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ANALYSIS OF THE DIFFERENTIAL EQUATIONS FOR THE EXCLUSIVE PROCESSES AND EXPLANATION FOR THE "MYSTERY" OF THE GAMMA-DISTRIBUTION

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1. Introduction

The processes for the generation of the different type hadron systems (components or resonances) are studied very intensively during last 15 years (see, e.g., ([1-11])).

When discussing of physical characteristics for the exclusive processes

(1) $a + b \Rightarrow n_1 + n_2 + \ldots + n_\nu,$

in which $n_1, n_2, \ldots, n_\nu$ hadrons of ν-types take a part, the following hypothesis by Koba, Nielsen and Olesen (KNO) ([1])'appears to be mentioned first of all: Let $\sigma(n_1, \ldots, n_\nu)$ and σ are topological and inclusive (total) cross-sections for the prosesses (1). Then 1) their combination with the average multiplicities $\langle n_i \rangle$ leads to the following multidimensional scaling

(2) $$\prod_{i=1}^{\nu} \langle n_i \rangle \cdot \sigma(n_1, \ldots, n_\nu) / \sigma = f(z_1, \ldots, z_\nu),$$

where $z_i = n_i / \langle n_i \rangle$;

2) from (2) it also follows a one-fold KNO-function

(3a) $$\langle n_c \rangle \cdot \sigma_{n_c} / \sigma \equiv \psi(z_c) = \int_0^\infty dz_2 \ldots \int_0^\infty dz_\nu \, f(z_c, z_2, \ldots, z_\nu)$$

for the $n_1 = n_c$ charged particles in the semi-inclusive reaction

(3b) $$a + b \Rightarrow n_c + X$$

3) The relation (3) means a "invariance under the resonanace decay (emmision?)"

We do not quite catch the third item of this hypothesis. First, according to the well know Caushy formula for ν-fold quadrature the expression (3a) is reducing to one-fold

(4) $$\psi(z_c) = \frac{1}{(\nu-1)!} \int_{z_c}^{\infty} (t-z_c)^{\nu-2} f(t)dt .$$

if the arguments of the function (2) are additive as $t = \sum z_i$. Thus, e.g., ψ may depend on ν. This may also take place when the resonances are produced in the limited amounts and distributing (or decaying) as the Kroneker $\delta(z_i - \alpha_i)$ function, where α_i are the decay parameters with $i=2, \ldots, \nu$. Then quadratures do not accur at all, but f may depend on ν and, consequently, the relation (3a) is broken just as that takes place for the central pseudorapidity windows $|\eta|$ ([14,15]).

Secondly, in some works, where it was possible to catch these effects of hadronization, unfortunately, the two-component models are considered only ([8,9,11]). In order to show incorrectness of this restriction, we give the following example. It is not difficult to obtain the correlation relation between the mean number of the neutral particles $\langle n_0(n_c)\rangle$ and the number of the charged one n_c

$$(5) \qquad \frac{\langle n_0(n_c)\rangle}{\langle n_0\rangle} = \frac{1}{\nu-1}\,\frac{\int_{z_c}^{\infty}(t-z_c)^{\nu-1} f(t)\,dt}{\int_{z_c}^{\infty}(t-z_c)^{\nu-2} f(t)\,dt}\,.$$

Here we substitute the function

$$(6) \qquad f(t) = t^{a-\nu}\exp[-(a/\nu)t],$$

which will be called the generalized gamma(Γ)-distribution. Then at $\nu<a$ the formula (5) describes the negative correlation (i.e. the right part is the decreasing function of z_c). When $\nu>a$ the saturation of the positive correlation is reached. Such a behaviour is in agreement with the experimental data (see, e.g., ([7,10,13])), that denotes once more the possibility of the correlated component number fluctuation.

The question now arises how well this number is defined ([13]) and of whether this is a reason for the "mystery" of the well known negative binomial distribution (i.e. the origin of KNO scaling violation?).

Speaking a priori, the answer is positive: in term of KNO scaling explanation of the enigma are the generalized Γ-distribution (6) and uncertainty principle between the number ν and the correlation intensity.

Generally speaking, the methods of the investigation of (1) have reduced to solution of the problems of algebraic ([4]), differential (D) ([6,10]) and differential-difference (DD) ([5,8]) equations. DD-eqs.(see ([11,12]) and refs. therein) are being investigated most intensively in the last years. Giovannini has derived them 10 years ago with the assumption that the gluon-gluon fussion, gluon bremsstrahlung and quark-antiquarke emmision processes have the stochastic (Markovian) nature. The authors of refs.([11,12]) seem to transpose these eqs. from sub-hadronic (quark-gluon) level to hadronic one without any changes.

At present the mentioned above quark-gluon subprocesses are exactly calculated in the α^3 order of QCD ([16]) and N=1,2 supersymmetric QCD ([17,20]) thanks to the modern powerfull methods ([18,21-23]). It should be noted, that the higher order calculations of the squared matrix element for the gauge invariant set of the Feynman diagrams require full automatization of all computing steps and as long as that cannot be accomplished by means of the universal system REDUCE-3 ([22]). Attempts to automate for the hadron exclusive processes of the α^5 order obeying the well-known "quark counting rule" ([19]), have been made by Farrar and Neri ([23]) in the programming language "C". We do not know the details of the calculations and below we should use some results from ([18]), which consist in determining a common structure for the parton squared matrix elements in the α^n order. Thus, using the principle of automodelity ([19]), we are starting from D-eq. for the homogenous function. Its degree is exactly defined by simple dimentional analysis. Further in sect.2 on the basis of the assumption of generating several hadronic components (resonances) by means of the given number of quarks, we make averaging with the Bogolubov method ([24]) that leads to the chain of the D-eqs. for mean multiplicities with matrix of the multiplicity correlation . Similarity with Bogolubov equations appears more in possibility of parametrization of this matrix with the aim of cutting the chain.

In sect.3 we solve the Caushy problem by the moment method and by the combination of the characteristic equations. It should be mentioned that the arising correlation parameters obey the principle of "universality" ([26]), according to which the parameters do not depend on the "time" variable τ and the number ν. However, as it will be see below, we do not refer to a standard renormalization scheme ([25,26]). Moreover, we do not see any disagreement with the previous results ([6,10,26]).

2. Chain of the D-eqs. with the correlated hadronic components

Recently a common structure of the differential cross-section for the n-parton processes was established in QCD

([18]).This squared matrix element for n-gluon processes is able to reproduce, for instance, the known formulae of the 4 - and 5 - gluons processes and has the following structure

(7a) $|M|^2_{ng} \approx \sum (p_i p_j)^4 \sum C_i / [(p_1 p_2)(p_2 p_3)\dots(p_n p_1)]$,

where C_i are independent isotopical constants.

Gluon replacement by means of quarks, as we have verified in the α^3 order ([17]) leads to the negligible change of the factor in (7a) so

(7b) $\sum (p_i p_j)^4 \Rightarrow \sum (p_i p_j)^{\gamma^1} (p_k p_\ell)^{\gamma^2}$,

where $\gamma^1 + \gamma^2 = 4$.

Testing of formulae (7) in the higher order of the perturbative theory is not possible by the standard methods of the quantum field theory ([25]) and computer algebra ([21]) because of the absence of complete automatization of all computing steps ([22]). So let us restrict ourselves by formula (7) when considering of n-quark tree diagrams with gluon exchange too. The exclusive hadron scattering at the large transferred momenta ([19,23]) is taken to be realised by these diagrams.

The expression $|M|^2$ as a function of the momentum $p_1,\dots,p_n$ satisfies the automodelity principle, i.e. under momentum scale transformation $p_i \Rightarrow \lambda p_i$ has to trasform as homogeneous function of corresponding dimension ([19]). It is easy to determine this dimension as m=-(n-4). In other words, $|M|^2$ satisfies the following D-eq.

(8) $\sum_{i=1}^{n} p_i \frac{\partial}{\partial p_i} |M|^2 = -m\, |M|^2$.

Now turn to the hadronic level. 1. Let us suppose that the given number n of quarks is so large that we can generate the several (ν) hadron components with multiplisities $n_1,\dots,n_\nu$ so that

(9) $n = \sum_{i=1}^{\nu} N_i n_i >> 4, \quad m \cong -2n$,

where N_i are the quark number in i-type hadron;

2. Integrate (8) over "entra" momenta of quarks in such way, that some of them remain for composing one (or two) in the final (initial) state;

3. Make averaging over some type of multiplicities and define from physical point of view the additional (and initial) condition for solving the chain of D-eqs.

It is easy to prove that integrating of the left part of (8) over (n - 1) - momenta gives

(8a) $[\, p_1 \frac{\partial}{\partial p_1} - (n - I)\,] \int |M|^2 \prod_{i=2}^{\nu} dp_i$.

If interpreting the appearing integral as a differential cross-section of the "quasiexclusive" processes

(10a) $a + b \Rightarrow c(p) + n_1 + n_2 + \dots + n_\nu$,

where c(p) is the single separated hadron with given momentum $p=p_1$, from (8) and (9) we get the following characteristic eq.:

(10b) $\frac{d}{d\tau} \sigma(n_1,\dots,n_\nu) = -(\sum N_i n_i)\cdot\sigma(n_1,\dots,n_\nu)$.

The physical meaning of time variable τ can be obtained if $p^\mu = p^\mu_0 e^t$ is a solution of eq. $d\,p^\mu = p^\mu\, dt$, so $\tau = \ln(pp_0)/p_0^2$. When momentum p is linked with initial particles in the reaction $a + b \Rightarrow n_c + X$, then $\tau = \ln s/2m_p^2$.

And now it is not difficult to obtain a characteristic eq. for the inclusive processes $a + b \Rightarrow c(p) + X$ differential cross-section ([26])

(11) $\frac{d}{d\tau} \sigma = -(\sum_{i=1}^{\nu} N_i \langle n_i\rangle)\sigma$,

and for mean (associated) multiplicities for all $i=1,\dots,\nu$

(12) $\frac{d}{d\tau} \langle n_i\rangle = -\sum_{k=1}^{\nu} N_k D_{ik}$,

where values $D_{ik} = \langle n_i n_k\rangle - \langle n_i\rangle\langle n_k\rangle$ form the matrix of correlation (dispertion) between multiplicities of the different type hadrons also. D_{ik} are linked to the higher multiplicity moments equation of the following form:

(13b) $$\frac{d}{d\tau} C(q_1,\dots,q_\nu) = \sum_{i=1}^{\nu} \Big\{ N_i \langle n_i\rangle \Big[C(q_1,\dots,q_{i+1},\dots,q_\nu) - C(q_1,\dots,q_i,\dots,q_\nu)\big[I + q_i D_{ii}/\langle n_i\rangle^2\big]\Big]\Big\}$$,

Here normalized multiplicities moments are defined as

(13a) $$C(q_1,\dots,q_\nu) = \int_0^\infty f(z_1,\dots,z_\nu) \prod_{i=1}^{\nu} z_i^{q_i}\, dz_i \equiv \langle n_1^{q_1}\dots n_\nu^{q_\nu}\rangle / \langle n_1^{q_1}\rangle\dots\langle n_\nu^{q_\nu}\rangle.$$

We are going to make a procedure of splitting (cutting) starting from DD-eqs. (13a).

In order to compare we write the simple form of DD-eq. given by Giovannini et al. ([5,8,11])

(14) $\frac{d}{dt} P_n = -A\cdot n\cdot P_n + \tilde{A}\cdot(n-I)P_{n-1}$,

where P_n denotes the normalized cross-section for the n-gluon sub-processes, A and $\tilde{A}$ are arbitrary constants. We see that the right-hand side of eq. (14) differs from (10b) one mainly by second terms. However, the hadronization in our approach more resembles the chain of Bogolubov than Markov's one. It is interesting to note that analogous eqs., in our opinion, may result from the renormalization group analysis in the framework of QCD .

3. Solution of the Cauchy problem by methods of recurrence and combination

Let us show that nontrivial and the physically interested solutions of eqs. (10)-(13) occur only for correlated components. We start from the noncorrelated case, where $D_{ik}=0$ when $i\neq k$, i.e. matrix of correlation

(15a) $$D = \begin{Bmatrix} D_{11},\dots D_{1\nu}, \\ \dots\dots\dots\dots \\ D_{\nu 1},\dots D_{\nu\nu}, \end{Bmatrix},$$

is diagonal. We line the remainder elements as

(15b) $$D_{ii}= \frac{1}{a_i}(\langle n_i\rangle-\alpha_i)^2$$

and consider the constants a_i и α_i as "universal", i.e. independent of the parameters of the theory: τ and ν. This law is well known, for instance, for charged particles ([27]).

As all $\langle n_i\rangle$ are independent and, moreover,

$C(I,\dots,I) = I$, i.e. $\frac{d}{d\tau} C(I,\dots,I) = 0$,

from eq.(13b) we get solution with index 2:

$C(2,I,\dots,I) = \{I+[I- (2\alpha_i/\langle n_i\rangle)+(\alpha_i^2/\langle n_i\rangle^2]\}$.

So as to continue the recurrence, we have else to suppose, that $\langle n_i\rangle \gg \alpha_i$ for all $i=1,\dots,\nu$. Then it easily can be checked that for any set of indices the normalazed multiplicity moments do not depend on τ and we can obtain them recursively in the factorized form

(13c) $$C(q_1,\dots,q_\nu) = \prod_{i=1}^{\nu}\left\{ \Gamma(a_i+q_i)/\Gamma(a_i) \right\}\cdot$$

We inverse (13a) as the Mellin transformation and see,

consequently that the multidimensional KNO-function (2) is also restoring as a production of the one-component Γ-distributions ([26]):

(6a) $$\Psi(z_i) = \frac{a_i^{a_i}}{\Gamma(a_i)} z_i^{a_i-1}\exp(-a_i z_i)\cdot$$

This function has been used most intensively for the last 5 years for approximation of the experimental data in the large range of the energies and for the dirrefent intervals of the pseudorapidity $|\eta|$. However, as was mentioned in Introduction, the parameter a_i figuring therein is "mysteriously" changed ([12,13]). This points the way to go out from the one-component regime sets and to obtain a physically correct solution for the function (2).

As it has been seen, DD-eq. (13b) is recurrently solvable at the very limited conditions and it does not help us in solving this problem. Therefore we turn to the immediate analysis of the chain of the D-eqs. (10)-(12). In the general case, by linearizing matrix (15a), we can give it in the nondiagonal form with the different wheights (intensities) of the correlations

(15c) $$D_{ik}= \frac{1}{a_{ik}}\langle n_i\rangle\langle n_k\rangle$$

for all $i,k=1,\dots,\nu$.

But the case of the saturation is conveniently considerable with the same intensities

$a_{ik} = a$ for all $i,k = I,\dots,\nu$.

Consequantly, the system (12) composed from ν-eqs. may be reduced to one

(16) $$\frac{d\langle x\rangle}{d\tau} = - D^2(x),$$

where

(16a) $x = \sum_1^\nu N_i n_i$, $\langle x\rangle = \sum_1^\nu N_i\langle n_i\rangle$, $D^2(x) = \langle x^2\rangle-\langle x\rangle^2 \Rightarrow \frac{1}{a}\langle x\rangle^2$.

For excluding $d\tau$ let us divide (10b) and (11) by (16). Thus we get

(17a) $$\frac{d}{d\langle x\rangle}\sigma(n_1,\dots,n_\nu) = \frac{ax}{\langle x\rangle^2}\sigma(n_1,\dots,n_\nu)$$

(17b) $$\frac{d}{d\langle x\rangle}\sigma = \frac{a}{\langle x\rangle}\sigma\cdot$$

They are easy integrated

(18) $$\sigma(n_1,\dots,n_\nu) = C_1\exp(-a\frac{x}{\langle x\rangle}),\qquad \sigma = C_2\langle x\rangle^a,$$

where C_1 and C_2 are first integrals.

Let us formulate the Cauchy problems so that the combination of (18) at $\tau=0$ must satisfy the following function

(19) $F(Z)=Z^{a-\nu}\exp(-aZ)$

as an initial condition, i.e.

(20) $$\left\{\langle x\rangle^{\nu}\,\frac{\sigma(n\ ,\dots,n\ )}{\sigma}\right\}_{\tau=0} = F(Z),$$

with argument $Z=x/\langle x\rangle$. Then we get equality

$$\left\{\frac{C_1}{C_2}\right\}_{\tau=0} = x^{a-\nu},$$

Inserting the first integrals (18) into the last expression, it is not difficult to check up, that relation (20) is fulfilled for all values of τ and one may give it the view of (2).

Since

$$\frac{d}{d\tau}\langle n_1\rangle = -\langle n_1\rangle\langle x\rangle ,$$

$\langle n_1\rangle/\langle n_1\rangle = C_1$ are the first integrals too. And now we can determine them introducing the explicit form of (20) into the standard definition $\langle n_1\rangle$. So we get the relations

$$N_1\langle n_1\rangle = N_1\langle n_1\rangle, \quad \langle x\rangle = \nu\, N_1\langle n_1\rangle, \quad Z = \textstyle\sum_1^{\nu} z_1/\nu,$$

which give to (20) the necessary form as KNO-scaling (2) ([10])

(21) $$f(z_1,\dots,z_\nu) = f(\textstyle\sum z_1) = A(\sum z_1)^{a-\nu}\exp[-\frac{a}{\nu}(\sum z_1)]$$

where

$$A = \left[\frac{a}{\nu}\right]\frac{\Gamma(\nu)}{\Gamma(a)}.$$

Finally, we should explain the Γ-distribution "enigma" mentioned in Introduction by means of (21). Let us consider precisely the case when $\nu>1$ quadratures do not arise due to the Kronecker delta function $\delta(z_1-\alpha_1)$. Then the KNO-function (3) for $n_1 = n_c$, according to (21), is parametrized in the generalized Γ-distribution form

(22) $$\Phi(z_c) \Rightarrow (z_c+\alpha)^{\nu(a_e-1)}\exp[-a_e(z_c+\alpha)]\,.$$

Emphasize the means of the parameters arising here : $a_e = a/\nu$ is effective intensity of all types of correlations, constants ν and α characterise the birth and decay processes of resonances, accordingly. The comparison of the formulas (22) with the experimental data ([15]) confirms increasing of the component number ($\nu = 2,3$) in the central pseudorapidity $|\eta|$ windows and, correspondingly, decreasing of a_e so that their production will be constant, i.e. $\nu\, a_e = a \Rightarrow 4$.

This uncertainty principle between number ν of correlated components and intensity of their correlation was also observed ([7,10]) at the approximation of the experimental data by means of formula (5) for the neutral-charge particles correlation. If $\delta(z_1-\alpha_1)$ is absent, the shape of the Γ-distribution is strongly modified by the averaging over the multiplicities $n_2,\dots,n_\nu$ ([7]).

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References

1. Z. Koba, H. B. Nielsen and P. Olesen: Nucl.Phys., B 40, 317 (1972).
2. F. T. Dao and J. Whitmore: Phys.Lett., 46 B, 252 (1973).
3. A. A. Logunov, M. A. Mestvirishvili and O. A. Khrustalev: Particles and Nuclei, 3, 3 (1972);
   S. P. Kuleshov, V. A. Matveev and A. N. Sissakian: Fizika, 5, 67 (1973);
   V. G. Grishin e.a.: Lett. Nuovo Cimento, 8, 590 (1973);
   Yu. A. Budagov e.a.: Czechoslovak J.Phys., B 26, 1271. (1976).
4. G.Thomas: Phys.Rev., D 8, 3042 (1973).
5. A. Giovannini: Nucl.Phys., B 161, 429 (1979);
   M. Anselmino e.a.: Nuovo Cimento, 62 A, 253 (1981);
   V. M. Maltsev, N. K. Dushutin and S. I. Sinegovsky: Yad. Fiz., 22, 590 (1975).
6. Ya. Z. Darbaidze, A. N. Sissakian, L. A. Slepchenko: Preprint JINR P2-80-615, Dubna (1980); Proceedings Internat. Seminar HEP and QFT, Protvino (1980), vol. 1, 304;
   Ya. Z. Darbaidze and N. V. Machaldiani: Preprint JINR P2-80-160, Dubna (1980);
7. N. S. Amaglobeli e.a.: Preprint JINR E2-82-107, Dubna (1982);
   Ya. Z. Darbaidze e.a.: Bull. Acad. Sci. GSSR, 111, 497 (1983);113, 289 (1984); 114, 285 (1984).

8. M. Biyajima and N. Suzuki: Phys.Lett., B 143, 463 (1984);
B. Durand and I. Sarcevic: Phys.Lett., B 172, 104 (1986).
9. Ya. Z. Darbaidze, A. N. Sissakian, L. A. Slepchenko and G. T. Torosian: Yad. Fiz., 34, 844 (1981); preprint JINR P2-82-297, Dubna (I982); preprint JINR P2-83-3I2 (I983).
10. Ya. Z. Darbaidze, A. N. Sissakian, L. A. Slepchenko and G. T. Torosian: Fortsch. Phys., 33, 5, 299 (1985).
11. N. Suzuki and M. Biyajima: Ann. Inst. Statist. Math. 40, 229 (1988); Phys. Rev., D 37, 1824 (1988).
12. A. Giovannini and L. Van Hove: Z. Phys., C 30, 391 (1986);
L. Van Hove: Proc. Shandong Workshop, Jinan, China (1987);
R. Szwed e.a.: Mystery of the Negative Binomial Distribution, Warsaw Univ. preprint IFD/3/87 (1987).
13. Chou Kuang-chao, Liu Lian-sou and Meng Ta-chung: Phys. Rev. D 28, 1080 (1983); Cai Xu, Chao Wei-qin and Meng Ta-chung: Phys. Rev. D 33, 1287 (1986);
14. W. Thome e.a.: Nucl.Phys., 129 B, 365 (1977).
15. G. Arnison e.a.: Phys.Lett., 107 B, 20 (1981);
G. J. Alner e.a.: Phys.Lett., 160 B, 193 (1985); 160 B, 199 (1985); 167 B, 476 (1986);
G. J. Alner e.a.: Phys.Rep., 154, 247 (1987);
R. E. Ansorge e.a. (UA5 Collab.): Charged Particle Multiplicity Distributions at 200 and 900 GeV C.M. Energy, CERN-EP/88-172 (1988).
16. F. A. Berends e.a.: Phys.Lett., 103 B, 124 (1981) .
17. Ya. Z. Darbaidze, V. A. Matveev, Z. V. Merebashvili and L. A. Slepchenko: Phys. Lett., 177 B, 188 (1986); 191 B, 179 (1987); 206 B, 127 (1988); preprint JINR P2-88-I29 (I988), to be publeshed in Proceedings Internat. Seminar "Quarks 88" (World Scientific).
18. S. J. Parke and T. R. Taylor: Phys. Rev. Lett., 56, 2459 (1986);
M. Mangano, S. J. Parke and Z. Xu: Nucl. Phys., B 298, 653 (1988);
F. A. Berends and W. Gielle: Nucl. Phys., B 294, 700(1987).
19. V. A. Matveev, R. M. Muradian and A. N. Tavkhelidze: Lett. Nuovo Cimento, 7, 719 (1973);
S. J. Brodsky and G. R. Farrar: Phys. Rev. Lett., 31, 1153 (1973);

20. S. J. Parke and T. R. Taylor: Phys. Lett., 157 B, 81 (1985);
M. T. Grisaru and H. N. Pendleton: Nucl. Phys., B 124, 81. (1977).
21. A. C. Hearn: REDUCE User's Manuel, Version 3.2, Rand Publ. CP78 (7/85) (1985);
A. A. Bogolubskaya, V. A. Rostovtsev and I. E. Zhidkova: REDUCE-2 System of Programming, B1-11-83-512, Dubna (1983).
22. V. P. Gerdt, O. V. Tarasov and D. V. Shirkov: Sov. Phys. Usp., 130, II3 (1980);
O. V. Tarasov: Proceedings of the Internat. Conference on Computer Algebra and its Application in Theoretical Physics, JINR DII-80-I3, Dubna (1980), p. 150; JINR D11-85-79I, Dubna (I985), p. 2I4;
Ya. Z. Darbaidze, Z. V. Merebashvili and V. A. Rostovtsev: preprint JINR P2-88-769, Dubna (I988).
23. G. R. Farrar and F. Neri: Phys. Lett., 130 B, 109 (1983);
V. L. Chernyak and A. R. Zhitnitsky: Phys. Rep., 112, 173 (1984).
24. N. N. Bogolubov: Problebms of the Dynamical Theory in Statistical Physics, M., Gostechizdat (1946).
25. N. N. Bogolubov and D. V. Shirkov: Introduction to the Theory of Quantized Fields, M., Nauka (1984).
D.V. Shirkov: Nucl.Phys., B 62, 194 (1973).
26. W. Ernst and I. Schmitt: Nuovo Cimento, 31 A, 109 (1976); 33 A, 493 (1976);
Ya. Z. Darbaidze, N. V.Machaldiani, A. N. Sissakian and L. A. Slepchenko: Teor. Mat. Fiz., 34, 303 (1978).
27. A. Wroblewski: Acta Phys. Polon., 4 B, 857 (1973)

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