## $9-306$



## объединенный <br> ИНСТИТУт <br> ядерных исслөдовании <br> дубна

E2-89-306
L.P.Kaptari, A.I.Titov, E.L.Bratkovskaya*,
A. Yu.Umnikov*

MESON EXCHANGE CORRECTIONS
IN DEEP INELASTIC SCATTERING
ON DEUTERON

Submitted to "Nuclear Physics A"
*Far-East State University, Vladivostok, USSR

## 1. Introduction

Despite a great amount of studies of structure functions of deep inelastic scattering (DIS), the keen interest is still shown in this problem, in particular, owing to recent experimental data obtaned by EMC [1] on DIS of polarized protons. It has been found that the fraction of proton spin carried by the quarks is consistent with zero. However, this conclusion essentially depends on the assumption of the behaviour of quark distributions $q(x)$ and structure functions $F_{1,2}^{A}(x)$ at small $x$ [2]. Analogous situation should be, obviously, take place in DIS on polarized nuclei with a spin $J>1 / 2$. In this case, the number of structure functions (independent invariants) increases [3], and the determination of each of them depends on the choice of the corresponding asymmetries and nuclear structure functions $F_{1,2}^{A}(x)$. All these necessitates a more detaled study of quark distributions in nuclei. All the existing theoretical approaches describing nuclear structure functions $F_{1,2}^{A}(x)$ and explaining their difference from the corresponding functions of free nucleons can be conventionaly divided into two classes [4]. The first of them assumes the change of quark distributions of nuclear nucleons at the expence of a possible change in a nucleus of the conditions of $Q^{2}$-evolution of nucleon structure functions ( $Q^{2}$-rescaling) [5], the fermi motion of nucleons being negleted. In this approach, the nuclear structure functions at point $x$ and $Q^{2}$ are expressed in terms of nucleon functions $F_{1,2}^{N}(x)$ at the same $x$ but at another $Q^{2}$, i.e. $F_{1,2}^{A}\left(x, Q^{2}\right)=F_{1,2}^{N}\left(x, \xi Q^{2}\right)$. A free parameter $\xi$ at present can neither be calculated theoretically nor determined from independent experiments. Therefore, $\xi$ is chosen from the condition of the best fit of experiments (EMC, BCDMS, SLAC ...). And vica versa, the second class explains the difference between $F_{1,2}^{A}$ and $F_{1,2}^{N}$ at the cost of interclear motion of nucleons with off-mass-shell effects taken into account, leaving the quark distributions of the nucleons themselves in a nucleus without change. This approach is based on the known fact of nuclear physics that the properties of quasiparticles-nucleons, differ from the free nucleons. Specifically, the bound nucleons have an effective mass that depends on the shell energy. This leads to renormalization of the scaling variable $x \rightarrow m / m^{*} x$ ( $x$-rescaling) [6]. This approach seems to be more preferable as it does not contain a free parameter. However, owing to the nucleons being bound here arises the problem with breaking of the energy sum rule (SR). It turns out that the fraction of the momentum carryed by quarks in a bound nucleons is smaller than in a free nucleon. An attempt may be undertaken to restore the SR trough taking meson degrees of freedom in nuclei into account because qualitatively it is clear that mesons should contribute to $F_{1,2}^{A}(x)$ at small $x\left(x \leq \mu_{\pi} / m\right)$ [7]. Until now the contribution of meson exchange currents to DIS was taken into account phenomenologically since model parameters were fixed from the same experiments. Therefore, the problem on an absolute contribution of meson dergees of freedom to the DIS cross section and ability of meson currents to restore the SR is still open. This paper is aimed at solving this problem by a common description of the off-mass-shell effects of nucleons and meson degrees of freedom in DIS of leptons on nuclei. A nucleus is represented as a system of interacting nucleon and meson fields; an equation for the nucleon wave function in a nucleus and main meson-exchange diagrams contributing to the structure function $F_{2}^{A}(x)$ are defined in a self-consistent way. We consider the simplest "exactly solvable" nuclear system, the deuteron, for which this problem can be solved. Self-consistency means that the same mesons (with their constans of meson-nucleon interaction) define both the Shrodinger equation with a OBE-potential (for instance, in the potential of the Bonn group [8], these are $\pi-, \omega-, \sigma-\cdots$-mesons) for the deuteron wave function and main characteristics
of DIS: structure functions, a contribution of exchange meson currents [9], boundedness effects, the energy SR.

## 2. Amplitude of DIS on a deuteron

Nonrelativistic reduction of the system of equations of interacting fields and separation of the contribution of meson exchange currents in various elastic and quasielastic processes have been developed by many authors [10,11]. Being different, their theoretical methods and used approximations, nevertheless, produce almost the same results. A common way for calculating matrix elements of an operator $\hat{O}$ over states of a nucleus (in our case, of the deuteron) is as folows:

1. transition to a nonrelativistic limit in the equations of motion of interacting mesonnucleon fields and derivation of an equation for the wave function of the nucleus;
2. a covariant relativistic representation of the operator $\hat{O}$ in terms of nucleon and meson fields;
3. nonrelativistic reduction of the operator $\hat{O}$ and computation of the corresponding matrix elements.

Further we shall follow a method elaborated in ref. [10] for the description of elastic and quasielastic electron-deuteron scattering. Prior to recall the method, note that all the computations will be made accurate to $g^{2}(g$ - is the meson NN interaction constant), following ref. [10]. Beside, for simplicity, we will make calculations only for pseudocalar isovector mesons (pions), as the contribution of other (vector and scalar) mesons is computed in a similar manner, and the relevant expression are reported in the Appendix.

Classical equations of motion of nucleons interacting with mesons are of the form:

$$
\begin{gather*}
(i \hat{\partial}-\mathrm{m}) N(x)=i g \gamma_{5} \Phi(x) N(x)  \tag{la}\\
\left(\square+\mu_{\pi}^{2}\right) \Phi_{i}(x)=-i g \bar{N}(x) \gamma_{5} \tau_{i} N(x), \tag{1b}
\end{gather*}
$$

where $N(x)$ and $\Phi(x)$ are nucleon and pion fields, respectively $\left(\Phi(x) \equiv \tau_{i} \Phi^{i}(x), \mathrm{i}=1,2,3\right)$, m and $\mu$ are the corresponding masses. Once small components $\phi(x)$ are eliminated from nucleon spinors $N(x)$ we arrive at an equation of motion for large components $f(x)$. The functions $f(x)$ have no probabilistic interpretation [12] owing to the violation of normalization conditions and sum rules for the baryon charge. Therefore another function $\psi(x)=(\hat{I}+\hat{F}) f(x)$ is introduced which satisfies the required conditions. The equation of motion for $f(x)$ and normalization conditions for $\psi(x)$ result in an equation of motion for the field $\psi(x)$, that in the leading order of the expantion in powers of the operator $\hat{F}$ coincidies with the Schrodinger equation with the Yukava potential:

$$
\begin{equation*}
i \dot{\psi}(\mathbf{x})=\frac{\Delta}{2 m} \cdot \psi(\mathbf{x})-\frac{g^{2}}{4 m^{2}} \int d^{3} \mathbf{y} \partial_{\mathbf{x}} Y(\mathbf{x}-\mathbf{y}) \partial_{\mathbf{y}}\left(\psi^{+}(\mathbf{y}) \tau_{i} \sigma \psi(\mathbf{y})\right) \tau^{i} \sigma \psi(\mathbf{x}) \tag{2}
\end{equation*}
$$

where the equal-time Green function $Y(\mathbf{x}-\mathbf{y})$ is defined by (1b) provided that no pion fields are present in an initial and final state:

$$
\begin{equation*}
Y(\mathbf{x})=\frac{\exp \left(-\mu_{\pi} \mathbf{x}\right)}{4 \pi \mathbf{x}} \tag{3}
\end{equation*}
$$

In the second quantization representation, $\psi(x)$ and $\psi(x)$ in (2) are operators of creation and annihilation of nucleons (unlike the relativistic equations (1), there are no antinucleons) and they can be used to obtain any multinucleon states. Analogous creation and annihilation operators exist for meson fields.

The ground state of the deuteron, $\mid D>$, is given by an expansion over states with two bare nucleons and $n$ mesons ( $n=1,2, \ldots$ ) which are obtaned by action of the above creation operators onto the vacuum:

$$
\begin{equation*}
\left|D>=\sqrt{1-Z} \alpha_{0}\right| N N>+\alpha_{1}\left|N N \pi>+\alpha_{2}\right| N N \pi \pi>+\cdots \tag{4}
\end{equation*}
$$

where the normalization constant $Z$ is defined from the condition $\langle D| D>=1$, and the expantion coefficients $\alpha_{i}$ are determined from the equation:

$$
\begin{equation*}
\left(H_{0}+H_{\text {int }}\right)\left|D>=M_{D}\right| D>. \tag{5}
\end{equation*}
$$

Here the Hamiltonian $H_{i n t}$ is expressed in terms of the nucleon and pion fields as follow:

$$
\begin{equation*}
H_{i n t}=i g \int d x \bar{N}(x) \gamma_{5} N(x) \Phi(x) \tag{6}
\end{equation*}
$$

Projecting (5) onto various states (4) we may determine the wave functions $\alpha_{i}$ explicitly. It turns out that $\alpha_{0}$ obeys the Schrodinger equations with a one-boson exchange potential, and $\alpha_{1,2}$ are functions of $\alpha_{0}$ and matrix elements $H_{i n t}$. The general form of functions $\alpha_{i}$ $(i=0,1,2)$ can be found in ref. [10]. In that follows we will assume that the wave function $\alpha_{0}$ is a solution to the Schrodinger equation, with a one-boson exchange potential taken to be a realistic meson exchange potential of the Bonn group [8]. Then matrix elements of operators describing DIS processes will be completely determined by parameters of the corresponding mesons (coupling constants, masses, vertex form factors).

Recall that a central quantity measured in DIS reactions is the nuclear (deuteron) tensor $W_{\mu \nu}^{A(D)}$ related by the optical theorem with the imaginary part of the forward elastic scattering amplitude of virtual gamma-quantum by nucleus target:

$$
\begin{equation*}
M_{D} W_{\mu \nu}^{D}=I m T_{\mu \nu}^{\gamma} \tag{7}
\end{equation*}
$$

$T_{\mu \nu}$ are the matrix elements of some operator $\hat{\mathrm{O}}_{\mu \nu}^{D}$ over the deuteron ground state. In our case this operator may be represented by a sum of two operators, $\hat{\mathrm{O}}_{\mu \nu}^{D}=\hat{\mathrm{O}}_{\mu \nu}^{N}+\hat{\mathrm{O}}_{\mu \nu}^{\pi}$, describing the Compton scattering on a nucleon and meson respectively.

$$
\begin{align*}
\langle D| \hat{\mathrm{O}}_{\mu \nu}^{D} \mid D> & =\frac{m}{M_{D}} \cdot T_{\mu \nu}^{D}\left(p_{D}, q\right) \\
\langle N| \hat{\mathrm{O}}_{\mu \nu}^{N}|N\rangle & =\frac{m}{\mathrm{E}} \cdot T_{\mu \nu}^{N}(p, q)  \tag{8}\\
\langle\pi| \hat{\mathrm{O}}_{\mu \nu}^{\pi}|\pi\rangle & =\frac{m}{\omega} \cdot T_{\mu \nu}^{\pi}(k, q)
\end{align*}
$$

Normalization in (8) is chosen so that the hadron tensors in (7) be of the same dimension. Let us construct operators $\hat{\mathrm{O}}_{\mu \nu}^{N, \pi}$ as quadratic forms in the relevant fields. Considering that for a polarized nucleon the relation [13]:

$$
\begin{equation*}
\hat{W}_{\mu \nu}^{N}(p, q, s)=W_{\mu \nu}^{N}(p, q) \frac{\bar{u} \hat{q} u}{p q} m, \quad \bar{u} u=1 \tag{9}
\end{equation*}
$$

holds valid, we obtain

$$
\begin{align*}
\hat{\mathrm{O}}_{\mu \nu}^{N} & =\sum_{s} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \sqrt{\frac{m^{2}}{E_{1} E_{2}}} T_{\mu \nu}^{N}\left(p_{2}, q\right) \frac{m}{p_{2} q} \cdot\left[a^{+}\left(p_{1}, s_{1}\right) \bar{u}\left(p_{1}, s_{1}\right)+\right. \\
& \left.+d\left(p_{1}, s_{1}\right) \bar{v}\left(p_{1}, s_{1}\right)\right] \hat{q}\left[a\left(p_{2}, s_{2}\right) u\left(p_{2}, s_{2}\right)+d^{+}\left(p_{2}, s_{2}\right) v\left(p_{2}, s_{2}\right)\right] \\
\hat{\mathrm{O}}_{\mu \nu}^{\pi} & =\frac{1}{2} \int \frac{d k_{1} d k_{2}}{(2 \pi)^{6}} \sqrt{\frac{m^{2}}{\omega_{1} \omega_{2}}} T_{\mu \nu}^{\pi}\left(k_{2}, q\right)\left[b^{+}\left(k_{1}\right) b\left(k_{2}\right)+b^{+}\left(k_{2}\right) b\left(k_{1}\right)\right] \tag{10}
\end{align*}
$$

Then, the nonrelativistic reduction of the operator $\mathrm{O}_{\mu \nu}^{N}$ gives

$$
\begin{equation*}
\mathrm{O}_{\mu \nu}^{N}(q)=\int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{m}{p_{20}+p_{2 z}}\left(1+p_{2 z} / m\right) T_{\mu \nu}^{N}\left(p_{2}, q\right)\left[a_{\sigma}^{+}\left(p_{1}\right) u_{\sigma}^{+} a_{\rho}\left(p_{2}\right) u_{\rho}\right], \tag{11}
\end{equation*}
$$

where $a_{\sigma, \rho}^{+}\left(a_{\sigma, \rho}\right)$ are operators of creation (annihilation) of a nucleon, $\sigma, \rho$ are spin-isospin indices $u_{\sigma, \rho}$ are spin-isospin functions; and besides we made use of the approximation $q p \approx q_{0}\left(p_{0}+p_{z}\right)$, where the axis $Z$ is directed opposite to the vector $\mathbf{q}$. From formulae $(4),(8),(10)$, and (11) we may derive the amplitude $T_{\mu \nu}^{D}$ to second order in the coupling constant $g^{2}$ :

$$
\begin{align*}
T_{\mu \nu}^{D} & =\frac{M_{D}}{m}\left(<N N\left|\alpha_{0}^{+} \hat{\mathrm{O}}_{\mu \nu}^{N}(q) \alpha_{0}\right| N N>-Z<N N\left|\alpha_{0}^{+} \hat{\mathrm{O}}_{\mu \nu}^{N}(q) \alpha_{0}\right| N N>+\right. \\
& +<N N\left|\alpha_{1}^{+} \hat{\mathrm{O}}_{\mu \nu}^{N}(q) \alpha_{1}\right| N N>+<N N \pi\left|\alpha_{1}^{+} \hat{\mathrm{O}}_{\mu \nu}^{\pi}(q) \alpha_{1}\right| N N \pi>+ \\
& \left.+<N N \pi \pi\left|\alpha_{2}^{+} \hat{\mathrm{O}}_{\mu \nu}^{\pi}(q) \alpha_{0}\right| N N>+<N N\left|\alpha_{0}^{+} \hat{\mathrm{O}}_{\mu \nu}^{\pi}(q) \alpha_{2}\right| N N \pi \pi>\right), \tag{12}
\end{align*}
$$

where various terms correspond to nonrelativistic diagrams in fig.1: a) impulse approximation (IA) b) renormalization of the wave function, c) nuclear recoil, d) and e) exchange diagrams.


Fig. 1

In formulae (10) and (12) we have not yet specified "elementary" amplitudies $T_{\mu \nu}^{N(\pi)}$ of the Compton scattering of a $\gamma$-quantum on bound nucleons and virtual pions. In the following we shall assume that the amplitudes of scattering on bound nucleons and virtual
mesons in the deuteron slightly differ from the free amplitudes; the latter will be taken from experiment.

## 3. Impulse approximation

Instead of the tenzor $W_{\mu \nu}$, it is more convenient to employ the structure functions $F_{1,2}(x)$ directly measured in experiment and connected with $W_{\mu \nu}$ by the relations: $F_{2}(x)=$ $-m x g_{\mu \nu} W_{\mu \nu}(p, q), F_{2}(x)=2 x F_{1}(x)$. A main contribution to (12) comes from the impulse approximation diagram of Fig. 1a. Recall that a standard computation of the nuclear structure function in IA gives a wrong behaviour throughout the whole range of variation of kinematic variable $x$ [14]. Modification of the IA consists in the inclusion of effects due to the nucleons being bound. This will provide a satisfactory description for the nuclear structure functions in the region of intermediate $x(x \geq 0.3)$ [6]. In our case the structure function $F_{2}$ is given explicitly by:

$$
\begin{equation*}
F_{2 D}^{I A}\left(x_{D}\right)=2 \int \frac{d p}{(2 \pi)^{3}}\left|\Psi_{D}(p)\right|^{2}\left(1+p_{z} / m\right) F_{2}\left(m x /\left(p_{0}+p_{z}\right)\right) \tag{13}
\end{equation*}
$$

where $x_{D}=-q^{2} / 2 M_{D} \nu, \quad x=-q^{2} / 2 M \nu$, and for convenience, instead of the deuteron wave function $\alpha_{0}$ we have introduced a more conventional notation, $\Psi_{D}$; boundedness effects are contained in $p_{0}$ and $p_{z}$. Note that expression (13) we have here obtained differs from the one in [6] by the factor $\left(1+p_{z} / m\right)$ that in our case appears automatically owing to the nonrelativistic reduction of the operator $\hat{\mathrm{O}}_{\mu \nu}^{N}$ (see (10),(11)). The necessity of inclusion of this factor into computation was discussed in ref.[15]. This factor does not change the general normalization of the function $\Psi_{D}$ and, consequently, does not break the conservation law of the baryon charge [ $13,15,16]$. Formula (13) can be written in a convolution form:

$$
\begin{equation*}
\frac{1}{2} F_{2 D}^{X A}(x)=\int d \xi F_{2 N}(x / \xi) f_{N / D}(\xi) \tag{14}
\end{equation*}
$$

where the function $f_{N / D}(\xi)$ means the probability of nucleon distribution in the deuteron with the momentum fraction $\xi$ :

$$
\begin{equation*}
f_{N / D}(\xi)=\int \frac{d p}{(2 \pi)^{3}}\left|\Psi_{D}(p)\right|^{2}\left(1+p_{z} / m\right) \delta\left(\xi-\left(p_{0}+p_{z}\right) / m\right) \tag{15}
\end{equation*}
$$

Now we are able to determine a mean momentum carried by quarks inside the deuteron and to compare it with the corresponding momentum of DIS on a free nucleon. So, from (15) we get

$$
\begin{equation*}
\left.\frac{2 m \int d x F_{2 D}^{I A}(x)}{M_{D} \int d x F_{2 N}(x)}=\frac{2 m<\xi\rangle}{M_{D}} ;<\xi\right\rangle \equiv \int f_{N / D}(\xi) \xi d \xi \approx 1+\frac{\langle\varepsilon\rangle}{m}-\frac{\left\langle p^{2}\right\rangle}{6 m^{2}} \tag{16}
\end{equation*}
$$

where $\langle\varepsilon\rangle$ is the binding energy in the deuteron and averaging is performed over deuteron ground state. From (16) it is seen that the off-mass shell effect leads to decreasing of the mean momentum carried by quarks in a nuclear medium compare to that in a free nucleon. In a "nucleon-meson" model, the mesons that make a nucleus bound also carry a certain portion of a momentum away. To estimate that portion and to compare it
with a lacking momentum in (16) we should compute meson corrections to the impulse approximation (diagrams(b)-(e) of Fig. 1.).

## 4. Meson corrections and energy sum rules

In the litetrature, diagrams b) and c) in Fig. 1 are conventionally called the renormalization and nuclear-recoil diagrams. Their explicit form is given in Appendix. We shall further neglect corrections for these diagrams for the following reasons: First, their total contributions to $\mathrm{SR}(16)$ is proportional to $g^{4}$ whereas our calculations are accurate up to $g^{2}$; second, analysis of their contribution to elastic and quasielastic processes performed by various authors $[10,11]$ has shown that it is either identical zero (the diagrams "cancel out"), or vanishes in the limit of small momenta transfer. As result the only exchange diagrams produced by exchange meson currents (MEC) are remained. So, for the pion we get:

$$
\begin{align*}
\frac{1}{2} F_{2 D}^{\pi}= & \int d \eta f_{\pi}(\eta) F_{2}^{\pi}(x / \eta) \\
f_{\pi}(\eta)= & \frac{g^{2}}{2 m^{2}} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{1}{\Omega \Omega^{4}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right)\left(\sigma_{1} k\right)\left(\sigma_{2} k\right)\left(\tau_{1} \tau_{2}\right) \Psi_{D}^{M}\left(p_{2}\right) \\
& \cdot\left(k_{0}+k_{z}\right) \delta\left(\eta-\left(k_{0}+k_{z}\right) / m\right) \Theta\left(k_{0}+k_{z}\right) \tag{17}
\end{align*}
$$

where $k=p_{1}+p_{2}$ is the pion moment um, and $\Omega^{2}=\left(k_{0}^{2}-\omega^{2}\right)$. Analogous expressions follow for other exchange mesons ( $\sigma, \omega, \ldots$ ). Now the conservation law for the total momentum carried by quarks and gluons in deuteron becomes follow:

$$
\begin{align*}
<x_{q}>_{D}=\frac{2 m}{M_{D}} & \left\{<x_{q}>_{N}<\xi>+<x_{q}>_{\pi}<\eta_{\pi}>+<x_{q}>_{\sigma}<\eta_{\sigma}>+\right. \\
& \left.+<x_{q}>_{\omega}<\eta_{\omega}>+\cdots\right\} \\
<x_{g}>_{D}=\frac{2 m}{M_{D}} & \left\{<x_{g}>_{N}<\xi>+<x_{g}>_{\pi}<\eta_{\pi}>+<x_{g}>_{\sigma}<\eta_{\sigma}>+\right. \\
& \left.+<x_{g}>_{\omega}<\eta_{\omega}>+\cdots\right\} \tag{18}
\end{align*}
$$

where the notation is obvious. Adding the right- and left-hand sides of (18) and using the equality $2 m / M_{D} \approx 1-<\varepsilon>/ 2 m$, we arrive to the sum rule for average momenta of the deuteron constituents:

$$
\begin{equation*}
1=1+\delta_{N}+\left\langle\eta_{\pi}\right\rangle+\left\langle\eta_{\sigma}\right\rangle+\left\langle\eta_{\omega}\right\rangle+\cdots ; \quad \quad \delta_{N}=\frac{\langle\varepsilon\rangle}{2 m}-\frac{\left\langle p^{2}\right\rangle}{6 m^{2}} \tag{19}
\end{equation*}
$$

Examine if the right hand side of (19) is equal unit. As an example, we assume that the deuteron contaits only pions. Then, instead of (19), we should obtain the equality: $\left.<\delta_{N}\right\rangle=-\left\langle\eta_{\pi}\right\rangle$ where in accordance with (17):

$$
\begin{equation*}
<\eta_{\star}>=\frac{g^{2}}{2 m^{2}} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6} 3} \frac{1}{\Omega^{4}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right)\left(\sigma_{1} k\right)\left(\sigma_{2} k\right)\left(\tau_{1} \tau_{2}\right) \Psi_{D}^{M}\left(p_{2}\right)\left(k_{0}+k_{z}\right)^{2} / m \Theta\left(k_{0}+k_{3}\right) \tag{20}
\end{equation*}
$$

Really, taking into account that $g^{2}\left(\sigma_{1} k\right)\left(\sigma_{2} k\right)\left(\tau_{1} \tau_{2}\right) / 4 m^{2} \omega^{2}$ represent the potential of pion nucleon interaction $V(k)$, and using the relation:

$$
\begin{equation*}
V(k) k^{2} / \omega^{2}=V(k)\left(1-\mu_{\pi}^{2} / \omega^{2}\right)=\left(1-\frac{1}{2} k \frac{d}{d k} V(k)\right) \tag{21}
\end{equation*}
$$

and the virial theorem $\left\langle r d V(r) / d r>=<2 T>\right.$, then in the static limit ( $k_{0} \approx 0, \Omega=\omega$ ) and $k_{z}^{2} \sim k^{2} / 3$ we get

$$
\begin{equation*}
\left.\left\langle\eta_{\pi}\right\rangle=-(<V\rangle+\langle V\rangle / 2+<T>\right) / 3 m \tag{22}
\end{equation*}
$$

where $T$ is the kinetic energy of nucleons in the deuteron. Substituting $\langle V\rangle=\langle\varepsilon\rangle$ $-\langle T\rangle$, into (22) we find that:

$$
\begin{equation*}
\delta_{N}=-\left\langle\eta_{\pi}\right\rangle \tag{23}
\end{equation*}
$$

i.e. the sum rule is exactly fulfilled in this case. It may be shown that (22)-(23) also hold valid for other mesons (for scalar mesons relation (23) has been derivated in ref.[13]); with the exception that for each of time, the virial theorem will contain an appropriate part of the potential, and as a result, we again get (19). So, in the static limit, the consideration of meson exchange corrections to the structure functions of DIS completely restores the energy sum rule broken in IA by the boundedness of nucleons. In the realistic case ( $k_{0} \neq 0$ ) this may be not valid. Note also that formula (23) is derived under the assumption that $g^{2}$ does not depend on $k^{2}$.

## 5. Numerical results and conclusions

In Fig. 2, we present the calculation ratio $R^{D / N}(x)$ of the deuteron structure function to the isoscalar-nucleon structure function. Curve $I$ is the impulse approximation; and curve 2 , is the calculation with the deuteron structure function by formulae (13) and (17). The deuteron wave function was determined by solving the Schrodinger equation with the "Bonn" potential [8], and the parameters of contribution of $\pi-, \omega-$ and $\sigma-$ mesons in meson exchange diagrams were taken the same as the NN-potential. Thus, the consideration is self-cosistent. A slight deviation from self-consistency is common to this approach and consist in that the main diagrams and Schrodinger equation for the function $\Psi_{D}$ are derivated in the $g^{2}$ approximation, however the wave function $\Psi_{D}$ in (17) and (20), in fact, contain all powers of $g^{2}$. Besides, in the realistic NN-potential we use


Fig.2. Ratio of the structure functions of the deuteron and free nucleon ( 1 - impulse approximation 2IA + MEC contribution ). the phenomenological meson-nucleons vertex form factors, i.e. $g^{2} \Rightarrow g^{2} \Lambda^{2} /\left(\Lambda^{2}+k^{2}\right)$. Therefore, even the contribution from the IA diagram depends on $\Lambda$ and $k^{2}$; and is thus to be taken into account in the exchange diagrams. This may violate relation (23) For numerical calculations we have employed the following expressions for the structure functions of DIS on free nucleons and mesons, $F_{2 N}$ and $F_{2}^{\mu}$

$$
\begin{align*}
F_{2 N}(x) & =\frac{5}{18}\left\{x^{0.58}\left(2.69(1-x)^{2.7}+1.56(1-x)^{3.7}\right)+0.8(1-x)^{7}\right\}  \tag{24}\\
F_{2}^{\mu}(x) & =\frac{5}{18}\left\{1.5 x^{0.5}(1-x)+40 \cdot 0.075(1-x)^{5}\right\} \tag{25}
\end{align*}
$$

From Fig. 2 one can see that the qualitative behaviour of the ratio $R$ is the same as for heavy nuclei, however, the depth of a minimum at $x \sim 0.6$ is much smaller owing to nuclear effects being weak in the deuteron. The calculated contribution of various mesons to the structure function is plotted in Fig.3. As can be seen, a dominant contribution comes from the pion exchange diagrams. Contribution of heavier mesons is suppresed by their large mass that enters into relevant propagators and by effects of scalar ( $\sigma$-meson) and vector ( $\omega$-meson) couplings. Numerical computation of sum rule (19) gave the following result: $\delta_{N}=0.0036$ and the total contribution of all the mesons $\sum_{\mu}<\eta_{\mu}>=0.0022$. So, it means that meson correction numerically only partially ( $\approx 60 \%$ ) restore the energy sum rule.
Formally, this discrepancy with (23) can be explained by going beyound the framework of the static approximation and by the $k^{2}$ dependence of form factors $g\left(k^{2}\right)$ that has not been taken into account in the derivation of (23). As a


Fig.3. The meson-exchange currents contribution to the deuteron structure function. Curves $1-3$ corespond to the contributions of $\omega-, \sigma-$, and $\pi$-mesons respectively; curve 4 is the sum over all the mesons. matter of fact, a completely consistent analysis of restoration of the energy sum rule requires to take into account

1. subsequent terms of the expansion in $g^{2}$;
2. the contribution of non-nucleon (multiquark) degrees of freedom in the deuteron. In doing so, one should remember that part of this contribution is phenomenologically taken into account in the realistic wave function of the deuteron;
3. the contribution of heavier mesons (which may imitate the $k^{2}$-dependence of vertex form factors). For instance, they can be meson-like objects, neutral with respect to the electromagnetic interaction (like "glueballs"[17]) contributing to the potential but not detected in DIS.

The authors are grateful to Burov V.V., Lukyanov V.K., Levin E.M., Dostovalov V.N. and Reznik B.L. for useful discutions.

Appendix

1. Renormalization and nuclear-recoil diagrams. a) renormalization diagrams:

$$
\begin{gather*}
\begin{array}{l}
W_{\mu \nu}^{\text {ren. }}(p, q)= \\
\quad-\frac{g^{2}}{4 m^{2}} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{1}{3 \Omega^{3}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right)\left(\sigma_{1} k\right)\left(\sigma_{2} k\right)\left(\tau_{1} \tau_{2}\right) \Psi_{D}^{M}\left(p_{2}\right) . \\
\text { b) recoil diagrams: }
\end{array} \int \frac{d p}{(2 \pi)^{3}}\left|\Psi_{D}(p)\right|^{2} \frac{\left(m+p_{z}\right)}{\left(p_{0}+p_{z}\right)} W_{\mu \nu}^{N}(p, q)
\end{gather*}
$$

$$
\begin{align*}
W_{\mu \nu}^{\text {ret. }}(p, q)= & \frac{g^{2}}{4 m^{2}} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{1}{3 \Omega^{3}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right)\left(\sigma_{1} k\right)\left(\sigma_{2} k\right)\left(r_{1} \tau_{2}\right) \Psi_{D}^{M}\left(p_{2}\right) . \\
& \frac{\left(m+p_{1 z}\right)}{\left(p_{10}+p_{1 z}\right)} W_{\mu \nu}^{N}(p, q) \tag{.42}
\end{align*}
$$

Considering that $<T>/ m \sim<V>/ m \sim<\varepsilon>/ m \sim g^{2}$ it may be shown that a total contribution to the energy sum rule from (A1) and (A2) is proportional to $g^{4}$.
2.Meson currents for $\sigma$ - and $\omega$-mesons: $\sigma$-mesons
$f_{\sigma}(\eta)=2 g_{\sigma}^{2} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{1}{3 \Omega_{\sigma}^{4}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right) \Psi_{D}^{M}\left(p_{2}\right) \cdot\left(k_{0}+k_{z}\right) \delta\left(\eta-\left(k_{0}+k_{z}\right) / m\right) \Theta\left(k_{0}+k_{z}\right)$
$\omega$-mesons
$f_{\omega}(\eta)=-2 g_{\omega}^{2} \int \frac{d p_{1} d p_{2}}{(2 \pi)^{6}} \frac{1}{3 \Omega_{\omega}^{4}} \sum_{M} \Psi_{D}^{M+}\left(p_{1}\right) \Psi_{D}^{M}\left(p_{2}\right) \cdot\left(k_{0}+k_{z}\right) \delta\left(\eta-\left(k_{0}+k_{z}\right) / m\right) \Theta\left(k_{0}+k_{z}\right)$
In (A3) and (A4) the propagator $\Omega$ is proportional to the corresponding meson mass and, as $\mu_{\omega}, \mu_{\sigma} \geqslant \mu_{\pi}$, the contribution of $\omega$ - and $\sigma$-mesons to the DIS cross section (and to the energy sum rule) is small as compared with the pion contribution.

## References

[1] EMC Collaboration. J. Asluman et al. Phys. Lett. 206B, 364 (1988).
[2] F.E. Close, R.G. Roberts. Phys. Rev. Lett. 60, 1471 (1988).
[3] P. Hoodbhoy, R.L. Jaffe, A. Monohar. Nucl. Phys. B312, 571 (1989).
[4] R.L. Jaffe. Nucl. Phys. A478, 3 (1988).
[5] F.E. Close, R.G. Roberts, G.G. Gross. Phys. Lett. 129B, 346 (1983); R.L. Jaffe, F.E. Close, R.G. Roberts, G.G. Gross. Phys. Lett. 134B, 449 (1984), Phys. Rev. D31, 1004 (1985); J. Cleymans, R.L. Thews. Phys. Rev. D31, 1014 (1985).
[6] S.V. Akulinichev, S.A. Kulagin, G.M. Vagradov. Phys. Lett. 158B, 475 (1985); JETP Lett. 42, 105, (1985); J. Phys. G11, L245 (1985).
[7] C.H. Llewellyn Smith. Phys. Lett. 128B, 107 (1983); M. Ericson, A.W. Thomas. Phys. Lett. 128B, 112, (1983); A.I. Titov J. Nucl. Phys. ISSR, 40, 76 (1984); E.L. Berger, F. Coester, R.B. Wiringa. Phys. Rev. D29, 398 (1984).
[8] R. Machleid et al. Phys. Rep. 149, 1 (1987).
[9] L.P. Kaptari, B.L. Reznik, A.I. Titov, A.Yu. Umnikov. JETP Let.t. 47(9), 428 (1988).
[10] M. Rosa-Clot, M. Testa. Nuovo Cim. 78A, 113 (1983).
[11] M. Cherntob. In: Mesons in Nuclei, Amsterdam, North-Hoiland, 555 (1976); G.E. Brown, A.D. Jackson. The Nucleon-Nucleon Interaction, New York (N.Y. 1976) M. Gari, H. Hyuga. Z. Phys. A277, 291, (1976); M. Kirchbach, E. Truglik. Particles and Nuclei, 17, 224 (1980); V.V. Burov, V.N. Dostovalov. Z. Phys. Atoms and Nuclei, 326, 245 (1987).
[12] A. Akhiezer, V. Berestetskii. Quantum Electrodynamics, New York (N.Y. 1965).
[13] B.L. Birbrair, E.M. Levin, and A.G. Shuvaev. Preprint LNPI-1227, Leningrad (1987).
[14] A. Bodek, J.L. Ritchie. Phys. Rev., D23, 1970 (1981), Phys. Rev., D24, 1400 (1981); L.L. Frankfurt, M.I. Strikman. Phys. Rep., 76C, 215 (1981); G. Berlad et al. Phys. Rev., D22,1547 (1980).
[15] L.L. Frankfurt, M.I. Strikman. Phys. Lett., 183B, 254 (1987).
[16] G.L. Li, K.F. Liu and G.E. Brown. Phys. Lett., 213B, 531 (1988) H. Jung, G.A. Miller. Phys. Lett., 200B, 351 (1988).
[17] Zhu Wei, Shen Jian Guo. Comm. Theor. Phys., v9, 51 (1988).

```
Received by Publishing Department
    on May 3, 1989.
```

