vec{k}^2+m_\psi^2(t)}$ has infinite number of branching points. So, the total amplitude can be expressed as a sum of amplitudes, each of which corresponds to some definite contour enclosing its own branching point $\vec{k}^2+m^2(t)=0$. The positions of two branching points within the period $\frac{2\pi}{\omega}$: $t_{1,2}=t'_{1,2}+t''_{1,2}$ are defined by the expression:

$$ch^2 t'_1 + ch^2 t'_2 = \frac{m_0^2+m_1^2+\vec{p}^2}{2m_1^2} + \left[\left(\frac{m_0^2+m_1^2+\vec{p}^2}{2m_1^2}\right)^2 - \frac{m_0^2}{m_1^2}\right]^{1/2}$$

$$sh\, t''_{1,2} = -\frac{m_0}{m_1}\, ch^{-1} t_1 .$$

When passing over the branching point energy sign changes. That is why, the amplitudes corresponding to the branching points shifted by a period differ in their phases: $A_{n+2}/A_n = \exp(2i\alpha)$ where $\alpha=\int\limits_0 dt\sqrt{\vec{k}^2+m_\psi^2(t)}$. The neighbouring amplitues have phase difference $A_{n+1}/A_n=\exp(2i\beta)$ where $\beta=\beta_1+\beta_2$ consists of two pieces, one equal to the phase increase along the real axes $\beta_1=\int dt\sqrt{\vec{k}^2+m_\psi^2(t)}$, the other appearing due to the phase increase along the cut $\beta_2 = 2\,\mathrm{Re}\int\limits_0^{t''} d\tau\, i\sqrt{\vec{k}^2+m_\psi^2(t)}$.
The module of the amplitude is defined by the real part of $\sqrt{\vec{k}^2+m_\psi^2(t)}$ when integrated over the cut

$$Q = -2\,\mathrm{Im}\int\limits_0^{t''_1} i\,d\tau\,\sqrt{\vec{k}^2+m^2(t_1+i\tau)}$$

$$|A_k| = (2\pi)^3\,\delta(\vec{k}_1+\vec{k}_2)\exp(-2Q).$$

Summation over all branching points results in $\delta(\alpha-\pi l)$ (where l is an integer) reflecting energy conservation law.

A. In the case when m_0 is small, $g\chi_0>\omega$, fermions creation rate per unit volume is

$$\dot{N}_\psi = \frac{2\omega^2}{\pi}\sum_n \int \frac{d^3p}{(2\pi)^3} \exp\left\{-\frac{4\sqrt{\vec{p}^2+m_1^2(t)}}{\omega}\left[K(\gamma)-E(\gamma)\right]\times\right.$$
$$\times\, \delta\left[n\omega - \frac{4}{\pi}\sqrt{\vec{p}^2+m^2}\; E\left(\sqrt{1-\gamma^2}\right)\right], \tag{14}$$

where $\gamma = p/\sqrt{p^2+m^2}$, and $K(\gamma)$, $E(\gamma)$ are the complete elliptic functions (see Gradshtein and Ryzhik 1962), and $m_1(t) = g\chi_0$ is a slowly decreasing with time function.

In the limiting case when $m_1 >> \omega$ we obtain

$$W = \sum_{n > \frac{4m}{\pi\omega}} \frac{\omega^2 m_1^{3/2}\sqrt{n\omega - 4m/\pi}}{(2\pi)^{3/2}\left(\ln\left(\frac{16 m_1}{\pi n\omega - 4m_1}\right)\right)^{3/2}} \cdot \exp\left\{-\frac{\pi^2\left(n - \frac{4m}{\pi\omega}\right)}{\ln\left[\frac{16 m_1}{\pi n\omega - 4m_1}\right]+1}\right\}.$$

As $\sqrt{n\omega - \frac{4m}{\pi}} = O(\omega)$, then in the limit, when $m_1 >> \omega$ and $\ln(m/\omega) >> \pi^2$ we obtain: $W = \omega^{5/2} m_1^{3/2}/\left(2\pi^{7/2}\ln^{1/2}\left(\frac{m}{\omega}\right)\right)$
So the χ- decay is:

$$\Gamma_\chi' = \dot{N}_\psi / N_\chi = \frac{g^2\omega^{3/2}}{\pi^{7/2} m_1^{1/2}\ln^{1/2}(m/\omega)} = \frac{4\Gamma_\chi}{\pi^{5/2}}\sqrt{\frac{\omega}{m_1 \ln\frac{m_1}{\omega}}}\,. \tag{15}$$

Where Γ_χ is the rate, calculatedin perturbation theory. The decay rate decreases because of the increase of the effective fermion mass at large m_1. This suppression is not an exponential one, but a power law one in terms of the ratio ω/m_1. This is due to the fact that fermion mass oscillates about zero level (or about $m_0 << \omega$), and as a result its average value vanishes, $\bar{m}_\psi = 0$.

Let us note, that after the formal substitution $\omega >> m_\psi$ into expression (14) (antiquasiclassical limit) the result coincides in form with that deduced by perturbation theory up to a numerical coefficient equal to $1/5$.

B. In the case when the fermion mass is large in comparison with m_1, $m_0 > g\chi_0$, the creation probability is strongly suppressed. Then we can solve this problem in analogy with the quantum mechanical problem of over barrier reflection. The probability is exponentially small: $W_p \sim \exp\left(-2\sqrt{m_0^2+\vec{p}^2}/\omega\right)$.

In this case it's easier to calculte creation probabilities for a little bit different form of the variable mass $m_\varphi^2(t) = m_0^2 + g^2\chi^2(t)$.

Such dependence is realized when χ field decays to bosons φ due to the interaction of the form $g^2\chi^2|\varphi^2|$. Qualitatively, these results will be valid for fermions, as well.

Formula (1) can express creation probability in this case as well if p is substituted by $\sqrt{\vec{p}^2+m_\varphi^2}$ leaving d^3p not altered . In the limit of large m one can obtain

$$N_\chi = \sum_n \frac{m_0\omega^2}{2\pi^3}\sqrt{m_0(n\omega-2m_0)}\,\exp\left(-\frac{2m_0}{\omega}\ln\frac{16m_0(n\omega-m_0)}{e\,m_1^2}\right)\sim$$
$$\sim \omega^{5/2}m_0^{3/2}\left(\exp\frac{16}{e}\frac{m_0^2}{m_1^2}\right)^{-\frac{2m_0}{\omega}}.$$

5. Reheating

Let us use the results obtained for the evaluation of the Universe reheating temperature after the particle creation by the inflaton field. To the moment when $\chi(t)$ starts to oscillate about its equilibrium point the Universe expansion is described by the nonrelativistic equation of state $p=0$ (matter dominance stage) and the Hubble parameter is

$$H = \frac{2}{3}\frac{1}{t+t_0}. \qquad (17)$$

The variation of $\chi(t)$ at this stage is determined by the expression

$$\chi(t) = \frac{m_{Pl}}{\sqrt{3\pi}\,m_\chi}\frac{\sin m_\chi(t+t_0)}{t+t_0}, \qquad (18)$$

where $m_{Pl} \simeq 10^{19}\,GeV$ is the Planck mass and m_χ is the mass of field χ . The decrease of $\chi(t)$: $\chi(t)\sim(t+t_0)^{-1}$ is connected with the Universe expansion. The effects connected with the particle production are not considered. The numerical coefficient in expression (18) is defined by the condition that at the initial moment $t=0$ the energy density of χ is equal to the closure density ρ_c . It is assumed that $m_\chi > H$. This allows in particular to use the obtained above results for particle production rate . The decrease $\chi\sim(t+t_0)^{-1}$ is taken into account adiabatically.

The energy density of the created relativistic particles satisfies the equation

$$\dot{\rho}_f = \Gamma_\chi\rho_\chi - 4H\rho_f. \qquad (19)$$

The total energy density evidently remains equal to the closure density. Thus during the period when ρ_χ dominates and the non-relativistic expansion law (17) is valid the following equation is fulfilled

$$\rho_\chi + \rho_f = \frac{m_{PL}^2}{6\pi (t+t_0)^2} . \qquad (20)$$

Substituting it into eq. (19) and integrating the latter with the initial condition $\rho_f(0)=0$ we obtain at MD -stage

$$\rho_f = \frac{\Gamma_\chi^2 m_{PL}^2}{6\pi} \frac{\exp(-\Gamma_\chi t)}{[\Gamma_\chi (t+t_0)]^{2/3}} \int_0^{\Gamma_\chi t} dx\, x^{2/3} \exp(x) . \qquad (21)$$

This result is approximately valid till the moment when $\rho_f \simeq \rho_\chi$ which is realized at $\Gamma_\chi t \simeq 1.3$. The reheating temperature in this case is

$$T_r \simeq \left(\frac{3}{50\pi^5 k}\right)^{1/4} g \sqrt{m_\chi m_{PL}} , \qquad (22)$$

where k is the number of effective degrees of freedom in the primeval plasma. It is assumed of course, that thermal equilibrium is established. Expression (11) for Γ_χ was used here. Substitution of the quasiclassical result (15) leads to a smaller reheating temperature.

Note that our result is about two times larger than the naive estimation $T_r = (30\rho_c / \pi^2 k)^{1/4}$ where ρ_c corresponds to the moment $t = \Gamma_\chi^{-1}$.
Let us note, that at the earlier stage, when fermion energy is small $\rho_f << \rho_\chi$, the plasma temperature may be higher, thanks to the highher total energy density, which is proportional to $(t+t_0)^{-2}$. Formally, as it can be seen from the condition $\dot\rho_f = 0$, the energy density of fermions has its maximum at $t/t_0 = O(1)$. However, one can speak about particle creation only when $t > m_\chi^{-1}$. Hence the maximum value of ρ_f must be estimated at $t\, m_\chi \simeq 1$:

$$\rho_{f\,max} \simeq g^2 \frac{m_{PL}^2 m_\chi^2}{24\pi^2} O(1) ,$$

which corresponds to temperature $T_{max} \simeq (5/4\pi^4 k)^{1/4} \sqrt{g m_{PL} m_\chi}$ Yet, one must keep in mind, that the part of the plasma energy contained in fermions is quite small $\rho_f / \rho_{tot} \simeq (g^2/4\pi)$. The value of the constant g is restricted by the condition $(g^2/4\pi)^2 < 10^{-12}$ which is necessary to obtain the small value of the inflation self interaction coupling constant $\lambda \sim g^4$. Otherwise, at great λ the Universe density perturbations are known to be too large. By the same reasons the condition $m_\chi / m_{PL} < 10^{-5}$ sould be fulfilled. These constraints lead to quite a low reheating temperature, which is fatal for the bariogenesis processes in the Universe at the GUT scale $E \sim 10^{14} - 10^{15}$ GeV .

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