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ANTIPARTICLES

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1. Introduction of the Minkowski space (MS) led to the unification of space and time into a sole four-dimensional manifold, thus reinforcing the interrelation between space and time coordinates following from the Lorentz transformations. It was as if time had acquired the properties of an ordinary coordinate. Naturally, the properties of the MS are not, generally speaking, equivalent to those of the ordinary Euclidean space. A direct consequence of introduction of the MS, and of the related condition of relativistic covariance, was the prediction of objects with negative energies, moving backward in time. Actually, symmetry of the MS with respect to time reversal /1/ served as the basis for establishing this fact, while application of the Stückelberg - Feynman reinterpretation principle* /2,3/ made it possible to identify such objects with the observed antiparticles.

We shall discuss the peculiarities of the indicated approach as well as the consequences following from it. We shall also touch upon the interrelation between space reflection and time reversal.

2. Symmetry of the MS and antiparticles. Symmetry of the MS with respect of reflections represents a consequence of the laws of Nature being (to a high degree) invariant relative to spatial inversion, P , and time reversal, T . What concerns spatial inversion, for instance specular reflection, the latter is quite a common notion. We are absolutely not used to the operation of time reversal, however, since in the macroworld time flows in a definite direction.

Naturally, the fact itself that to each elementary process there corresponds a reversed (inverse) process was known long ago. T-invariance was, generally, considered a common property inherent in all motions governed by any forces.

A result peculiar to the theory of relativity, however, is that within its framework the energy p^0 of an object is defined as

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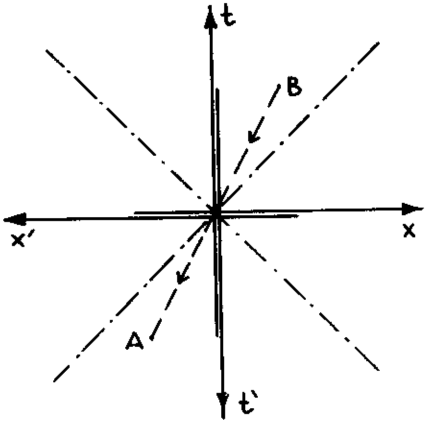
[*] *See also Ref.* /4/ *for References related to this issue.*

$$p^\circ = m\frac{dx^\circ}{d\tau}, \quad c = 1, \tag{1}$$

where m is the object's mass, $x^\circ = t$, τ is the proper time. Therefore, $t = -|t|$ will correspond to the object moving with negative energy, $p^\circ = -|E|$, backward in time, similar to motion in the negative direction along the x-axis with a momentum $p^1 = -|p^1|$. In the general case we will, obviously, have motion in the opposite 4-direction of a particle of momentum $p^i = -|p^i|$.

To better apprehand the physical meaning of the last result we turn to the Figure. In it the space-time picture is presented of the motion of a particle (in a plane MS) of energy E from point $A(-t, -x)$ to point $B(t, x)$; the arrows indicate the direction of the particle spin s_{BA} (the helicity $\lambda = -1/2$). Let the charge of the particle be q^-. Now we shall pass to the (t', x')-map which is derived from the initial (t, x)-map with the aid of 2-inversion* . In the (t', x')-map the particle moves from $A(t', x')$ to $B(-t', -x')$ exhibiting an energy of $-E$. Thus, at point A the energy increases by the quantity E, the spin s_{AB} and a charge q^+ appear; at point B the energy decreases by the same quantity and the spin s_{BA} appears together with the charge q^-. But such a picture of a particle of negative energy moving backward in time is in absolute disagreement with our every-day macroscopic experience based on the existence of "time arrows". Indeed, since we pertain to the macroscopic world, we are not even capable of "seeing" a particle moving backward in time. Therefore, taking advantage of usual language (the reinterpretation principle) we shall interpret this phenomenon as follows.



At the moment of time $-t'$ a particle of energy E and charge q^+ leaves point $B(-x')$; at the time t' it arrives at point $A(x')$, its helicity is $\lambda = 1/2$. But this is just

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* *In the general case this will be 4-inversion.*

what we usually call an antiparticle. Hence it is evident that its other characteristics (mass, lifetime, magnetic moment, etc.) must necessary remain intact. The above transition is in some way similar to T-inversion. It is, however, obvious that here the directions of the t'- and x'-axes remain unaltered, as do the directions of the dual E'- and p'-axes. In accordance with the reinterpretation procedure the initial and final states are exchanged (the source A and the detector B exchange roles), which leads to a change in sign of the energy, momentum, charge and helicity of the particle.

Of course, it must be admitted that backward motion in time itself is not something totally unusual. It is sufficient to recall backward movies. The same can be said about negative energy. Here the principle of detailed balance may serve as an example, the test of which in strong interactions, for instance, is performed with the aid of direct and inverse reactions:

$$^{24}Mg + d \leftrightarrows {}^{25}Mg + p. \tag{2}$$

What is important here is that while the direct reaction involves release of energy $(E_{init} = 0, E_{fin} = |E|)$ then the inverse one involves its consumption $(E_{init} = 0, E_{fin} = -|E|)$*

However, the backward motion in time of an object, which, besides, exhibits negative energy, is totally at variance with our every-day experience. We shall, of necessity, perceive such motion to be "reinterpreted". Figuratively, this is like when the images of objects seen by the eye are turned upside down (reinterpreted).

3. *The Dirac vacuum.* Evidently, the above considered approach essentially represents an alternative to the conventional concept based on the existence of the Dirac vacuum. Indeed, within this approach there is no need of artificial introduction of a non-observable "endless sea" of occupied states (from nearly the entire spectrum of elementary particles, at least, of its fermion part) with negative energies. This is not to mention the fact that the electron vacuum initially introduced by Dirac had to have infinite charge. The latter

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**By analogy with the above the preceding approach can be considered a sort of application of the principle of detailed balance to the motion of an individual particle.*

could, for example, be compensated by the infinite charge of the proton vacuum. But what is one to do with the infinite charge of the muon vacuum?... Another difficulty is related to the decay of non-stable fermions. It seems one can avoid it, if one considers all the possible states for the decay products to be occupied, and so on. Truly, it is not easy to renounce the notion that charged particles occupying the negative levels make up a complicated system, since they interact electrically (and not only electrically) between themselves. But the most essential difficulty seems to be that the Dirac vacuum must possess infinite mass and, in particular, an infinite continuous spectrum of state of negative energy.

At the same time the following must be noted. The introduction of states of negative energy is actually based on the presence of a negative sign in the relativistic formula for energy: $E = \pm\sqrt{m^2 + (\vec{p}\,)^2}$. Now, since this formula holds also for bosons, we should admit the existence of such a "sea", for example, in the case of pions. By the way, the discovery of the antideuteron may serve as an argument in favour of the above. As a result the Dirac vacuum should expand infinitely. Moreover, since the Pauli principle no longer restricts the number of particles occupying a sole state, bosons will undergo transitions to levels of negative energy and be continuously accelerated in doing so, i.e. their energy $E \to -\infty$.

Luckily, it seems that, in accordance with modern concepts, part of the indicated difficulties (for instance, the last one) can be overcome. Indeed, in accordance with the quark model of hadron structure all the non-leptonic part of the "traditional" Dirac vacuum is merely reduced to a "sea" of quarks with negative energies. Nevertheless, even such a drastic simplification of the structure of the Dirac vacuum does not eliminate all difficulties. Maybe, the most essential difficulty consists, for example, in that, as before, the "simultaneous" production is forbidden of two or more particle-antiparticle pairs with identical energy-momentum characteristics. This is a consequence of there existing only a single quark with given p^1 in the "sea", since the existence of several "seas" of identical quarks should be forbidden by the Pauli principle. At the same time within the framework of such lepton-quark vacuum there still remain difficulties related to its infinite negative charge, mass and energy.

However, the existence itself of an infinite continuum of particles with negative energies does give rise to a certain dissatisfaction, since this actually signifies non-symmetry of the dual MS with respect to the charge in sign of E . With account of formula (1), establishing a rigorous relationship between the direction in which time flows and the sign of the energy, this ultimately should signify, also, non-symmetry of the initial, or fundamental, MS.

It seems that the only way to eliminate the difficulties listed above, as well as others, consists in the application of the above considered based on the interpretation of antiparticles as objects moving backward in time with negative energy.

4. Non-conservation of lepton charge. It must be pointed out that the prediction of antiparticles was essentially already made in the fundamental work published by Minkowski in 1908 /5/ * , in which, in particular, formula (1) was introduced. Truly, it is anyhow difficult to do without the reinterpretation principle. But, maybe the most essential thing is the following: what we term antiparticles simply reflects the joint influence of the properties of the MS proper and of its conjugate energy-momentum space. Therefore, questions concerning such issues as the equality of the masses of particles and antiparticles, their lifetimes and so on (in the conventional approach such issues require special proofs) do not even arise at all.

As follows from the above consideration, the T- operation essentially leads to antiparticles which differ from the respective particles by the sign of one or another charge. Therefore, it might seem that violation of T-invariance should lead to the "introduction" itself of antiparticles being impossible. We recall that the observed non-conservation of the combined CP-parity in the decays of K°-mesons actually signifies violation of T-invariance in weak interactions. Therefore, the previous assertion is evidently first of all related to particles only participating in these interactions. But neutrinos are such particles, and they differ from antineutrinos by the sign of their lepton charge. From the above it follows that violation of T-invariance must be accompanied

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[*] *That is long before the well-known article by Dirac* /6/ .

by violation of the lepton charge conservation. A characteristic example here can be provided by the $K_{\ell 3}$-decays:

$$K_L^\circ \to \pi^- + \ell^+ + \nu_\ell \qquad (\ell = e, \mu) \tag{3a}$$

$$K_L^\circ \to \pi^+ + \ell^- + \bar{\nu}_\ell \, . \tag{3b}$$

Non-conservation of the lepton charge signifies the possibility of the corresponding antineutrino $\tilde{\nu}_\ell$ being produced in the first reaction and of the neutrino ν_ℓ in the second reaction. Naturally, there exists a temptation to admit a "strong" violation of the law of lepton conservation, when, for instance, in K_{e3} -decays there will be produced $\nu_\mu (\tilde{\nu}_\mu)$, and vice versa. In any case, a check of the noted possible non-conservation of the lepton charge would obviously be interesting. This is so, even though the expected effect should be quite small owing to the extremely weak violation of CP-symmetry.

5. *The interrelation between the* P- *and* T*-operations* following from relativity theory has already been discussed previously /7/ . However, owing to the importance of this problem we shall once again take it up, and, taking into account the latest results, we shall also touch upon the charge C-operation.

Consider a (pseudo) scalar wave function $\Psi'(t',x')$ describing, for example, an object at rest. Now perform (for the special case of $t' = 0$) inversion of the space coordinate:

$$\Psi'(0,x') \to \mp \Psi'(0,-x'). \tag{4}$$

From the point of view of another inertial reference system S moving with respect to the initial one, S′ , this procedure will, on the basis of the Lorentz transformations, look like the following:

$$\Psi(t,x) \to \mp \Psi(-t,-x). \tag{5}$$

Consider a certain interaction (process) to take place as a result of which the S′-observer sees a mixture of states appearing with a spatial parity (P) differing from the initial one. In other words, the interaction proceeds with violation of P-parity. From the point of view of the S-observer, however, the appearance of a mixed state may be related either to the violation of spatial or time parity, or to the combined action of both factors. In the general case ($t' \neq 0$) spatial inversion in its own right is not a covariant operation from the point of view of an S-observer.

Moreover, we recall the representation of t and x through the operational coordinates t_- and t_+^* ,

$$t = \frac{1}{2}(t_+ + t_-), \quad x = \frac{1}{2}(t_+ - t_-). \tag{6}$$

From (6) it is obvious that a charge in the sign of x indicates transition to $-t_+$ and $-t_-$, while this, in turn, means a change in the direction of time (inversion of the time coordinate t), i.e. essentially a 2-inversion. In the general case, for example, within the framework of the "Cartesian model" /9/ this will represent a 4-inversion. In this case, however, it seems absolutely not understandable that the violation of T-invariance is significantly weaker than of P-invariance. Truly, a certain explanation may consist in that the interaction depends on the reversed coordinate. Than, as it follows from (6), an agreement with experiments can be achieved for $t_+ \approx t_-$. But at any rate one should (once again as a result of the introduction of the MS) speak about relativistic reflection (**R**) , i.e. of the joint operation PT** .

It must be underlined that the issue of the validity of parity conservation, naturally, is broader than the framework of relativity theory. However, as it follows from the preceding arguments pointing to the close interrelation between the P - and T- operations, within the frame of relativity theory non-conservation of spatial parity must necessarily lead to a violation of T-invariance. Actually, this fact was confirmed by the well-known experiment of Ref. /11/ , in which the non-conservation of combined parity was first observed.

Let us once again turn to the Figure. Again we perform the **R** -operation, i.e. pass to (t', x') -map. But, besides this, we shall, in accordance with the C-operation, change the direction of the "charge axis", therefore at A we shall now have q^+ (and helicity $\lambda = 1/2$). On the basis of the reinterpretation principle, and taking account of the results of

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* *Or variables of the light front, which in the case of a plane MS have a very simple physical meaning: they represent the departure time and the receiving time of a light signal in a radar experiment (see, for example, Ref.* /8/ *).*

** *Usually one is said to deal with a strong reflection of the time space* $\mathbf{R}_s$ *(see, for example, Ref.* /10/ *).*

section 2, we shall interpret the corresponding process as follows. A particle of energy E , charge q^- and helicity $\lambda = -1/2$ departs from point $B(-x')$ at the moment of time $-t'$. Now, this is just what we call a particle. Thus, while the R -operation led to the antiparticles, the C-operation brought us back to the initial state. The latter condition can be written down with the aid of the formula TPC =1. But, this, obviously, is just what is usually called the CPT -theorem /12/ , which was introduced only within the framework of quantum field theory.

6. Conclusion. As one of the principle consequences of the introduction of the MS and of its conjugate energy-momentum space we have considered the backward motion in time of particles with negative energies. With account of the reinterpretation principle this leads to the existence of antiparticles. The proposed approach exhibits significant advantages as compared to the conventional approach based on the existence of the Dirac vacuum. One of its most characteristic consequences consists in that the violation of CP-invariance must be accompanied by violation of the law of lepton charge conservation (for example, in $K_{\ell 3}$-decays). Relativization of the space-time coordinated also permitted to establish the interrelation between spatial and time reflections. This means, for instance, that non-conservation of space parity must necessarily be related to the violation of T-invariance. The CPT-theorem actually reflects the symmetry properties of the MS.

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