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NONLOCAL RENORMALIZATION "STOPPING" THE RUNNING GAUGE

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1. Introduction

The problem of renormalization scheme choice in QCD does not lose its vitality. It is well known that in practical calculations momentum subtraction schemes turn out to be more preferable than MS ones. However MOM schemes are gauge dependent. The renorm-group analysis of gauge dependence of UV predictions in MOM schemes yielded the conclusion[1,2,3] that this dependence may happen to be strong enough even to spoil the property of asymptotic freedom for effective coupling. This effect is tightly related to the fact that the gauge parameter a being constant in the classical Lagrangian in the quantum case due to renormalization becomes a momentum function and must be treated as a second running coupling $\bar{a}$.

There exists a solution of this trouble for matrix elements, say R, satisfying the gauge invariance condition $dR/da = 0$. Combining this condition with 2-coupling RG differential equations it is possible to get[4,5] effectively a 1-coupling group equation for R that contains the gauge parameter a as a fixed number. However this procedure does not influence the RG analysis of the UV behaviour of virtual quantities like propagators and vertices.

Quite recently the problem of scheme choice has been revived again by calculation of the 4-loop contribution to the process of e^+e^- hadronic annihilation. Here an anomalously large value of the numerical coefficient of α_s^3 term was found[7].

In this paper we propose an effective drug to cure the trouble of strong gauge dependencies in RG analysis of QCD based on a rather simple modification of the renormalization procedure.

2. Renormalization that stops the running gauge

As mentioned above for momentum subtraction schemes perturbation theory results in QCD explicitly depend on the constant gauge parameter a that correspond to the gauge fixing term $(\partial B)^2/2a$ in the Lagrangian.

According to general renorm-group ideology[8], it must be treated as a second coupling constant with the corresponding effective coupling (or rather effective gauge) $\bar{a}$ properly introduced:

$$\bar{a}(Q^2, \alpha, a) = a\, d^{-1}(Q^2, \alpha, a) . \qquad (1)$$

Here d is a dimensionless function describing radiative corrections to the transverse part of the gluon propagator that perturbatively can be written as follows

$$d = 1 + (\alpha/4\pi)\, d_1(Q, a) + (\alpha/4\pi)^2\, d_2(Q, a) + \dots .$$

The 1-loop coefficient in the massless approximation is equal

$$d_1 = \left\{\frac{2n}{3} + \frac{3a - 13}{2}\right\} \ln (Q^2/\mu^2) + c_1 . \qquad (2)$$

Here n is the flavor number and c_1, a scheme-dependent constant.

Let us assume now that the gauge fixing term in the Lagrangian contains the function dependence

$$(f(\Box)(\partial B))^2/2a \qquad (3)$$

that is equivalent to the transition from a constant gauge parameter to a gauge function

$$a \to a(Q^2) = a\,\varphi(Q^2) , \quad \varphi = 1/f^2 . \qquad (4)$$

This operation modifies Eq.(1) and can be used to provide the condition

$$\bar{a}(Q^2, \dots) = a = \text{const}. \qquad (5)$$

to fulfil with a proper choice of φ . On a 1-loop level this corresponds to

$$\varphi = \varphi(Q^2,\alpha,a) = 1 + (\alpha/4\pi)\, d_1(Q^2,a) + O(\alpha^2) . \qquad (6)$$

However in the next order one must take into account that operation (4) modifies the Feynman rule for the gluon propagator.

In other words the 1-loop correction to the longitudinal gluon propagator, as described by Eqs. (4) and (6), changes results in higher order calculations. Hence the second order term in r.h.s. of (6) must compensate the modified 2-loop contribution $\tilde{d}_2$ to the transverse part of the gluon propagator. This $O(\alpha^2)$ term again must be taken into account in the course of a next order calculation and so on. Hence the "proper choice" can be written as

$$\varphi = \tilde{d} . \qquad (7)$$

This change of the renormalization procedure in gauge theories is of a general nature. It is formulated for covariant gauges in the same way for different renormalization schemes. We shall refer to it as to the "stopping gauge" renormalization or SG modification.

The nonlocality of the used counterterms does not lead to the loss of the theory locality as they can be absorbed by a special gradient transformation of a nonlocal form with the parameter function depending on the vector field operator

$$B^c_\mu(x) \to \tilde{B}^c_\mu = B^c_\mu + F(\Box)\, \partial_\mu(\partial B^c) . \qquad (8)$$

Let the initial potential B be related to a covariant gauge specified by the constant parameter a. Then, as can be easily shown[9], the new one $\tilde{B}$ will correspond to gauge (3) with

$$f(k^2) = [1 + k^2 F(k^2)]^{-1}. \qquad (9)$$

In other words, nonlocal transformation (8) is equivalent to insertion of corresponding nonlocal counterterms .

In the case $k^2 F(k^2) = b$ = const our transformation results in a numerical change of the gauge parameter: $a \to a(1+b)^2$. It is clear that such a transformation (like generally (8)) cannot change the sign of the gauge parameter. This means that the transversal gauge $a = 0$ is a singular one. It separates the positive and negative gauge domains each of which can be considered as a projection of the orbit of gauge transformation.

This observation is complementary to the reasoning on the existence of a singularity at $a = 0$ based upon UV behaviour arguments [12,3].

3. The Electron Propagator in QED

As a simplest illustration, consider the electron propagator in quantum electrodynamics. It is well known that the UV asymptotics of the electron wave function factor has the form

$$s(Q^2,\dots) = (Q^2)^{\nu\alpha}, \quad \nu = a/4\pi . \qquad (10)$$

As was first shown by Logunov[8] this result can be obtained by a standard renorm-group technique using two effective couplings

$$\bar{\alpha} = \alpha\, d(Q^2, \alpha, a), \quad \bar{a} = a\, d^{-1} . \qquad (11)$$

Starting with the 1-loop perturbative input in the MOM scheme

$$s(Q^2,\alpha,a) = 1 + \frac{a\alpha}{4\pi} l + O(\alpha^2), \quad l = \ln (Q^2/\mu^2)$$

one obtains the Lie equation for $s(Q^2)$;

$$\frac{d\ln s}{dl} = \frac{\bar{a}\bar{\alpha}}{4\pi} . \qquad (12)$$

Taking into account that according to (11) the r.h.s. is just the constant $\bar{a}\,\bar{\alpha} = a\,\alpha$ after integration we get (10).

Let us now use the SG modification, in which we have the same 1-loop input but another differential equation

$$\frac{d\ln s}{dl} = \frac{a\bar{\alpha}}{4\pi} \qquad (13)$$

instead of (12). By solving it we arrive at the expression

$$s(Q^2,\dots) = (\bar{\alpha}/\alpha)^{3a/4} = [1 - \frac{\alpha}{3\pi} \ln(Q^2/\mu^2)\ ]^{-3a/4} \qquad (14)$$

that is quite different from (10). As we see, even at the 2-loop level leading logarithms are different! Power expansion of (14) contains the contribution

$$\alpha^2\, a\, l^2/24\,\pi^2$$

that is absent in the usual perturbation expansion summed up in (10). However its origin is very transparent. As follows from general discussion in Section 2, in SG MOM calculations we have to use, in Feynman rules, a modified photon propagator that looks like

$$k^2 D^{SG}_{\mu\nu}(k) = k^2 D_{\mu\nu}(k) + P^{long}_{\mu\nu}(k)\, a\, (\frac{\alpha}{4\pi} d_1(k) + O(\alpha^2)) .$$

Due to this all perturbation calculation results starting from the second order ones are changed. For the UV asymptotics one can take $d_1 = (4/3) \ln (k^2/\mu^2)$. Inserting this α-order correction to D into the 1-loop diagram for the electron propagator we get precisely the mentioned contribution.

4. Application to QCD

Consider now the QCD case. For propagator scalar factors one has to start with the perturbation input

$$s(Q^2,\alpha, a) = 1 + s_1(a)\alpha\, l + q(a)\alpha^2 l^2 + s_2(a)\, \alpha^2 l + O(\alpha^3)$$

and the corresponding Lie equation for the SG case

$$\frac{d\ln s}{dl} = s_1(a)\bar{\alpha} + s_2(a)\, \bar{\alpha}^2, \qquad (15)$$

where for the QCD effective coupling $\bar{\alpha} = \bar{\alpha}_s$ one has to use the 1-loop RG summed expression $\bar{\alpha}_1 = \alpha\, (1 + \alpha\, \beta_1 l)^{-1}$ in the second term and the 2-loop one[13]

$$\bar{\alpha}_2 = \alpha\, \{1 + \alpha\, \beta_1 l + c_2\alpha\, \ln(1 + \alpha\, \beta_1 l)\}^{-1}, \quad c_2 = \beta_2/\beta_1$$

for the first term in the r.h.s. Integration of Eq. (15) yields

$$s(Q^2, \alpha, a) = [\bar{\alpha}_2(Q^2)/\alpha]^{-\sigma_1(a)} \exp\, (\, \alpha Z_2\, [\, \bar{\alpha}_1(Q^2) - \alpha\,]) \qquad (16)$$

with $\sigma_1(a) = s_1(a)/\beta_1$ and Z_2 depending linearly on 2-loop coefficients β_2, c_2.

For the gluon propagator according to (2) we have

$$s_1 = 3(a - a_*)/8\pi\ , \quad a_* = (39 - 4n\,)/9.$$

Generally, in SG modified calculations it is possible to obtain RG improved expressions with an explicit gauge dependence in contrast to the usual 2-coupling RG procedure where we have solutions on the phase plane (α, a) given implicitely.

5. Discussion

It is important for us that the SG modification turns the 2-coupling renormalization group in the gauge dependent QCD into the 1-coupling one. As a result the 2-loop coefficient β_2 of the group generator in every SG modified MOM scheme ceases to depend on the scheme and on the gauge parameter being equal to its "usual" invariant β_2^{MS} value. Hence the beta-function gauge dependence is moved aside to the 3-loop level. However the β_3 coefficient in each scheme must be calculated anew with account of the mentioned Feynman rule SG modification .

Here it must be said also that the gauge dependence can still arise in the QCD beta-function on the 2- and even 1-loop level due to the inclusion of heavy quark masses along the line developed in[14].

We see that the use of the SG trick provides an elegant solution of all troubles with a strong gauge dependence of perturbative QCD predictions in MOM schemes. It is clear that such "gauge phantoms" as the loss of asymptotic freedom for effective $\bar{\alpha}$ observed in papers[1,2] can be avoided not only for physical quantities R as in the aforementioned procedure [4,5] but for all quantum field functions simultaneously.

Simple estimates reveal that the gauge dependence of the 3-loop coefficient β_3 can still lead to appearance of a fixed point in some SG MOM schemes. However, for reasonable values of the gauge parameter, say $|a|<10$, it lies far away from the weak coupling region.

In this connection we would like to notice that in QCD the 3-loop approximation for $\bar{\alpha}$ turns out[15] to be physically irrelevant as the 3-loop contribution to the $\bar{\alpha}$ becames essential

only in the sufficiently low Q^2 region where perturbative QCD cannot be used for the description of strong-interaction physics due to higher twists and hadron mass effects.

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