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E.E.Entralgo, V.V.Kuryshkin*

ON THE CLASSICAL THEORY OF A POINT PARTICLE WITH SPIN

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*Peoples' Friendship University, Moscow, USSR

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Attempts to construct a classical theory of a point particle with spin have been made repeatedly (see for example the review in work /1/). The classical realizations of the notion of spin proposed by such constructions have been always related to some independent variables "hidden" from the direct observation. More than that, these "unobservable" variables, which had no explicit physical meaning, turned to be included in the mathematical formalism of the proposed classical theories. This makes their analysis and the necessary experimental verification highly difficult.

While investigating (works /1-3/) the questions of the mathematical description of structural point objects and of their motion in external force fields there was discovered a new classical model of spin which did not demand of any physically empty "hidden" variables. This model permits to formulate /1/ a phenomenological classical theory of a point particle with spin without "unobservable" variables.

In the discussed theory a point particle with spin possesses the following constant physical characteristics: mass $m>0$, charge e, gyromagnetic factor g, characteristic length $\ell>0$, characteristic frequency $\omega>0$.

The theory supposes that in stationary gravitational, electric and magnetic fields with the potentials Φ, φ, $\vec{A}$ and the intensities $\vec{G}=-\nabla\Phi$, $\vec{E}=-\nabla\varphi$, $\vec{H}=[\nabla\times\vec{A}]$ correspondingly the particle moves along a twice differentiable trajectory $\vec{q}(t)$ so that one can introduce into the consideration the velocity $\dot{\vec{q}}(t)$ and the acceleration $\ddot{\vec{q}}(t)$.

It is supposed also that a point particle with spin possesses, together with the constant m, e, g, ℓ, ω, the variable physical characteristics: spin $\vec{S}$, electric dipole moment $\vec{d}$, ingerent $\vec{s}$, orbital $\vec{L}$ and total $\vec{J}$ mechanical moments, inherent magnetic moment $\vec{\mu}$, energy $\mathcal{H}$.

The mathematical formalism of the discussed classical theory for a point particle with spin is defined by the following statements.

1. As the independent variables one can choose the multitude a of physical quantities,

$$a = (a_i,\ i \in \overline{1,15}) = (\vec{q}, \vec{p}, \vec{d}, \vec{f}, \vec{S}), \qquad (1)$$

where $\vec{p} = m\dot{\vec{q}}$ is the kinetic momentum, $\vec{f} = \dot{\vec{d}}$ is the speed of the dipole moment changement.

2. The independent variables a satisfy the system of equations of the motion:

$$\dot{\vec{q}} = \frac{1}{m}\vec{p}, \qquad (2a)$$

$$\dot{\vec{p}} = m\vec{G} + e\vec{E} + \frac{e}{mc}[\vec{p}\times\vec{H}] + \frac{1}{c}[\vec{f}\times\vec{H}] + \nabla(\vec{d}\vec{E} + g\vec{S}\vec{H}) +$$

$$+ \frac{1}{mc}(\vec{d}\nabla)\left\{[\vec{p}\times\vec{H}] + \frac{e}{\ell c^2}[\vec{f}\times\vec{H}] + \frac{2gm}{\ell}\left(g - \frac{e}{2mc}\right)\nabla(\vec{S}\vec{H})\right\} +$$

$$+ \frac{1}{2m\ell c^2}(\vec{d}\nabla)(\vec{d}\nabla)\left\{m\vec{G} + e\vec{E} + \frac{e}{mc}[\vec{p}\times\vec{H}]\right\}, \qquad (2b)$$

$$\dot{\vec{d}} = \vec{f}, \qquad (2c)$$

$$\dot{\vec{f}} = -\omega^2\vec{d} + \ell c^2\vec{E} + \frac{\ell c}{m}[\vec{p}\times\vec{H}] + \frac{e}{mc}[\vec{f}\times\vec{H}] + \frac{e}{m^2c}(\vec{d}\nabla)[\vec{p}\times\vec{H}] +$$

$$+ \nabla\left\{\vec{d}\vec{G} + \frac{e}{m}\vec{d}\vec{E} + 2cg\left(g - \frac{e}{2mc}\right)\vec{S}\vec{H}\right\}, \qquad (2d)$$

$$\dot{\vec{S}} = g\left[\vec{S}\times\left\{\vec{H} + \frac{2}{\ell c}\left(g - \frac{e}{2mc}\right)(\vec{d}\nabla)\vec{H}\right\}\right]. \qquad (2e)$$

3. The variable physical characteristics are related to each other by the bonds:

$$\vec{J} = \vec{L} + \vec{s}, \qquad \vec{L} = [\vec{q}\times\vec{p}], \qquad \vec{s} = \vec{S} + \frac{1}{\ell c^2}[\vec{d}\times\vec{f}], \qquad (3a)$$

$$\vec{\mu} = g\vec{S} + \frac{e}{2m\ell c^3}[\vec{d}\times\vec{f}] = g\vec{s} - \frac{1}{\ell c^2}\left(g - \frac{e}{2mc}\right)[\vec{d}\times\vec{f}], \qquad (3b)$$

$$\mathcal{H} = \frac{1}{2m}\vec{p}^2 + m\Phi + e\varphi + \frac{1}{2\ell c^2}(\vec{f}^2 + \omega^2\vec{d}^2) - \vec{d}\vec{E} - g\vec{S}\vec{H} -$$

$$- \frac{1}{2\ell c^2}(\vec{d}\nabla)\left\{\vec{d}\vec{G} + \frac{e}{m}\vec{d}\vec{E} + 4gc\left(g - \frac{e}{2mc}\right)\vec{S}\vec{H}\right\}. \qquad (3c)$$

The formulated above statements define a close classical theory for a point particle of mass m and charge e moving in external stationary fields Φ, φ, $\vec{A}$ along a trajectory $\vec{q}(t)$ and possessing, in contradistinction to the habitual charged mass point, the characteristics inherent to a nonpoint object - constants g, ℓ, ω, spin $\vec{S}$, moments $\vec{d}$, $\vec{s}$, $\vec{\mu}$ and so on.

The trajectory $\vec{q}(t)$ and the values of the variables $\vec{S}(t)$, $\vec{d}(t)$, ... for instant $t > t_0$ are uniquely determined by the equations of the motion (2) after the known values $\vec{q}_0$, $\vec{p}_0$, $\vec{d}_0$, $\vec{f}_0$, $\vec{S}_0$ of the independent variables at instant t_0. With that, in arbitrary stationary fields the two conservation laws are fulfilled:

$$\mathcal{H} = Const., \qquad \vec{S}^2 = Const. \qquad (4)$$

The justice of the last statement can be easily demonstrated by the differentiation of $\mathcal{H}$ and $\vec{S}^2$ with respect to time with the help of equations (2).

The gauge invariance of the theory is evident.

The correspondence principle is fulfilled as for vanishingly small ℓ, $\vec{d}$ and $\vec{S}$ the discussed formalism turns to the classical theory for a mass point.

It is essential to note that the theory does not contain any "unobservable" in principle variables and all the quantities included to its formalism have a clear physical meaning.

Thus, the standard interpretation of the gyromagnetic factor g follows from the contribution $g\vec{S}\vec{H}$ of a homogenious magnetic field to the energy of a particle. Just this factor is measured experimentally. It is to note only that here factor g is not a coefficient of proportionality between $\vec{\mu}$ and $\vec{s}$ since in accordance with (3b) $\vec{s}$, $\vec{\mu}$ and $\vec{S}$ are different in a general case non-proportional to each other quantities.

For a free particle from the equations (2) for $t_0 = 0$ we have:

$$\vec{q} = \frac{1}{m}\vec{p}_0 t + \vec{q}_0, \qquad \vec{d} = \vec{d}_0\cos\omega t + \frac{1}{\omega}\vec{f}_0\sin\omega t, \qquad \vec{S} = \vec{S}_0, \qquad (5)$$

i.e. ω is the frequency of an oscillation or of a rotation of the electric dipole moment of a free particle. Here $\langle\vec{d}\rangle = 0$.

For the motion in a homogenious electric field from the equations (2) for $t_0 = 0$ we obtain:

$$\vec{q} = \frac{e}{2m}\vec{E}t^2 + \frac{1}{m}\vec{p}_0 t + \vec{q}_0 , \qquad \vec{S} = \vec{S}_0 , \tag{6a}$$

$$\vec{d} = \vec{d}_0 \cos\omega t + \frac{1}{\omega}\vec{f}_0 \sin\omega t + \frac{2\ell c^2}{\omega^2}\vec{E}\sin^2\frac{\omega t}{2} , \tag{6b}$$

i.e. ℓ determines the polarizability of a particle in an electric field. Here $\langle\vec{d}\rangle = \ell c^2\vec{E}/\omega^2$. An analogous effect appears in a magnetic field.

As the conclusion we have to note that the proposed above classical theory for a point particle with spin may turn to be useful (for more details see [1]) for the description of a trajectorial motion of elementary particles and nuclei in external force fields.

References

1. Kuryshkin V.V., Entralgo E.E. Preprint JINR P2-89-198, Dubna, 1989, 15 p. (in Russian).
2. Kuryshkin V.V., Entralgo E.E. In: Gravitation and Hypothetic Interactions, Peoples' Friendship Univ., Moscow, 1988, p. 56-65 (in Russian).
3. Entralgo E.E. Communication JINR P2-88-879, Dubna, 1988, 15 p. (in Russian).

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