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# GAUGE MODELS OF DISCRETE STRINGS

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## 1. Introduction

One of the possible approaches to nonperturbative string theory is to consider discretized (lattice) strings. Most attempts up to now were applied to the world sheet discretization which is difficult to interpret in terms of hamiltonian dynamics (see e.g. the pioneer papers [1], recent reviews [2] and references therein). It would be of great interest to have some finite dimensional hamiltonian systems which imitate strings with their symmetries and for which one might hope to develop nonperturbative quantum dynamics.

Recently, one of us [3] has developed a gauge approach to constructing constrained theories of relativistic particles bound by harmonic forces, including a model of chain-like objects resembling in some aspects relativistic strings. Unfortunately, their relation to the continuous string is not clear, and here we propose a new class of discrete "string" models constructed in close analogy with the standard strings. Of particular importance for us is the chiral structure of the gauge group acting on the chiral phase space string variables (left and right movers) and the technique of quantization based on path integral methods for constrained systems. We introduce an exact discrete analog for the chiral variables as well as for the chiral gauge group of the string theory, and suggest one of the possible approaches to quantizing the models for arbitrary gauge groups. A possibility of including fermionic degrees of freedom is also pointed out. The fermionic discrete "strings" can possibly be applied either to hadron physics or to unified theories. At least, they can be used as finite-dimensional approximations to superstrings which have

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simpler dynamical structure making a nonperturbative approach to solving them more feasible.

The paper is organized as follows. In Sect.2 we develop the hamiltonian formulation of the new discrete "string" models. Sect.3 deals with quantizing these models with the use of the path integral methods. In particular, the heat kernel and the propagator are constructed. In conclusion (Sect.4), we summarize the results and outline problems and prospects for future development.

## 2. Classical discrete "strings"

The canonical action of the standard closed bosonic string may be written as (see [4],[5]):

$$S = \int_0^T dt \int_0^{2\pi} ds \; (\dot{q}_\mu p^\mu - \tfrac{1}{4} l_+ z_+^2 + \tfrac{1}{4} l_- z_-^2), \qquad (1)$$

where all variables are functions of $t$ and periodic functions of $s$ (more often these variables are known as $\tau$ and $\sigma$), the dot denotes $\partial_t$, $\mu$ is the D-dimensional Minkowski space-time index, and $z_\pm^\mu = p^\mu \pm \partial_s q^\mu$ are the usual chiral string variables in the phase space $(p,q)$. This theory is easily seen to be invariant under the gauge transformations which are most clearly expressed in terms of the chiral variables $z_\pm$:

$$\delta z_\pm^\mu = \partial_s f_\pm z_\pm^\mu; \quad \delta l_\pm = \dot{f}_\pm + [f_\pm, l_\pm]_\partial \equiv \dot{f}_\pm + f_\pm \partial_s l_\pm - l_\pm \partial_s f_\pm . \qquad (2)$$

Remind that the invariance of the action (1) requires certain boundary conditions on $f_\pm$ at $t = 0$, T. Commuting two successive transformations of $z_+$ or $z_-$,

$$[\delta_2, \delta_1] z = \partial f_3 z, \quad f_3 = f_1 \partial_s f_2 - f_2 \partial_s f_1 \equiv [f_1, f_2]_\partial ,$$

one finds that they form the representation of the algebra



$Vect(S^1) \otimes Vect(S^1)$, as the Lie bracket $\cdot[,]_\partial$ defines the Lie algebra of one-dimensional vector fields. In Eq.(2) the gauge potentials $l_\pm$ and infinitesimal functions $f_\pm$ may be thought of as symmetric matrices depending on the continuous indices $s', s''$, e.g. $(f_\pm)_{s's''} = \int ds \; f_\pm(t,s) \Gamma_{s,s's''}$ where $\Gamma_{s,s's''} = \delta(s'-s)\delta(s-s'')$. Then, $\partial_s$ is the skew-symmetric matrix, $(\partial_s)^{s's''} = \delta'(s'-s'')$, and the gauge transformations may be presented in a more standard form by identifying $(\partial_s f_\pm)^{s'}_{s''}$ with the generators $(F_\pm)^{s'}_{s''}$ of the gauge transformations while considering $(\partial_s l_\pm)^{s'}_{s''}$ as matrices of the gauge potentials $(A_\pm)^{s'}_{s''}$. Then, Eq.(2) can be rewritten in the standard form

$$\delta z_\pm = F_\pm z_\pm; \quad \delta A_\pm = \dot{F}_\pm + [F_\pm, A_\pm] \equiv \dot{F}_\pm + F_\pm A_\pm - A_\pm F_\pm . \qquad (3)$$

Now, we construct a discrete version of the string theory introducing canonical variables $p^a(t)$, $q_a(t)$ and a skew-symmetric matrix $\partial^{ab}$, $a,b,c = 1,2,\ldots,N$. This matrix is considered as a discrete analog of the derivative $\partial_s$. Accordingly, we introduce the chiral variables $z_\pm^a = p^a \pm \partial^{ab} q_b$ and the canonical action

$$S_1 = \int_0^T dt \; (\dot{q}_a p^a - \tfrac{1}{4} l_+^m \, z_+ \Gamma_m z_+ + \tfrac{1}{4} l_-^m \, z_- \Gamma_m z_-), \qquad (4)$$

where $\Gamma_m = \Gamma_{m,ab}$ are some symmetric $N \times N$ matrices, $m = 1,2,\ldots,M$. Defining the matrices similarly to the continuous case

$$T_m = (T_m)^a_b = \partial^{ac} \, \Gamma_{m,cb} , \quad T_m^* = (T_m^*)^b_a = \Gamma_{m,ac} \, \partial^{cb}, \qquad (5)$$

$$F_\pm = (F_\pm)^a_b = f^m(t)(T_m)^a_b , \quad A_\pm = (A_\pm)^a_b = l^m(t)(T_m)^a_b , \qquad (6)$$

it is easy to find that the gauge transformations (3) with these matrices $A_\pm$ and $F_\pm$ are closed if $\Gamma_m$ satisfy the commutation relations

$$\Gamma_m \, \partial \, \Gamma_n - \Gamma_n \, \partial \, \Gamma_m \equiv [\Gamma_m, \Gamma_n]_\partial = t^l_{mn} \, \Gamma_l , \qquad (7)$$

from which the standard Lie algebra relations follow for $T_m$, $T_m^*$

$$[T_m, T_n] = t^l_{mn}\, T_l, \quad [T^*_m, T^*_n] = t^l_{mn}\, T^*_l\ . \tag{8}$$

From the definitions (5) we see that $T_m$ and $T^*_m$ are traceless matrices and $(T^*_m)^b_a = -(T_m)^b_a$.

Thus, to construct our discrete model we need a skew-symmetric matrix $\partial$ and the symmetric matrices $\Gamma_m$ satisfying Eq.(7). The complete system of the $N \times N$ symmetric matrices clearly satisfies Eq.(7). One can see that in this case the matrices $T_m$ generate the real noncompact algebra $sp(N,\mathrm{R})$. Any other possible algebra must be represented as some subalgebra of $sp(N,\mathrm{R})$. If we wish to have a good analogy with the continuous string theory the number of generators in this subalgebra must be of the order of $2N$.

Now we discuss the hamiltonian structure of our system. The Poisson brackets for the canonical and chiral variables are

$$\{q_a, p^b\} = \delta^b_a; \quad \{z^a_+, z^b_-\} = 0; \quad \{z^a_\pm, z^b_\pm\} = \pm 2\partial^{ab}, \tag{9}$$

and the equations of motion have the form

$$\dot q_a = \tfrac{1}{2}\Gamma_{m,ab}(l^m_+ z^b_+ - l^m_- z^b_-); \quad \dot p^a = \tfrac{1}{2}\partial^{ac}\,\Gamma_{m,cb}(l^m_+ z^b_+ + l^m_- z^b_-), \tag{10}$$

$$T^\pm_m \equiv \pm\tfrac{1}{4}\Gamma_{m,ab}\; z^a_\pm\; z^b_\pm = 0. \tag{11}$$

Eqs.(11) are the constraints on the canonical variables. They form the Lie algebra (8) with respect to the Poisson brackets

$$\{T^\pm_m,\ T^\pm_n\} = t^l_{mn}\ T^\pm_l\ , \quad \{T^+_m,\ T^-_n\} = 0. \tag{12}$$

These first-class constraints generate the gauge transformations

$$\delta q_a = \tfrac{1}{2}\,\Gamma_{m,ab}(f^m_+ z^b_+ - f^m_- z^b_-); \ \delta p^a = \tfrac{1}{2}\,\partial^{ac}\,\Gamma_{m,cb}(f^m_+ z^b_+ + f^m_- z^b_-), \tag{13}$$

The action $S_1$ is invariant under these gauge transformations if $l^m_\pm$ transform as gauge potentials,

$$\delta l^m_\pm = \dot f^m_\pm + f^k_\pm\ t^m_{kn}\ l^n, \tag{14}$$

and if $f^m_\pm$ satisfy the boundary conditions

$$f^m_\pm(0) = f^m_0,\ f^m_\pm(\mathrm{T}) = f^m_{\mathrm{T}}. \tag{15}$$

Eqs.(13),(14) can easily be rewritten in the standard form (3).



Remark that the matrices $T_m$ act on momenta while $T^*_m$ act on coordinates similarly to the standard reparametrizations.

The equations of motion for $z_+$ and $z_-$ are independent,

$$\dot z^a_\pm = l^m_\pm\ (T_m)^a_b\ z^b_\pm\ , \tag{16}$$

and the Cauchy problem for them can formally be solved,

$$z^a_\pm(t) = V_\pm(t,t_0)^a_b\ z^b_\pm(t_0), \tag{17}$$

$$V_\pm(t,t_0) = \mathrm{Pexp}\left\{\int_{t_0}^{t} dt'\, l^m_\pm(t')T_m\right\}. \tag{18}$$

Taking into account that the finite gauge transformations corresponding to Eqs.(3) have the standard form, one can easily find the transformations of the evolution matrix

$$V_\pm(t,t_0) \rightarrow \exp(f^m_\pm(t)T_m)\ V_\pm(t,t_0)\ \exp(-f^m_\pm(t_0)T_m), \tag{19}$$

$$z^a_\pm(t_0) \rightarrow \exp(f^m_\pm(t_0)T_m)\ z^a_\pm(t_0). \quad z^a_\pm(t) \rightarrow \exp(f^m_\pm(t)T_m)\ z^a_\pm(t).$$

These finite gauge transformations form the gauge group $\mathcal{G}\otimes\mathcal{G}$ corresponding to the group $G$ generated by the Lie algebra of the matrices $T_m$. It is analogous to the chiral group of the continuous theory $Vect(S^1)\otimes Vect(S^1)$. This completes our construction of the classical discrete "string" models. Above, we have considered general canonical coordinates. To obtain a closer correspondence with the relativistic strings one introduces the relativistic phase space $(q_a,\ p^a) = (q^\mu_a,\ p^{a\mu})$ where $\mu$ is the D-dimensional space-time index, $\mu = 0,1,\ldots,D-1$. By contracting these indices in Eq.(4) one trivially obtains the Lorentz-invariant theory. To add space-time translation invariance, consider the transformation

$$q^\mu_a(t) \rightarrow q^\mu_a(t) + c^\mu\Sigma_a,$$

where $c^\mu$ and $\Sigma_a$ are $t$-independent. The action is invariant under these transformations if $\Sigma_a\partial^{ab} = 0$, i.e. $\partial^{ab}$ is degenerate and $\Sigma_a$ is an eigenvector with zero eigenvalue.

## 3. Quantizing discrete "strings"

Following the rules for quantizing constrained hamiltonian systems [6,7] consider the path-integral representation for the transition amplitude (propagator)

$$\mathfrak{D}[q^f,q^i] = \int D\mu\ \exp\left\{ i\int_0^T dt\ \left(\dot q_a p^a - \tfrac{1}{4} l_+^m\ T_m^+ - \tfrac{1}{4} l_-^m\ T_m^-\right)\right\}, \qquad (20)$$

$$D\mu = \prod_{0\leqslant t\leqslant T} Dp^a\ Dq_a\ Dl_+^m\ Dl_-^m\ [\Delta_{FP}\ \Pi_{gf}],$$

where the integration is performed over all Lagrange multipliers $l_\pm^m(t)$ and all phase-space trajectories $p^a(t)$, $q_a(t)$ with fixed coordinates at the boundaries of the evolution interval

$$q_a(0) = q_a^i, \quad q_a(T) = q_a^f .$$

We also include in the definition of the integration measure the Faddeev-Popov determinant $\Delta_{FP}$ and the gauge-fixing term $\Pi_{gf}$.

We fix the gauge by choosing $l_\pm^m(t)$ independent of $t$,

$$l_\pm^m(t) = \tfrac{1}{T}\hat l_\pm^m . \qquad (21)$$

In this gauge the evolution matrix $V_\pm(T,0)$ is simply $\exp(\hat l_\pm^m\ T_m)$, see Eq.(18). If the end-point values of $f_\pm^m(t)$ vanished, all $\hat l_\pm^m$ would be invariant under gauge transformations (19). In fact, as $f_0^m$ and $f_T^m$ in Eq.(15) are arbitrary parameters, there are residual transformations of $\hat l_\pm^m$, $q(0)$, and $q(T)$ which can be obtained from Eqs.(17)-(19)

$$\exp(\hat l_\pm^m\ T_m) \to \exp(f_T^m\ T_m)\ \exp(\hat l_\pm^m\ T_m)\ \exp(-f_0^m\ T_m), \qquad (22)$$

$$q(0) \to \exp(f_0^m\ T_m)\ q(0), \quad q(T) \to \exp(f_T^m\ T_m)\ q(T).$$

The transformations (22) are automorphisms of the group $G \otimes G$ which generate a subgroup $G_0$ in $G \otimes G$. Therefore, the invariant combinations of the parameters $\hat l_\pm^m$ may be considered as coordinates on the coset space $(G\otimes G)/G_0$. The transformations of the end-point coordinates are analogous to reparametrizations of the boundary contours in the string theory, and the invariant combinations of the parameters $\hat l_\pm^m$ correspond to the Teichmuller parameters.



Our gauge condition (21) is implemented by setting

$$\Pi_{gf} = \Pi_+\Pi_-;\ \Pi_\pm = \int d\hat l_\pm \prod_{t,\mu} \delta(l_\pm^m - \tfrac{1}{T}\hat l_\pm^m), \qquad (23)$$

where $d\hat l_\pm$ is the left-invariant measure over the Lie group $G$. Using the standard technique [6] we now present $\Delta_{FP}$ in the form

$$\Delta_{FP} = \det(\partial_t - l_+^m\ \tilde T_m)\ \det(\partial_t - l_-^m\ \tilde T_m) = \Delta_+\Delta_- = \qquad (24)$$

$$= \int D\mu_g\ \exp\left\{ i\int_0^T dt\ \left[B^+(\partial_t - l_+^m\ \tilde T_m)C_+ - B^-(\partial_t - l_-^m\ \tilde T_m)C_-\right]\right\},$$

where $D\mu_g$ is an integration measure for the standard ghost variables $B_n^\pm$, $C_\pm^n$, and the matrices $\tilde T_m$ realize the adjoint representation of our algebra

$$(\tilde T_m)^l_n = t^l_{mn}. \qquad (25)$$

Following [7] we extend the phase space by adding ghost terms to the action

$$S_2 = \int_0^T dt\ \left(\dot q_a p^a + i(B_m^+\dot C_+^m - B_m^-\dot C_-^m) - i\{l_+^m B_m^+,\Omega^+\} + i\{l_-^m B_m^-,\Omega^-\}\right), \qquad (26)$$

where $\Omega^\pm = C_\pm^m\ T_m^\pm \mp \frac{i}{2}\ B_l^\pm\ t^l_{mn}\ C_\pm^m\ C_\pm^n$ are the standard BRST charges corresponding to our constraints $T_m^\pm$, and the Poisson superbrackets are $\{B_m^\pm,C_\pm^n\} = \mp i\delta_m^n$, $\{B_m^\pm,C_\mp^n\} = 0$. The ghost equations of motion

$$\dot C_\pm^m = l_\pm^n\ t^m_{nl}\ C_\pm^l, \quad \dot B_m^\pm = -B_l^\pm\ l_\pm^n\ t^l_{nm} \qquad (27)$$

can be solved similarly to Eqs.(17),

$$C_\pm(t) = \tilde V_\pm(t,t_0)\ C_\pm(t_0), \quad B^\pm(t) = B^\pm(t)\ (\tilde V_\pm(t,t_0))^{-1}, \qquad (28)$$

where $\tilde V_\pm$ is obtained from $V_\pm$ by substituting $\tilde T_m$ for $T_m$ in Eq.(18).

To construct the heat kernel and the propagator for our system we change the ghost variables $B_m^\pm$ and $C_\pm^m$ to the standard canonical coordinates $\rho^m$, $\bar\rho_m$ and momenta $\pi_m$, $\bar\pi^m$ by using linear canonical transformations

$$B_m^\pm = a_\pm\bar\rho_m \pm ib_\pm\pi_m\ , \quad C_\pm^m = a_\mp\rho^m \pm ib_\mp\bar\pi^m \qquad (29)$$

where $a_+b_- + b_+a_- = 1$. The new ghosts have the canonical Poisson superbrackets $\{\rho^\mu,\pi_\nu\} = \{\bar\rho_\nu,\bar\pi^\mu\} = -\delta^\mu_\nu$, others being zero.

Returning to the propagator (20) we now write it in the form

$$\mathfrak{D} = \int d\hat{l}_+ d\hat{l}_- \int Dl^m_+ \, Dl^m_- \prod_{t,\mu} \delta(l^m_+ - \tfrac{1}{T}\hat{l}^m_+)\, \delta(l^m_- - \tfrac{1}{T}\hat{l}^m_-)\, \mathcal{K}_{fi}, \tag{30}$$

$$\mathcal{K}_{fi} = \mathcal{K}(q^e,\rho^e,\bar\rho^e) = \int D\tilde\mu \, \exp(iS_3), \; e = f,i. \tag{31}$$

Here, the measure $D\tilde\mu$ corresponds to integration over all paths $X(t)$ in the extended phase space $X = (q_a, p^a, \rho^m, \pi_m, \bar\rho_m, \bar\pi^m)$ with fixed end points in the coordinate subspace

$$q_a(t_e) = q^e_a;\;\; \rho^m(t_e) = \rho^{me};\;\; \rho_m(t_e) = \rho^e_m;\;\; t_i = 0,\; t_f = T. \tag{32}$$

The action $S_3$ is related to $S_2$ as follows

$$S_3 = S_2 + \bar\rho_m(T)\bar\pi^m(T) - \bar\rho_m(0)\bar\pi^m(0). \tag{33}$$

The functional $\mathcal{K}_{fi}$ implicitly depending on $l^m_\pm(t)$ is the kernel of the evolution operator (heat kernel) for our extended system. Performing the integrations in Eq.(30) one can obtain it in an explicit form.

With this aim we first find the classical trajectories $X^{cl}(t)$ satisfying the boundary conditions (32). This can easily be done by using the solutions of the Cauchy problem (see Eqs.(17), (18), (28)). Then, shifting the integration variables in the integral (31), $X(t) \to X(t) + X^{cl}(t)$, one can easily show that

$$\mathcal{K}_{fi} = Z \, \exp(iS^{cl}), \tag{34}$$

$$Z = \int D\tilde\mu_0 \, \exp(iS_3), \tag{35}$$

where $S^{cl}$ is the stationary value of the action $S_3$

$$S^{cl} = S\{X^{cl}\} = \tfrac{1}{2}[q^f_a p^a(T) - q^i_a p^a(0)) + (\bar\rho^f_m \bar\pi^m(T) - \bar\rho^i_m \bar\pi^m(0)]. \tag{36}$$

The measure $D\tilde\mu_0$ is obtained from $D\tilde\mu$ by restricting integrations to trajectories $X(t)$ with zero boundary conditions, i.e. in Eqs.(32) one has to set $q^e = 0$, $\rho^e = 0$, $\bar\rho^e = 0$. Using the solutions of the



Cauchy problem (see (17),(18),(28)) one can express $p^a(t_e)$ and $\bar\pi^m(t_e)$ in terms of the boundary values of the coordinates in (32). Substituting these expressions in Eq.(36) we obtain the final form of $S^{cl}$ entering into Eq.(34):

$$\begin{aligned} S^{cl} = {} & \tfrac{1}{2}[q^f(V_+ + V_-)(V_+ - V_-)^{-1}\partial q^f - 4q^i(V_+ - V_-)^{-1}\partial q^f + \\ & + q^i(V_+ - V_-)^{-1}(V_+ + V_-)\partial q^i] + \\ & + i[\bar\rho^f(c_+\tilde V_+ + c_-\tilde V_-)(\tilde V_- - \tilde V_+)^{-1}\rho^f - (c_+ + c_-)\bar\rho^f \tilde V_-(\tilde V_- - \tilde V_+)^{-1}\tilde V_+\rho^i - \\ & - (c_+ + c_-)\bar\rho^i(\tilde V_- - \tilde V_+)^{-1}\rho^f + \bar\rho^i(\tilde V_- - \tilde V_+)^{-1}(c_-\tilde V_+ + c_+\tilde V_-)\rho^i]. \end{aligned} \tag{37}$$

Here $c_\pm = a_\pm/b_\pm$, and the matrices $V_\pm$ and $\tilde V_\pm$ are defined in (18), (28), $V_\pm = V_\pm(T,0)$, $\tilde V_\pm = \tilde V_\pm(T,0)$ (note that the matrices $V_\pm$ are defined in our original representation (5) while $\tilde V_\pm$ depend on the generators of the adjoint representation (25)). The inverse matrices in Eq.(37) may have zero eigenvalues which have to be treated in a usual way. The necessary modifications of this formula depend on the detailed group structure of the model and are not considered in this letter.

To finish the calculation of the heat kernel we have to evaluate the path integral $Z$ in Eq.(35) which is independent of the end-point coordinates (32). This can be done directly but a more transparent calculation may be based on the fundamental convolution property of the kernel ("sewing" formula)

$$\mathcal{K}_{31} = \int dq_2 d\rho_2 d\bar\rho_2 \, \mathcal{K}_{32}\, \mathcal{K}_{21} \equiv \mathcal{K}_{32} * \mathcal{K}_{21} \tag{38}$$

where the subscripts correspond to respective initial and final coordinates in Eq.(31) and the integration is performed over all intermediate coordinates denoted by the subscript 2. Substituting Eqs.(34), (37) in Eq.(38) one obtains the equation determining $Z$. To stress the analogy with the continuous string we write the solution of this equation for the model in which the coordinates $q_a$ are vectors in the D-dimensional Minkowski space

$$Z = \det{}^{D/2}[(V_+ - V_-)^{-1}\partial]\ \det(\tilde{V}_+ - \tilde{V}_-), \tag{39}$$

where we have used that $\det(\tilde{V}_\pm) = 1$ (this follows from the tracelessness of $\tilde{T}$). Remark that $\det(\tilde{V}_+ - \tilde{V}_-)$ emerges from the Faddeev-Popov determinant and its zero eigenvalues have to be treated in the standard way.

To obtain the propagator $D_{fi}$ we perform the integrations over the Lagrange multipliers $l_\pm$ in Eq.(30):

$$D_{fi} = \int d\hat{l}_+ d\hat{l}_-\ \det{}^{D/2}[(V_+ - V_-)^{-1}\partial]\ \det(\tilde{V}_+ - \tilde{V}_-)\ \exp(iS^{cl}), \tag{40}$$

where $V_\pm = \exp(\hat{l}^m_\pm T_m)$, $\tilde{V}_\pm = \exp(\hat{l}^m_\pm \tilde{T}_m)$. For infinite-dimensional gauge algebras one has to regularize the determinants in Eq.(39). One of the regularization methods in the theory of closed bosonic strings $(G = Diff(S^1))$ was considered in Ref.[5]. To treat some general infinite-dimensional discrete models one has to generalize such methods.

Using the transformations (22) and the corresponding ones for the ghost coordinates

$$\rho^e(0) \to \exp(f^m_0\, \tilde{T}_m)\ \rho^e(0), \quad \tilde{\rho}^e(T) \to \exp(f^m_T\, \tilde{T}_m)\ \rho^e(T) \tag{41}$$

we can present the integral in Eq.(40) as the integral over the Teichmuller space $(G \otimes G)/G_0$ and over the group $G_0$ which is the group of reparametrization of the boundaries $\{q^e, \rho^e, \bar{\rho}^e\}$, see Eqs.(22), (41). To complete the calculation of the propagator we have to determine the unique measure of integration over the coset space $(G \otimes G)/G_0$. This requires a careful analysis of the global gauge structure of the theory and is analogous to finding the moduli space and the measure on it in the continuous string theory. This problem is easy to solve for the simplest example of the algebra $sp(2,R)$, or $sl(2,R)$, but the general case requires a special investigation which now is in progress.



## 4. Discussion

In conclusion we would like to stress that the proposed finite dimensional gauge models, being in many aspects analogous to continuous strings, should not be regarded as some simple discretizations. Each model is in fact a hamiltonian dynamics which can be treated on its own. One can try to apply these models to bound states of quarks or to imitate some properties of string-based unified theories. Before such applications become possible, one has first to include the spin degrees of freedom, to find the spectrum and to construct the vertices (interacting discrete "strings"). It will not be difficult to find the spectrum of our models as soon as we succeed in calculating the propagator in an explicit form (integration over the moduli). Constructing the vertices is a more difficult problem which can hardly be solved without more complete knowledge of the propagators for different groups and representations. A more remote goal is to construct a field theory of interacting discrete "strings" allowing one to approach nonperturbative calculations.

The easiest thing to do is to construct a discrete analog of the fermionic string. A simplest approach to this consists in adding fermionic (grassmanian) degrees of freedom $\xi^m_\pm$ with canonical superbrackets $\{\xi^m_\pm, \xi^n_\pm\} = -ih^{mn}$. For example, if the matrices $\Gamma_{m,ab}$ in addition to Eq.(7) satisfy the identity

$$(\Gamma_{m,ab}\,\Gamma_{n,cd} + \text{cyclic}(a,b,c))h^{mn} = 0,$$

there exists a superextension of the corresponding bosonic model. This extension is equivalent to replacement of the group $G$ by some supergroup for which $\Gamma_{m,ab}$ are the structure constants appearing in the anticommutator of odd generators. We will elaborate this remark in a separate publication.

Finally, we note that it might be interesting to investigate some infinite-dimensional discrete models based on the Kac-Moody or Krichever-Novikov algebras. It would also be interesting to consider the limit of infinite dimension either of the group $G$ or of the representation of the finite-dimensional group.

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