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MICROSCOPIC CALCULATION METHOD OF pd ELASTIC SCATTERING IN QUARK MODEL

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It is obvious that the deuteron cannot be treated as an n-p system if the internucleon distance is too small. In quark models which fit the baryon spectrum this leads to a large probability for a nucleon to be part of a six-quark cluster. Many theorists are seeking unique signals for such multiquark clusters. Electron scattering and other electromagnetic processes should be particularly good for studying the quark structure of a nucleus, since gluonic processes do not directly participate in electron scattering. The microscopic calculation of pd elastic scattering at high energies seems to be the best source of information about a short-distance nuclear structure.

The two- and three-body systems are most accessible to microscopic calculations. A great deal of experimental data on elastic pd scattering is available, and theoretical calculations in the framework of the multiple scattering theory are practically lacking.

In the paper one of the authors /1/ has evaluated the pd scattering cross section within the framework of the constituent quark model in Glauber formalism with allowance for the six-quark state admixture, using symmetrical Gaussian wave functions.

In general, calculating the elastic pd scattering one must take into account antisymmetrization effects and realistic wave functions of the deuteron.

Traditionally antisymmetrization of the deuteron wave function performed in Jacobian coordinates and matrix elements are calculated using Wheeler's method or generator coordinate method /2-4/ . The use of these functions for calculation of one-body operators, e.g. profile-operators, is very cumbersome.

Moreover, in Glauber's theory the amplitude of pd elastic scattering in the hybrid quark-nucleon model is determined by the sum of $2^{18}-1$ terms representing different rescattering processes. Among these terms there are many similar terms, that is why the amplitude is actually determined by a smaller number of essentially different terms. Reduction of similar terms in the scattering amplitude was shown in paper /1/ for the case of Gaussian wave functions. It requires any Gaussian representation for the realistic pairs wave function.

Three main purposes of the present work are description of the total antisymmetric deuteron wave function in one-particle coordinates, construction of a Gaussian representation for pairs wave function and microscopic calculation of the elastic pd-scattering cross section at high energies.

1. THE DEUTERON WAVE FUNCTION

At present the most adequate method for investigation of two-nucleon system at short distances is the resonating group method[5]. One can write down the wave function of the two-nucleon system in the form

$$\Psi_{NN} = \frac{1}{N}\Big(1 - \sum_{\alpha=1}^{3}\sum_{\beta=4}^{6}\hat{P}_{\alpha\beta}\Big)\Psi_N(r_1,r_2,r_3)\,\Psi_N(r_4,r_5,r_6)\,\chi(R). \qquad (1)$$

Here

$$N^2 = 10\,(1-\delta),$$

$$\delta = \langle\Psi_N(r_1,r_2,r_3)\,\Psi_N(r_4,r_5,r_6)\,\chi(R)|\hat{P}_{14}\,|\Psi_N(r_1,r_2,r_3)\Psi_N(r_4,r_5,r_6)\,\chi(R)\rangle - \qquad (2)$$

and $P\alpha\beta$ – is the quark permutation operator, Ψ_N is the quark wave function of a nucleon; $\chi(R)$ describes the relative motion of nucleon clusters; N is the normalization constant.

The total wave function of the deuteron with the admixture of the six-quark state can be written as:

$$\Psi_d = \alpha\Psi_{NN} + \beta\Psi_{6q}\,. \qquad (3)$$

Here Ψ_{6q} is the six-quark bag wave function. The space part of the quark wave function of the nucleon is taken here in the form of the ground state of the oscillatory model

$$\Psi_{n(p)} = \Big(\frac{\pi}{\alpha}\Big)^{3/2}\Big(\frac{1}{3}\Big)^{1/4} e^{-\frac{\alpha}{2}\sum_{i\in\{1,2,3 \atop 4,5,6\}}(r_i - R_{n(p)})}\,, \qquad (4)$$

and $\Psi_N = \Psi_{n(p)}\cdot\Psi_c$, Ψ_c is total antisymmetric colour function of a nucleon. The following approximations are most popular at present for relative motion wave function of nucleons

$$\chi(R) = \frac{1}{\sqrt{4\pi}} \frac{U(R)}{R} \chi_{1M} + \frac{W(R)}{R} \Sigma\,(2m\,1\mu\,|\,1M)\, Y_{2m}(\hat{R})\, \chi_{1\mu}\,, \tag{5}$$

where

$$\chi_{1\mu} = \Sigma\,(\frac{1}{2}\sigma_1\,\frac{1}{2}\sigma_2\,|\,1\mu)(\frac{1}{2}\tau_1\,\frac{1}{2}\tau_2\,|\,00)\,\Phi^{\frac{1}{2}\,\frac{1}{2}}_{\sigma_1\tau_1}\,\Phi^{\frac{1}{2}\,\frac{1}{2}}_{\sigma_2\tau_2}, \tag{6}$$

and $\Phi^{ST}_{\sigma\tau}$ is the symmetrical spin-isospin function of a nucleons.

The space part of the S- and D-state wave functions are taken here in the form /6/

$$U(R) = \sum_j C_j\, e^{-m_j R}, \tag{7}$$

$$W(R) = \sum_j D_j\, e^{-mR}(1 + \frac{3}{m_j R} + \frac{3}{m_j^2 R^2}). \tag{8}$$

The space configuration S^6 can be described by the only antisymmetrical spin-isospin-colour (SIC) wave function of the six-quark bag with the deuteron quantum numbers /7/. It can be written as follows

$$\Psi^{sic}_{6q} = (N^{sic}_{6q})^{-1}(1 - \sum_{\alpha=1}^{3}\sum_{\beta=4}^{6}\hat{P}_{\alpha\beta})\,\Psi^{sic}_{N}(1,2,3)\,\Psi^{sic}_{N}(4,5,6)\,F^{si}_{d}\,,\; N^{sic}_{6q} = 10/3. \tag{9}$$

Here Ψ^{SIC}_{N} is the SIC part of the nucleon wave function; F^{SI}_{d} is the spin-isospin part of the WF describing relative motion of three-quark clusters in a deuteron.

Using formulae

$$\frac{e^{-mr}}{r^n} = \frac{m^{n-1}}{(n-2)!}\int_1^\infty dR\,(R-1)^{n-2}\,\frac{e^{-mRr}}{r}\,, \tag{10}$$

we can write (7), (8) in the Gaussian representation; for example:

$$\frac{e^{-mr}}{r} = \int_0^\infty \frac{dx}{x^{3/2}} e^{-m^2 x - \frac{r^2}{4x}}$$

$$\frac{e^{-mr}}{r^2} = \frac{1}{4} \int_0^\infty \frac{dx}{x^2} \operatorname{erfc}(m\sqrt{x})\, e^{-\frac{r^2}{4x}},$$

$$\frac{e^{-mr}}{r^3} = \frac{1}{4\sqrt{\pi}} \int_0^\infty \frac{dx}{x^{5/2}} e^{-m^2 x - \frac{r^2}{4x}} - \frac{m}{4}\int_0^\infty \frac{dx}{x^2} \operatorname{erfc}(m\sqrt{x})\, e^{-\frac{r^2}{4x}}.$$

Next, using the expression

$$e^{-\frac{R^2}{x}} = \left(\frac{x}{\pi}\right)^{3/2} \int e^{-2i\beta R - \frac{\beta^2}{x}} d^3\beta, \qquad (11)$$

for the space part of $\chi(R)$ and with the help of relations

$$R\sin\theta\, e^{\pm im\phi} = (X \pm iY)^m, \quad R\cos\theta = Z, \qquad (12)$$

$$R = (r_1 + r_2 + r_3 - r_4 - r_5 - r_6)/3, \qquad (13)$$

we can write Ψ_d in the factorized form and antisymmetrization is usually carried out.

2. CALCULATION OF THE ELASTIC SCATTERING AMPLITUDE

According to the basic principles of Glauber's approximation, the differential cross section of nucleon-deuteron scattering is determined by the following expression

$$\mathfrak{F}(q) = \frac{iP}{2\pi} \int d^2\vec{b}\, e^{iq\vec{b}} \langle \Psi_p \Psi_d | [1 - \prod_{i=1}^{3} \prod_{j=1}^{6} (1 - \gamma(\vec{b} - \vec{s}_i + \vec{r}_j))] | \Psi_d \Psi_p \rangle, \qquad (14)$$

where P is the momentum at the projectile nucleon; q is the transverse momentum; Ψ_p is the wave function of nucleon, $\gamma(b)$ is the amplitude of elastic quark-quark scattering in the impact parameter representation. $\{s_i\}$, $\{r_i\}$ are the coordinates of the quarks of deuterons and protons within the plane of the impact parameter b.

As is seen from (14), the scattering amplitude is determined by the sum of $2^{18}-1$ terms representating different rescattering processes.

Among these terms there are many similar terms, that is why the amplitude is actually determined by a smaller number of essentially different terms. Using the algorithm for reduction of similar terms developed by one of the authors in/8,9/ we take 665 essentially different terms from the total of 162643.

Inserting (7) into (14) we obtain for the direct member:

$$\mathcal{F}_1(q) = \sum_{i,j=1}^{36} \int_0^\infty \frac{\mathcal{F}(x)}{\langle N \rangle} \frac{\pi k_i \cdot k_j}{|\mathrm{Det}\, W|} \left(-\frac{6a}{2\pi}\right)^{k_i+k_j} e^{-\frac{q^2}{4}\frac{|\mathrm{Det}\, Q|}{|\mathrm{Det}\, W|}} dx,$$

where $\langle N \rangle$ is the normalization integral, k_i is the number of black points in the scattering diagram/9/,

$$\mathcal{F}(x) = \sum_{k,\ell} \Big\{ \frac{3D_k D_\ell}{16\sqrt{\pi}} \left[\frac{1}{m_k} - \frac{m_k}{m_\ell^2}\right] \frac{1}{x^{5/2}} e^{-(m_k+m_\ell)^2 x} + \frac{9D_k D_\ell}{32\sqrt{\pi}} \times$$

$$\times \frac{1}{m_k m_\ell} \frac{1}{x^{7/2}} e^{-(m_k+m_\ell)^2 x} + \left[\frac{1}{16}\left(\frac{3m_k^2}{m_\ell^2}+1\right) D_k D_\ell + \frac{1}{4} C_k C_\ell\right] \times$$

$$\times \frac{1}{x^2} \operatorname{erfc}\left[(m_k+m_\ell)\sqrt{x}\right] - \frac{D_k D_\ell}{16 m_k^2} \frac{1}{x^3} \operatorname{erfc}\left[(m_k+m_\ell)\sqrt{x}\right] + \tag{15}$$

$$+ \frac{9D_k D_\ell}{128} \frac{1}{m_k^2 m_\ell^2} \frac{1}{x^4} \operatorname{erfc}\left[(m_k+m_\ell)\sqrt{x}\right]\Big\},$$

$$Q_{ij} = \begin{pmatrix} G_1 & G_2 & G_3 & G_4 & G_5 & G_6 \\ G_2 & U_2 & U_3 & U_4 & U_5 & U_6 \\ G_3 & U_3 & H_3 & H_4 & H_5 & H_6 \\ G_4 & U_4 & H_4 & X_4 & X_5 & X_6 \\ G_5 & U_5 & H_5 & X_5 & Y_5 & Y_6 \\ G_6 & U_6 & H_6 & X_6 & Y_6 & V_6 \end{pmatrix} \tag{16}$$

$$G_1 = -a\left[\frac{N_{11}^2(i)}{T_1} + \frac{N_{11}^2(j)}{T_4} + \frac{N_{21}^2(i)}{T_2} + \frac{N_{21}^2(j)}{T_5} + \frac{N_{31}^2(i)}{T_3} + \frac{N_{31}^2(j)}{T_6}\right] + D_1$$

$$G_2 = -a\left[\frac{N_{11}(i)\,N_{12}(i)}{T_1} + \frac{N_{11}(j)\,N_{12}(j)}{T_4} + \frac{N_{21}(i)\,N_{22}(i)}{T_2} + \right.$$

$$\left. + \frac{N_{21}(j)\,N_{22}(j)}{T_5} + \frac{N_{31}(i)\,N_{32}(i)}{T_3} + \frac{N_{31}(j)\,N_{32}(j)}{T_6}\right]$$

$$G_3 = -a\left[\frac{N_{11}(i)\,N_{13}(i)}{T_1} + \frac{N_{11}(j)\,N_{13}(j)}{T_4} + \frac{N_{21}(i)\,N_{23}(i)}{T_2} + \right.$$

$$\left. + \frac{N_{21}(j)\,N_{23}(j)}{T_5} + \frac{N_{31}(i)\,N_{33}(i)}{T_3} + \frac{N_{31}(j)\,N_{33}(j)}{T_6}\right]$$

$$G_4 = 0$$

$$G_5 = \frac{N_{11}(i)}{T_1} + \frac{N_{21}(i)}{T_2} + \frac{N_{31}(i)}{T_3}$$

$$G_6 = \frac{N_{11}(j)}{T_4} + \frac{N_{21}(j)}{T_5} + \frac{N_{31}(j)}{T_6}$$

$$U_2 = -a\left[\frac{N_{12}^2(i)}{T_1} + \frac{N_{12}^2(j)}{T_4} + \frac{N_{22}^2(i)}{T_2} + \frac{N_{22}^2(j)}{T_5} + \frac{N_{32}^2(i)}{T_3} + \frac{N_{32}^2(j)}{T_6}\right] + D_2$$

$$U_3 = -a\left[\frac{N_{12}(i)\,N_{13}(i)}{T_1} + \frac{N_{12}(j)\,N_{13}(j)}{T_4} + \frac{N_{22}(i)\,N_{23}(i)}{T_2} + \right.$$

$$\left. + \frac{N_{22}(j)\,N_{23}(j)}{T_5} + \frac{N_{32}(i)\,N_{33}(i)}{T_3} + \frac{N_{32}(j)\,N_{33}(j)}{T_6}\right]$$

$U_4 = 0$

$$U_5 = \frac{N_{12}(i)}{T_1} + \frac{N_{22}(i)}{T_2} + \frac{N_{32}(i)}{T_3}$$

$$U_6 = \frac{N_{12}(j)}{T_4} + \frac{N_{22}(j)}{T_5} + \frac{N_{32}(j)}{T_6}$$

$$H_3 = -a\left[\frac{N_{13}^2(i)}{T_1} + \frac{N_{13}^2(j)}{T_4} + \frac{N_{23}^2(i)}{T_2} + \frac{N_{23}^2(j)}{T_5} + \frac{N_{33}^2(i)}{T_3} + \frac{N_{33}^2(j)}{T_6}\right] + D_3$$

$H_4 = 0$

$$H_5 = \frac{N_{13}(i)}{T_1} + \frac{N_{23}(i)}{T_2} + \frac{N_{33}(i)}{T_3}$$

$$H_6 = \frac{N_{13}(j)}{T_4} + \frac{N_{23}(j)}{T_5} + \frac{N_{33}(j)}{T_6}$$

$$X_4 = -\frac{2}{9x}$$

$X_5 = 1$

$X_6 = 1$

$$Y_5 = -\left[\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3}\right] - 9x$$

$Y_6 = 0$

$$V_6 = -\left[\frac{1}{T_4} + \frac{1}{T_5} + \frac{1}{T_6}\right] - 9x$$

$$W_{ij} = \left(\begin{array}{cc} & W_1 \\ & W_2 \\ & W_3 \\ Q_{ij} & W_4 \\ & W_5 \\ & W_6 \\ \hline & W_7 \end{array}\right)$$

$W_1 = 0$

$W_2 = 0$

$W_3 = 0$

$W_4 = -1$

$W_5 = 1/2$

$W_6 = 1/2$

$W_7 = 0$

$E_1 = -a[N_{11}(i) + N_{12}(i) + N_{13}(i)] + \alpha$

$E_2 = -a[N_{21}(i) + N_{22}(i) + N_{23}(i)] + \alpha$

$E_3 = -a[N_{31}(i) + N_{32}(i) + N_{33}(i)] + \alpha$

$E_4 = -a[N_{11}(j) + N_{12}(j) + N_{13}(j)] + \alpha$

$E_5 = -a[N_{21}(j) + N_{22}(j) + N_{23}(j)] + \alpha$

$E_6 = -a[N_{31}(j) + N_{32}(j) + N_{33}(j)] + \alpha$

$D_1 = -a[N_{11}(i) + N_{21}(i) + N_{31}(i) + N_{11}(j) + N_{21}(j) + N_{31}(j)] + \alpha$

$D_2 = -a[N_{12}(i) + N_{22}(i) + N_{32}(i) + N_{12}(j) + N_{22}(j) + N_{32}(j)] + \alpha$

$D_3 = -a[N_{13}(i) + N_{23}(i) + N_{33}(i) + N_{13}(j) + N_{23}(j) + N_{33}(j)] + \alpha$

Using (12) and (13), one can write down the angular part of Ψ_d as

$(z_1 + z_2 + z_3 - z_4 - z_5 - z_6)(s_1 + s_2 + s_3 - s_4 - s_5 - s_6)$

where

$s = x \pm iy.$

Then after cumbersome calculation we obtain the following expressions for the angular (polynomial) part of the direct and exchange integrals of (14):

$$18(4s_i s_j z_k z_\ell - 2s_i z_j s_k z_k - 2z_i z_j s_k^2 -$$
$$- 2s_i s_j z_k^2 + s_k^2 z_k^2 + z_i^2 s_k^2).$$

In calculation of the exchange integrals representation (11) was used separately for functions Ψ_d and $\hat{P}_{14}\Psi_d$, matrices Q_{ij}, W_{ij} are determined as in the case of the direct integral.

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